Low-complexity Turbo Equalization and Multiuser Decoding for TD-CDMA

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Abstract

We propose a low complexity multiuser joint Parallel Interference Cancellation (PIC) decoder and Turbo Decision Feedback Equalizer for CDMA. In our scheme, an estimate of the interference signal (both MAI and ISI) is formed by weighting the hard decisions produced by conventional (i.e., hard-output) Viterbi decoders. The estimated interference is subtracted from the received signal in order to improve decoding in the next iteration. By using asymptotic performance analysis of random-spreading CDMA, we optimize the feedback weights at each iteration. Then, we consider two (mutually related) performance limitation factors: the bias of residual interference and the *ping-pong* effect. We show that the performance of the proposed algorithm can be improved by compensating for the bias in the weight calculation, and we propose a modification of the basic PIC algorithm, which prevents the ping-pong effect and allows higher channel load and/or faster convergence to

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the single-user performance. The proposed algorithm is validated through computer simulation in an environment fully compliant with the specifications of the time-division duplex mode of 3rd generation systems, contemplating a combination of TDMA and CDMA and including frequency-selective fading channels, user asynchronism, and power control. The main conclusion of this work is that, for such application, Soft-Input Soft-Output (SISO) decoders (e.g., implemented by the forward-backward BCJR algorithm) are not needed to attain very high spectral efficiency, and simple conventional Viterbi decoding suffices for most practical settings.

Keywords: Turbo Multiuser Detection, Turbo Equalization, CDMA.

1 Introduction and motivation

The recently proposed Universal Mobile Telecommunication Systems (UMTS) standard for the 3rd generation (3G) of mobile communication systems adopts Wideband Code Division Multiple Access (WCDMA) for the Frequency Division Duplex (FDD) mode and a combination of TDMA and CDMA (TD-CDMA) for the Time Division Duplex (TDD) mode [1].

In both FDD and TDD modes, the UMTS basic receiver scheme contemplates the use of conventional Single-User Matched Filtering (SUMF). Since Multiple-Access Interference (MAI) is treated as additional background noise [1], powerful and high-complexity channel coding such as 256-states convolutional codes and turbo codes [3] are envisaged in order to attain low Bit Error Rates (BER) at low decoder input signal-to-interference plus noise ratio (SINR). In any case, channel loads larger than 1 user/chip are practically very difficult if not impossible to attain by the SUMF front-end and single-user decoding [4].

On the other hand, Information Theory shows that much larger channel loads can be achieved provided that a *non-linear* multiuser joint detector and decoder is employed [4, 5]. This may range from the impractically complex optimal joint decoder to practically appealing successive interference cancellation approaches [6, 7].

In practice, successive interference cancellation must cope with decision errors, which prevent perfect cancellation of already decoded users. Then, several *iterative* schemes have been pro-

[1] Notice that the only difference between the SUMF and the linear MMSE filter is that the latter treats MAI as colored noise while the former treats it as white noise [2].

posed, which limit the deleterious effect of decision errors by feeding back soft-estimates of the detected symbols (see for example [8, 9, 10]). These schemes require Soft-Input Soft-Output (SISO) decoders, usually implemented by the forward-backward BCJR algorithm [11]. However, such SISO decoders represent a non-negligible factor in the complexity of whole receiver. In real CDMA applications for either TDD and FDD modes, the maximum achievable channel load is often limited by synchronization and channel estimation issues, rather than by the ultimate capability of the decoder itself [1]. Hence, it makes sense to investigate simpler joint detection and decoding schemes, which outperforms the conventional linear SUMF, MMSE and decorrelator, and non-linear Parallel Interference Cancellation (PIC) or Serial Interference Cancellation (SIC) receivers [12], and nevertheless yield performance similar to the SISO-based schemes at lower decoding complexity.

Driven by this consideration, this paper proposes a low complexity iterative multiuser receiver, where SISO decoders are replaced by simpler standard (i.e. hard-output) Viterbi decoders. The Viterbi hard decisions are weighted and fed back to the interference cancellation stage. By using large-system analysis of random CDMA [13] we optimize the feedback weights at each iteration such that the SINR at the decoders input of the next iteration is maximized.

We address the problem of bias of residual interference [14, 15] and of the *ping-pong* effect [12, 16] and we show that the performance of the proposed algorithm can be improved by compensating for the bias in the weight calculation and by modifying the basic PIC algorithm in order to prevent the ping-pong effect. These modifications allow higher channel load and/or faster convergence to the single-user performance at almost no additional computational cost.

We validate the proposed receiver algorithm in UMTS-TDD realistic scenarios, including asynchronous transmission, frequency selective fading channels and power control. In this regime, the proposed receiver performs very close to the single user (i.e., MAI-free) Matched Filter Bound (i.e., ISI-free) performance, even for large channel load.

The remainder of this paper is organized as follows: Section 2 gives a description of the system model. Section 3 describes the large system asymptotic analysis. Based on the latter, the feedback optimal weights are derived in Section 4 for synchronous and flat channels, taking into account the bias on the residual interference and ping-pong effect for large channel load. Section 5 deals with asynchronous transmission and frequency selective fading channels. Finally in Section 6, conclusions are pointed out.

2 System model

We consider the uplink of a DS-CDMA system where U users send *encoded* information to a common receiver. The baseband transmission chain for the u^{th} user is depicted in Figure 1(a). Source bits are channel-encoded and organized in codewords of length N bits. The codewords are then interleaved and modulated with transmitted energy per symbol $\mathcal{E}_u$. For simplicity we assume that all users make use of convolutional coding and BPSK modulation. Let $a_u[n]$ be the n^{th} symbol generated by the u^{th} user, is in the set $\{-1, +1\}$, and $\mathbf{a}_u = [a_u[0], a_u[1], \ldots, a_u[N-1]]^{\text{T}}$ represent the code word of user u *after interleaving*. The symbols are then spread by the spreading sequence $\mathbf{s}_u = [s_u[0], s_u[1], \ldots, s_u[L-1]]^{\text{T}}$. We assume all the users have the same spreading factor (number of chips per symbol) L and unitary energy sequences, that is, $\mathbf{s}_u^{\text{H}}\mathbf{s}_u = 1, \forall u = 1, \ldots, U$. If the chip pulse-shaping filter is $\psi(t)$ and is common to all the users, then the u^{th} baseband continuous-time transmitted signal is

$$x_u(t) = \sqrt{\mathcal{E}_u} \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} a_u[n] s_u[l] \psi(t - lT_c - nT) \tag{1}$$

where T_c is the chip period and $T = LT_c$ is the symbol period.
In this paper we focus on UMTS-TDD, where users are synchronous at the slot level but asynchronous at the chip level. The channels are considered random but slowly varying and constant over one slot. Moreover, for the sake of simplicity, we assume that the convolutional codewords span a single slot.
Thus, the u^{th} user signal is sent through the channel $c_u(t)$ with impulse response given by

$$c_u(t) = \sum_{p=0}^{P_u-1} c_{u,p} \delta(t - \tau_{u,p}) \tag{2}$$

where P_u is the number of resolvable paths, $\tau_{u,p}$ is the p^{th} path delay, and $c_{u,p}$ is the p^{th} path coefficient. The coefficients $c_{u,p}$ are assumed Gaussian complex circularly symmetric random variables with distribution $\mathcal{N}_{\mathcal{C}}(0, \sigma_{u,p}^2)$ where $\sigma_p^2 = \text{E}[|c_{u,p}|^2]$. This model takes into account also users' asynchronous transmission, where the relative delays between users are included into the channel path delays.

The signal at the receiver is given by

$$y(t) = \sum_{u=1}^{U} \int_{-\infty}^{+\infty} x_u(\tau) c_u(t-\tau)\, \mathrm{d}\tau + n(t) \tag{3}$$

$$= \sum_{u=1}^{U} \sum_{n=0}^{N-1} a_u[n]\, g_u(t-nT) + n(t) \tag{4}$$

where $n(t)$ is the Gaussian noise and $g_u(t)$ is defined by

$$g_u(t) = \sqrt{\mathcal{E}_u} \sum_{l=0}^{L-1} \sum_{p=0}^{P_u-1} s_u[l]\, c_{u,p}\, \psi(t - lT_c - \tau_{u,p}) \tag{5}$$

and represents the overall channel impulse response for the u^{th} user as depicted in Figure 1(b). The signal $y(t)$ is sampled by the receiver at rate W/T_c, an integer multiple of the chip-rate, where W is a suitable integer chosen in order to satisfy the Nyquist criterion.

The discrete time version of $g_u(t)$, sampled at rate W/T_c, may have an infinite support but most of its energy is concentrated in a finite interval. This interval depends on the channel maximum delay spread, on the pulse-shaping filter $\psi(t)$, and on the spreading factor L. After a suitable truncation we can represent the discrete-time channel impulse response as a vector $\mathbf{g}_u = [g_u[0], \ldots, g_u[N_g-1]]^{\mathrm{T}}$, $\forall u = 1, \ldots, U$ where N_g is the maximum discrete-time channel length of all user channels. Subject to these assumptions, the received discrete-time baseband signal can be written as follows

$$\mathbf{y} = \sum_{u=1}^{U} \mathbf{G}_u \mathbf{a}_u + \boldsymbol{\nu} = \mathbf{G}\mathbf{a} + \boldsymbol{\nu} \tag{6}$$

where

- $\mathbf{y} = [y[0], \ldots, y[(N-1)LW + N_g]]^{\mathrm{T}}$ is the vector of the received signal samples.
- $\mathbf{G}$ is a $((N-1)LW + N_g) \times UN$ matrix given by $\mathbf{G} = [\mathbf{G}_1, \ldots, \mathbf{G}_U]$ where the $((N-1)LW + N_g) \times N$ matrices $\mathbf{G}_u$ contain, in each column, a shift of the vector $\mathbf{g}_u$ as shown in Figure 2.
- $\mathbf{a} = [\mathbf{a}_1^{\mathrm{T}}, \ldots, \mathbf{a}_U^{\mathrm{T}}]^{\mathrm{T}}$ is the concatenation of the U user's codewords.
- $\boldsymbol{\nu} = [\nu[0], \ldots, \nu[(N-1)LW + N_g]]^{\mathrm{T}}$ contains the noise samples where the $\nu[i]$'s are i.i.d. circularly symmetric complex Gaussian random variables with distribution $\boldsymbol{\nu} \sim \mathcal{N}_{\mathcal{C}}(\mathbf{0}, N_0\mathbf{I})$. ($\mathbf{I}$ denotes the identity matrix)

For the sake of simplicity, the receiver front-end is constrained to be the conventional SUMF [2] given by the matrix $\mathbf{G}^{\mathrm{H}}$.[2] The matched filter performs at the same time pulse-shape matched filtering, channel matched filtering, and despreading. The output of the bank of SUMFs is given by [2]

$$\begin{aligned} \mathbf{z} &= \mathrm{Re}\left\{\mathbf{G}^{\mathrm{H}}\mathbf{y}\right\} \\ &= \mathrm{Re}\left\{\mathbf{G}^{\mathrm{H}}\mathbf{G}\mathbf{a} + \mathbf{G}^{\mathrm{H}}\boldsymbol{\nu}\right\} \\ &= \mathbf{R}\mathbf{a} + \mathbf{v} \end{aligned} \tag{7}$$

where

- $\mathbf{z} = [\mathbf{z}_1^{\mathrm{T}}, \ldots, \mathbf{z}_U^{\mathrm{T}}]^{\mathrm{T}}$ and $\mathbf{z}_u = [z_u[0], \ldots, z_u[N-1]]^{\mathrm{T}}$ for $u = 1, \ldots, U$ is the concatenation of the U matched filter outputs.
- $\mathbf{R}$ is the $UN \times UN$ cross-correlation matrix given by $\mathbf{R} = \mathrm{Re}\left\{\mathbf{G}^{\mathrm{H}}\mathbf{G}\right\}$ where $(\cdot)^{\mathrm{H}}$ denotes the Hermitian operator.
- the noise term, $\mathbf{v} = [\mathbf{v}_1^{\mathrm{T}}, \ldots, \mathbf{v}_U^{\mathrm{T}}]^{\mathrm{T}}$ is given by $\mathbf{v} = \mathrm{Re}\left\{\mathbf{G}^{\mathrm{H}}\boldsymbol{\nu}\right\}$ and has distribution $\mathbf{v} \sim \mathcal{N}(\mathbf{0}, \frac{N_0}{2}\mathbf{R})$. The vector $\mathbf{v}_u = \mathrm{Re}\left\{\mathbf{G}_u^{\mathrm{H}}\boldsymbol{\nu}\right\} = [v_u[0], \ldots, v_u[N-1]]^{\mathrm{T}}$ represents the additive colored noise contribution after the u^{th} matched filtering.

In particular, the SUMF output for the n^{th} symbol of the u^{th} user is given by

$$z_u[n] = z[uN+n] = \sum_{k=0}^{UN-1} \mathbf{R}[uN+n, k]a[k] + v[UN+n] \tag{8}$$

The matrix $\mathbf{R}$ is Hermitian and contains U^2 blocks of size $N \times N$ each. In the following, $\mathbf{R}_{u,v}$ indicates the $(u,v)^{\mathrm{th}}$ block given by

$$\mathbf{R}_{u,v} = \mathrm{Re}\left\{\mathbf{G}_u^{\mathrm{H}}\mathbf{G}_v\right\} \tag{9}$$

The $[i,j]$ entry of $\mathbf{R}_{u,v}$ is given by

$$\mathbf{R}_{u,v}[i,j] = \mathbf{R}[uN+i, vN+j] = \sum_{k=0}^{N_g} \mathrm{Re}\left\{g_u^*[k]g_v[k+(j-i)LW]\right\} \tag{10}$$

From the structure of $\mathbf{G}_u$, depicted in Figure 2, and from Equation (10) it is clear that the matrices $\mathbf{R}_{u,v}$ are banded with $2D+1$ non-zero diagonals where

$$D = \left\lfloor \frac{N_g}{LW} \right\rfloor \tag{11}$$

[2]Some works as for example [9] and [10] consider linear MMSE front-end.

Notice that since $\mathbf{R}_{u,v}[i,j]$ depends only on the difference $(j-i)$, $\mathbf{R}_{u,v}$ is uniquely defined by the vector $\mathbf{r}_{u,v} = [r_{u,v}[-D], \ldots, r_{u,v}[0], \ldots, r_{u,v}[+D]]^{\mathrm{T}}$ where

$$r_{u,v}[j-i] = \mathbf{R}_{u,v}[i,j]$$

Hence, using Equation (7) and the above considerations it is possible to rewrite Equation (8) as

$$\begin{aligned} z_u[n] &= \sum_{v=0}^{U-1}\sum_{k=0}^{N-1} \mathbf{R}_{u,v}[n,k]a_v[k] + v_u[n] \\ &= \sum_{v=0}^{U-1}\sum_{d=-D}^{D} \mathbf{R}_{u,v}[n,n+d]a_v[n+d] + v_u[n] \\ &= \sum_{v=0}^{U-1}\sum_{d=-D}^{D} r_{u,v}[d]a_v[n+d] + v_u[n] \\ &= |\mathbf{g}_u|^2\, a_u[n] + \underbrace{\sum_{\substack{d=-D \\ d\neq 0}}^{D} r_{u,u}[d]a_u[n+d]}_{\text{ISI}} + \underbrace{\sum_{\substack{v=0 \\ v\neq u}}^{U-1}\sum_{d=-D}^{D} r_{u,v}[d]a_v[n+d]}_{\text{MAI}} + v_u[n] \end{aligned} \quad (12)$$

where the first term is the useful symbol, the second term denotes the Inter Symbol Interference (ISI) the third term is the MAI and the last term is the colored noise.

The SUMF output feeds the proposed iterative multiuser detector-decoder depicted in Figure 3. After SUMF, the signal $\mathbf{z}$ passes through an Interference Cancellation (IC) stage that uses the estimates $\hat{a}_u^{(m)}[n]$ of the symbols $a_u[n]$ to remove ISI and MAI (the superscript $^{(m)}$ denotes m^{th} iteration). The hard decisions $\hat{a}_u^{(m)}[n] \in \{\pm 1\}$ are provided by a bank of Viterbi decoders, whose outputs are weighted by a factors $\beta_u^{(m)} \in [0,1]$. Thus, the signal at the output of the IC stage in the m^{th} iteration is given by

$$\begin{aligned} z_u^{(m)}[n] &= z_u[n] - \left(\sum_{\substack{d=-D \\ d\neq 0}}^{D} r_{u,u}[d]\beta_u^{(m)}\hat{a}_u^{(m)}[n+d] + \sum_{\substack{v=0 \\ v\neq u}}^{U-1}\sum_{d=-D}^{D} r_{u,v}[d]\beta_v^{(m)}\hat{a}_v^{(m)}[n+d] \right) \\ &= |\mathbf{g}_u|^2\, a_u[n] + \zeta_u^{(m)}[n] + v_u[n] \end{aligned} \quad (13)$$

where

$$\begin{aligned} \zeta_u^{(m)}[n] = &\sum_{\substack{d=-D \\ d\neq 0}}^{D} r_{u,u}[d]\left(a_u[n+d] - \beta_u^{(m)}\hat{a}_u^{(m)}[n+d]\right) \\ &+ \sum_{\substack{v=0 \\ v\neq u}}^{U-1}\sum_{d=-D}^{D} r_{u,v}[d]\left(a_v[n+d] - \beta_v^{(m)}\hat{a}_v^{(m)}[n+d]\right) \end{aligned} \quad (14)$$

is the residual MAI+ISI at iteration m, and $v_u[n] \sim \mathcal{N}\left(0, \frac{N_0|\mathbf{g}_u|^2}{2}\right)$.
Equation (13) shows that the IC stage performs at the same time MAI suppression and also non-causal Decision Feedback Equalization (DFE) by removing from the n^{th} symbol the contributions due to the past and future symbols of the same user.
At the first iteration, the initial estimated symbols are set to zero, $\hat{a}_u^{(0)}[n] = 0$, $u = 0, 1, \ldots, U-1$ and $n = 0, \ldots, N-1$ so that $z_u^{(0)}[n] = z_u[n]$. In the case of perfect symbol estimates and $\beta_u^{(m)} = 1$, equation (13) reduces to

$$z_u^{(m)}[n] = |\mathbf{g}_u|^2 \, a_u[n] + v_u[n] \tag{15}$$

where the MAI and ISI are completely removed and the single-user Matched Filter Bound (MFB) performance for user u is attained. For later use, we define the normalized received instantaneous channel energy for user u as $\gamma_u = \frac{|\mathbf{g}_u|^2}{\mathcal{E}_u}$, such that $\mathrm{E}[\gamma_u] = 1$, and the single-user MFB instantaneous SNR for user u as

$$\text{SNR}_u^{\text{MFB}} = \frac{2\mathcal{E}_u}{N_0}\gamma_u \tag{16}$$

Intuitively, the weighting factors $\beta_u^{(m)}$ should depend on the reliability of the estimated symbols $\hat{a}_u^{(m)}[n]$ and be equal to 1 or to 0 in the case of completely reliable or completely unreliable symbol estimates, respectively. In the next Section we make this statement precise by studying the performance of an idealized synchronous system in the large-system limit.

3 Large system asymptotic analysis

Rigorous asymptotic analysis of this iterative scheme in the large-system limit (i.e., for $N, U, L \to \infty$ with finite ratio $U/L = \alpha$ and $U/N \to 0$), for single-path channels and synchronous users is proposed in [14]. This analysis is based on the general approach of density evolution over graphs: a standard analysis tool for evaluating the asymptotic performance of message-passing iterative decoders [17], and makes use of results from the theory of large random matrices developed in [13] for the analysis of linear receivers with randomly spread CDMA in the large-system limit. In [14] it is also shown that this analysis holds only if the symbol estimates $\hat{a}_u^{(m)}[n]$ are functions of the decoder *extrinsic information* [18]. The decoder extrinsic information is defined for a SISO decoder based on the sum-product algorithm [19], such as the BCJR algorithm, but it is not defined for a ML decoder such as the Viterbi algorithm. Hence, we shall optimize the weights $\beta_u^{(m)}$ for a *fictitious* receiver where the Viterbi decoders are replaced by BCJR decoders and the hard decisions $\hat{a}_u^{(m)}[n]$ are obtained by one-bit quantization of the extrinsic likelihood ratios produced by the latter.

We hasten to say that the fictitious receiver is used in this paper as a reference but has no practical relevance, for the obvious reason that if BCJR decoders are used, then much more efficient soft-estimation of the interfering symbols as in [8, 9, 10] is possible. However, as it will be clear from the rest of this section, the weight optimization based on the asymptotic analysis of the fictitious receiver allows us to derive a very simple expression for the optimal weights, independent of the user sequences and their mutual correlations, and a very simple practical algorithm for calculating these weights.

We make the following assumptions:

- complex random spreading sequences formed by i.i.d. chips uniformly distributed over the QPSK constellation;
- users are slot and chip synchronous;
- equal received expected energy per symbol $\mathcal{E}$ for each user;
- frequency-flat identically distributed propagation channels;
- we let $N, U, L \to \infty$ with $U/L = \alpha$ (α denotes the *channel load*);
- the empirical distribution of the user received normalized powers, defined by [13]

$$F_\gamma^{(U)}(z) = \frac{1}{U}\sum_{u=1}^{U} 1\{\gamma_u \leq z\} \tag{17}$$

  converges weakly to a given non-random cumulative distribution function $F_\gamma(z)$, for $U \to \infty$.

With the above assumptions, for a user u randomly selected with uniform probability in the user population, we have $\mathrm{SNR}_u^{\mathrm{MFB}} = \frac{2\mathcal{E}}{N_0}\gamma_u$ where γ_u is a random variable distributed (in the limit of large U) according to $F_\gamma(z)$.

Under these conditions, the SINR at the decoders input in the m^{th} iteration for the fictitious receiver can be written as follows [14]

$$\mathrm{SINR}_u^{(m)} = \eta^{(m)}\mathrm{SNR}_u^{\mathrm{MFB}} = \eta^{(m)}\frac{2\mathcal{E}}{N_0}\gamma_u \tag{18}$$

where

$$\eta^{(m)} = \frac{1}{1 + \alpha \frac{\mathcal{E}}{N_0} \mu^{(m)}} \tag{19}$$

is the degradation factor payed by any user with respect to its single-user MFB performance, and where

$$\mu^{(m)} = \mathrm{E}\left[\gamma \left|a - \beta^{(m)} \hat{a}^{(m)}\right|^2\right] \tag{20}$$

is the average variance of the residual interfering symbols. In (20), a denotes a coded symbol of a generic user received at instantaneous power γ, and $\hat{a}^{(m)}$ denotes the hard decision relative to a at the decoder output. Expectation in (20) is with respect to $a \sim$ uniform over $\{\pm 1\}$, $\hat{a}^{(m)}$ distributed as the hard decision about a at the decoder output, and $\gamma \sim F_\gamma(z)$. We refer to $\eta^{(m)}$ as the Multiuser Efficiency (ME) [2] at iteration m. Clearly, the single-user MFB performance is achieved for all users if $\lim_{m\to\infty} \eta^{(m)} = 1$.

We can optimize the weighting factor $\beta^{(m)}$ as a function of the channel energy γ in order to minimize the expected interference variance $\mu^{(m)}$ at every iteration. Therefore, for each iteration m we seek the solution of the simple optimization problem

$$\min_{\beta=\beta(\gamma)} \quad \mathrm{E}\left[\left|a - \beta \hat{a}^{(m)}\right|^2 \middle| \gamma\right] \tag{21}$$

By expanding the above conditional expectation, we get

$$\begin{gathered} 1 + \beta^2 - 2\beta \mathrm{E}[a\hat{a}^{(m)}|\gamma] \\ 1 + \beta^2 - 2\beta\left[(+1)(1 - \epsilon^{(m)}(\gamma)) + (-1)\epsilon^{(m)}(\gamma)\right] \\ 1 + \beta^2 - 2\beta\left(1 - 2\epsilon^{(m)}(\gamma)\right) \end{gathered} \tag{22}$$

where $\epsilon^{(m)}(\gamma)$ is the Symbol Error Rate (SER) of a decoder with an input signal-to-noise ratio $\mathrm{SINR}^{(m-1)} = \eta^{(m-1)} \frac{2\mathcal{E}}{N_0}\gamma$. The solution of (21) is easily obtained as

$$\beta^\star(\gamma) = 1 - 2\epsilon^{(m)}(\gamma) \tag{23}$$

Assuming that the residual interference plus noise at iteration m is Gaussian [3] the SER is a known function $f(\cdot)$ of the decoder input SINR. We shall refer to $f(\cdot)$ as the *SER characteristic*

[3] In the large-system limit and under mild technical conditions this assumption is valid, as shown rigorously in [20].

of the user code (see Appendix A). Thus, for user u at iteration m the optimal weighting factor is given by

$$\beta_u^{(m)} = \beta^\star(\gamma_u) = 1 - 2f\left(\eta^{(m-1)}\frac{2\mathcal{E}}{N_0}\gamma_u\right) \tag{24}$$

The resulting minimized average interference variance is given by

$$\mu^{(m)} = 4\mathrm{E}\left[\gamma f\left(\eta^{(m-1)}\frac{2\mathcal{E}}{N_0}\gamma\right)\left(1 - f\left(\eta^{(m-1)}\frac{2\mathcal{E}}{N_0}\gamma\right)\right)\right] \tag{25}$$

where expectation is with respect to $\gamma \sim F_\gamma(z)$.

The behavior of the asymptotic system is described by the evolution of the ME and it can be represented by the one-dimensional non-linear dynamical system $\eta^{(m)} = \Psi(\eta^{(m-1)})$ with initial condition $\eta^{(0)} = \frac{1}{1+\alpha\mathcal{E}/N_0}$, and where mapping function $\Psi(\cdot)$ is defined by

$$\Psi(\eta) = \frac{1}{1 + 4\alpha\frac{\mathcal{E}}{N_0}\mathrm{E}\left[\gamma f\left(\eta\frac{2\mathcal{E}}{N_0}\gamma\right)\left(1 - f\left(\eta\frac{2\mathcal{E}}{N_0}\gamma\right)\right)\right]} \tag{26}$$

It can be easily checked that, since $f(\cdot)$ has range $[0, 1/2]$ and it is non-increasing, then $\Psi(\eta)$ has range $[\eta^{(0)}, 1]$ and it is non-decreasing. Therefore, the dynamical system defined by $\Psi(\eta)$ with initial condition $\eta^{(0)}$ has at least one stable fixed point in the interval $[\eta^{(0)}, 1]$, and the sequence $\{\eta^{(m)}\}_{m=0}^{\infty}$ is non-decreasing and upper bounded by 1. The limit $\eta^* = \lim_{m\to\infty}\eta^{(m)} \leq 1$ exists and is equal to the left-most stable fixed point of the system in the interval $[\eta^{(0)}, 1]$.

As an example, Figure 4 shows the function $\Psi(\eta)$ for channel load $\alpha = 2$, $E_b/N_0 = 5$dB, constant instantaneous received power (i.e. $\gamma_u = 1$ for all users), and the 4-states convolutional code of rate $1/2$ and octal generators $\{5, 7\}$ (denoted in the following by CC$(5, 7)$). For the sake of comparison, we show also the evolution of the same system when the BCJR decoder provides soft extrinsic estimates as proposed in [8]. The $\Psi(\eta)$ function in this case is derived in [14]. The limit η^* in this case is very close to 1, meaning that all users attain near-single-user performance. Notice that soft feedback assures a faster decoder convergence to the single-user performance.

When the channel load is increased, the Ψ curves are modified so that for a certain threshold channel load α, the curve corresponding to weighted hard decisions is tangent to the diagonal, as shown in Figure 5. This means that the system has reached its maximum load and is not able to converge to near-single-user performance. On the contrary, the system using soft decisions still converges to single user performance. This shows that SISO decoding and soft feedback also provides a higher threshold channel load, i.e., a larger overall maximum achievable spectral efficiency of the system.

4 Implementation of the proposed receiver

4.1 The basic algorithm

The performance of the proposed receiver depends on the computation of the weighting factors $\beta_u^{(m)}$ which do depend on a reliable SINR estimation. Driven by the asymptotic analysis of the previous section, we propose to compute the weighting factor for the u^{th} user at iteration m as

$$\beta_u^{(m)} = 1 - 2\, f\left(\widehat{\text{SINR}}_u^{(m-1)}\right) \tag{27}$$

where $f(\cdot)$ is the (known) SER code characteristics, and $\widehat{\text{SINR}}_u^{(m)}$ is the estimated SINR at the u^{th} decoder input of the m^{th} iteration. In order to estimate the input SINR, we can use the estimator proposed in [21], given by

$$\widehat{\text{SINR}}_u^{(m)} = \frac{1}{\frac{1}{N}\sum_{n=0}^{N}\left|\tilde{z}_u^{(m)}[n]\right|^2 - 1} \tag{28}$$

where $\tilde{z}_u^{(m)} = \frac{z_u^{(m)}}{|\mathbf{g}_u|^2}$.
In [21] it is shown that

$$\xi_u^{(m)} = \frac{1}{N}\sum_{n=0}^{N}\left|z_u^{(m)}[n]\right|^2 - |\mathbf{g}_u|^2$$

is an unbiased estimator of the residual MAI plus ISI plus noise variance at the decoder input if $\zeta_u^{(m)}[n]$ is uncorrelated with the desired variable $a_u[n]$. Remarkably, in this case, the MSE of this estimator is very close to that of the ML estimator assuming known useful symbols, given by $\frac{1}{N}\sum_{n=0}^{N}\left|z_u^{(m)}[n] - |\mathbf{g}_u|^2 a_u[n]\right|^2$ (which is not applicable here, since the symbols $a_u[n]$ are unknown).
When the symbol estimates $\hat{a}_u^{(m)}[n]$ are provided by a decision statistics *containing* the current observation interval, such as in the Viterbi decoder, then the residual interference term given by Equation (14) is conditionally biased given $a_u[n]$, that is,

$$\text{E}\left[\zeta_u^{(m)}[n]|a_u[n]\right] = \delta_u^{(m)} a_u[n] \tag{29}$$

where the bias coefficient $\delta_u^{(m)}$ is non-positive and depends on the system parameters and on the user and iteration indexes as shown in [14, 15]. The bias is not negligible especially for high channel load α even in the absence of ISI and reduces the energy of the useful signal at iteration m by a factor $\left(1 + \frac{\delta_u^{(m)}}{|\mathbf{g}_u|^2}\right)^2$.

On the contrary, if the symbol estimates $\hat{a}_u^{(m)}[n]$ are provided by estimates *not containing* the current observation interval, i.e., they are based on the decoder *extrinsic information* [14, 22], then in the limit for large block length (i.e., $N \to \infty$) and random interleaving, the residual interference is conditionally unbiased, i.e., $\mathrm{E}\left[\zeta_u^{(m)}[n]|a_u[n]\right] = 0$.
We can rewrite the input of the u^{th} decoder at iteration m given in Equation (13) as

$$z_u^{(m)}[n] = \left(|\mathbf{g}_u|^2 + \delta_u^{(m)}\right) a_u[n] + \widetilde{\zeta}_u^{(m)}[n] + v_u[n] \tag{30}$$

where $\widetilde{\zeta}_u^{(m)}[n]$ is uncorrelated with $a_u[n]$.
Hence, the true SINR in the presence of bias is given by

$$\mathrm{SINR_u}^{(m)} = \frac{\left(|\mathbf{g}_u|^2 + \delta_u^{(m)}\right)^2}{\mathrm{E}\left[\left|\widetilde{\zeta}_u^{(m)}[n] + v_u[n]\right|^2\right]} = \frac{\left(|\mathbf{g}_u|^2 + \delta_u^{(m)}\right)^2}{\sigma_\zeta^2} \tag{31}$$

where we define $\sigma_\zeta^2 = \mathrm{E}\left[\left|\widetilde{\zeta}_u^{(m)}[n] + v_u[n]\right|^2\right]$. Now, the SINR estimator proposed in (28), in the presence of bias and for large N, converges in probability to

$$\widehat{\mathrm{SINR}}_u^{(m)} \to \frac{1}{\mathrm{E}\left[\left|\frac{\widetilde{\zeta}_u^{(m)}[n]+v_u[n]}{|\mathbf{g}_u|^2}\right|^2\right] + \left(1 + \frac{\delta_u^{(m)}}{|\mathbf{g}_u|^2}\right)^2 - 1}$$

Since $\delta_u^{(m)} \leq 0$ (i.e., the bias tends to decrease the useful signal term), we conclude that the estimator (28) tends to overestimate the SINR at the decoder input. As a consequence, the weights $\beta_u^{(m)}$ computed according to (27) are mismatched in the presence of bias.

4.2 Estimation of the bias

In order to overcome this problem, a better SINR estimation taking into account the bias of residual interference is required. The term σ_ζ^2 in (31) appears also in the variance of the decoder input signal, given by

$$\sigma_{z_u}^{(m)\,2} = \mathrm{E}\left[\left|z_u^{(m)}[n]\right|^2\right] = \left(|\mathbf{g}_u|^2 + \delta_u^{(m)}\right)^2 + \sigma_\zeta^2 \tag{32}$$

and the bias term $\delta_u^{(m)}$ is contained in the expression of the correlation between the symbol estimates and the decoder input, that, recalling Equation (30), is given by

$$\phi_u^{(m)} = \mathrm{E}\left[z_u^{(m)}[n]\hat{a}_u[n]\right] = \left(1 - 2f\left(\mathrm{SINR_u}^{(m-1)}\right) + \Gamma\left(\mathrm{SINR_u}^{(m-1)}\right)\right)\left(|\mathbf{g}_u|^2 + \delta_u^{(m)}\right) \tag{33}$$

where $\Gamma(\cdot)$ is a known characteristic function of the convolutional code, proportional to the correlation between the symbol estimates and the interference (see Appendix A).

Using Equations (31) and (32) the correlation in equation (33) can be rewritten as

$$\phi_u^{(m)} = \left(|\mathbf{g}_u|^2 + \delta_u^{(m)}\right)\left[1 - 2f\left(\frac{\left(|\mathbf{g}_u|^2 + \delta_u^{(m)}\right)^2}{\sigma_{z_u}^{(m)2} - \left(|\mathbf{g}_u|^2 + \delta_u^{(m)}\right)^2}\right) + \Gamma\left(\frac{\left(|\mathbf{g}_u|^2 + \delta_u^{(m)}\right)^2}{\sigma_{z_u}^{(m)2} - \left(|\mathbf{g}_u|^2 + \delta_u^{(m)}\right)^2}\right)\right] \tag{34}$$

In practice, an approximation of $\sigma_{z_u}^{(m)2}$ and $\phi_u^{(m)}$ can be computed as follows

$$\begin{aligned} \sigma_{z_u}^{(m)2} &\approx \frac{1}{N}\sum_{n=0}^{N-1}\left(z_u^{(m)}[n]\right)^2 \\ \phi_u^{(m)} &\approx \frac{1}{N}\sum_{n=0}^{N-1} z_u^{(m)}[n]\hat{a}_u[n] \end{aligned}$$

and Equation (34) can be solved numerically for $\delta_u^{(m)}$. Let $\hat{\delta}_u^{(m)}$ be the estimated bias, solution of (34). Then the estimated SINR is given by

$$\widehat{\mathrm{SINR}}_u^{(m)} = \frac{\left(|\mathbf{g}_u|^2 + \hat{\delta}_u^{(m)}\right)^2}{\sigma_{z_u}^{(m)2} - \left(|\mathbf{g}_u|^2 + \hat{\delta}_u^{(m)}\right)^2} \tag{35}$$

Figure 6 refers to a flat non-fading system, (i.e. $\gamma_u = 1$ for all users), with $U = 32$ users and spreading factor $L = 16$ (corresponding to the channel load $\alpha = 2$), $E_s/N_0 = 5$dB using the convolutional code CC$(5,7)$. Users are chip-asynchronous and their relative delays are uniformly distributed over an interval spanning one symbol. The true and estimated SINR vs. the iterations for a given user are shown. The curve labeled "Viterbi (SINR est.)" refers to the proposed iterative receiver that estimates the SINR using equation (28) while the curve labeled "Viterbi (SINR-BIAS est.)" refers to the system that estimates both the bias and the SINR, using Equation (35). Notice that in the first few iterations the SINR estimator given by Equation (28) overestimates the true SINR up to several dBs. Instead, the SINR estimation given by Equation (35) is very close to the true SINR.

4.3 The ping-pong effect and its compensation

As it has been shown in the previous Sections, the proposed receiver allows system loads up to a certain threshold above which the system cannot approach the single user performance. In such high load situations, the system parameters as BER, SER, ME, and bias tend to oscillate between two convergence patterns [12]. This phenomenon is called *ping-pong* and it is related to the bias in the residual interference term. In fact, it does not appear when feedback is obtained from a SISO decoder extrinsic information [14]. [4]

A further investigation reported in [16] showed that such a bistable situation is due to a fixed subset of the estimated symbols that flip when passing from one iteration to the next, while the estimated symbols in the complementary subset do not change. A countermeasure to this problem proposed in [16] consists of introducing a perturbation into the bistable situation, by feeding back to the IC stage the average of the estimates obtained from the two previous iterations. Thus, the contribution of the flipping symbols (considered as not reliable) is mitigated. By considering this idea, Equation (20) is modified as

$$\mu^{(m)} = \mathrm{E}\left[\gamma \left| a - {\beta_1}^{(m)} \hat{a}^{(m)} - {\beta_2}^{(m)} \hat{a}^{(m-1)} \right|^2 \right] \tag{36}$$

where the two previous estimates are now weighted with the coefficients $\beta_1^{(m)}$ and $\beta_2^{(m)}$. Now, we can optimize the weighting factors $\beta_1^{(m)}$ and $\beta_2^{(m)}$ as functions of the channel energy γ in order to minimize the expected interference variance $\mu^{(m)}$ at every iteration. In analogy with what done before, we seek the solution of the optimization problem

$$\min_{\beta_1=\beta_1(\gamma),\beta_2=\beta_2(\gamma)} \quad \mathrm{E}\left[\left| a - \beta_1 \hat{a}^{(m)} - \beta_2 \hat{a}^{(m-1)} \right|^2 \middle| \gamma \right] \tag{37}$$

After straightforward algebra (we skip the details for the sake of space limitations), we obtain the solution

$$\begin{aligned} {\beta_1}^{\star}(\gamma) &= \frac{A - BC}{1 - C^2} \qquad (38) \\ {\beta_2}^{\star}(\gamma) &= \frac{B - AC}{1 - C^2} \qquad (39) \end{aligned}$$

where

$$\begin{aligned} A &= \mathrm{E}\left[a\hat{a}^{(m)}\right] = 1 - 2\epsilon^{(m)}(\gamma) \\ B &= \mathrm{E}\left[a\hat{a}^{(m-1)}\right] = 1 - 2\epsilon^{(m-1)}(\gamma) \\ C &= \mathrm{E}\left[\hat{a}^{(m)}\hat{a}^{(m-1)}\right] \end{aligned}$$

In practice, the weights $\beta_{1,u}^{(m)}$ and $\beta_{2,u}^{(m)}$ of user u at iteration m are computed as follows:

[4] For the extrinsic-based schemes, there exist obviously a threshold load above which single-user performance cannot be achieved, but no oscillatory behavior appears.

- for $m = 1$, $\beta_{2,u}^{(1)}$ is irrelevant (since $\hat{a}^{(0)} = 0$) and $\beta_{1,u}^{(1)} = 1 - 2\, f(\widehat{\mathrm{SINR}}_u^{(1)})$ where $\widehat{\mathrm{SINR}}_u^{(1)}$ is computed according to Equation (35).

- for $m = 2, 3, \ldots$, we let $C = \frac{1}{N}\sum_{n=0}^{N-1} \hat{a}_u^{(m)}[n]\hat{a}_u^{(m-1)}[n]$, $A = 1 - 2\, f(\widehat{\mathrm{SINR}}_u^{(m)})$, and $B = 1 - 2\, f(\widehat{\mathrm{SINR}}_u^{(m-1)})$. Thus, $\beta_{1,u}^{(m)}$ and $\beta_{2,u}^{(m)}$ are given by (38) and (39), respectively.

Figures 7 and 8 illustrate the bias and the true SINR plotted versus the iterations for a system using Viterbi decoding, with unfaded chip asynchronous flat channel, (i.e. $\gamma_u = 1\ \forall u$) $U = 40$, $\alpha = 2.5$ and in the same conditions of Figure 6. The "Viterbi (SINR est.)" and the "Viterbi (SINR-BIAS est.)" decoders do not converge to near-single-user performance and show the ping-pong effect in the bias and in the SINR. The receiver that weights the two previous iterations, denoted in the Figures by "Viterbi (2-feedback)", converges faster to near-single-user performance and does not show oscillations.

This behavior can also be seen in Figure 9 where the BER provided by the proposed receivers is shown, for an unfaded chip asynchronous flat channel, and the same conditions of Figure 8. The curve labeled "Viterbi ($\beta = 1$)" denotes the receiver that feeds-back hard decisions (i.e. assuming $\beta_u^{(m)} = 1$ for $u = 1, \ldots, U$ and $m = 0, 1, \ldots$) and the bold horizontal line represents the single user performance. The receiver "Viterbi (2-feedback)" converges to single user performance in 6 iterations. In these conditions, it converges even faster than employing BCJR decoders providing soft extrinsic output, denoted by "BCJR (soft EXT)".

Finally, Figure 10 compares the performance of several receivers in terms of speed of convergence to near-single-user performance for $E_b/N_0 = 5$dB, $L = 16$, and for $\gamma_u = 1\ \forall u$. As test of convergence, we consider the number of iterations required to reach an average ME larger than -0.1dB. The basic hard feedback Viterbi receiver has maximum channel load $\alpha = 1.5$. The weighted hard feedback Viterbi with SINR and bias estimation improves this limit to $\alpha = 2.4$. Eventually, the receiver "Viterbi (2-feedback)" outperforms any other considered receiver and has limit load $\alpha = 3.7$ (corresponding to a spectral efficiency of 1.85 bit/s/Hz). The dashed curves refer to the receivers employing BCJR decoders and extrinsic information feedback and are shown as a reference.

5 Performance in Frequency Selective Fading Channels

In this Section we consider frequency selective channels and asynchronous users. Thus, the ISI term appears in Equation (12). In presence of ISI, the system converges to near single-user Matched Filter Bound (MFB) performance [5] for a channel load α generally lower than the limit for flat synchronous channels. Indeed, the presence of ISI slows down the convergence to single user performance and allows lower channel loads. Two different cases are considered:

- constant instantaneous power for all users;
- constant average power for all users.

5.1 Constant instantaneous power

This case is representative of perfect fast power control that fully compensates for the instantaneous (slot-by-slot) effect of fading, so that $\gamma_u = 1$. Thus,

$$|\mathbf{g}_u|^2 = \mathcal{E} \ , \ \ \forall u = 1, ..., U \tag{40}$$

Under these conditions, we evaluated by simulation the degradation due to the ISI with respect to the results shown in the previous Section. Figure 11 shows the number of iterations needed to reach ME larger than -0.1dB, as a function of the channel load, for $E_b/N_0 = 5$dB, $L = 16$, the "Viterbi (2-feedback)" receiver, and for three different channels whose Multipath Intensity Profile (MIP) [23] is shown in Table 1. CH1 is specified as a standard UMTS test channel in [24] while CH2 and CH3 were created "ad hoc" in order to test the system in severe ISI conditions. CH1 shows a negligible degradation with respect to the single-path case while, for the channels CH2 and CH3, the maximum achievable channel load is decreased to $\alpha = 3.3$ and $\alpha = 2.9$, respectively. Notice also that, in the case of strong ISI and low channel loads, more iterations are required for the convergence with respect to the single-path case.

5.2 Constant average power

This case is representative of a slow power control system that cannot compensate for the instantaneous fading, but maintains a fixed *average* received power for each user. The average received power is assumed to be the same for all users, that is

$$\mathrm{E}[|\mathbf{g}_u|^2] = \mathcal{E} \ \ ; \ \ \forall u = 1, ...U \tag{41}$$

Figure 11 shows the number of iterations needed to reach ME larger than -0.1dB as a function of the channel load and for the channels given in Table 1. In this case, two contrasting phenomena characterize the convergence to single-user MFB performance. On one hand, faded users have little impact onto unfaded users, since they are received at much lower power. On the other hand, even if unfaded users can be reliably estimated and subtracted from the received signal, faded users have an instantaneous SINR that is too low for achieving a small SER, therefore, their contribution cannot be perfectly eliminated even in the absence of MAI. The fact that the performance in slow power control are worse than their fast power-control counterparts (see Figure 11) indicates that the effect of uncanceled faded users dominates the performance of the receiver. Moreover, Figure 11 shows also another interesting fact. Namely, the degradation of the receiver performance due to uncompensated fading is larger for channels with little multipath diversity (e.g., single-path, CH2 and CH3), while it is smaller for channels with rich multipath diversity (e.g., the UMTS test channel CH1). This is intuitively clear, since when the multipath diversity is large, then the random fluctuations of the instantaneous received power $|\mathbf{g}_u|^2$ are reduced, and the fraction of users that can be reliably estimated and subtracted is larger. In general, the average bit error rate (BER) for a finite-dimensional system can be computed by Monte Carlo simulation, by averaging over a large number of frames and channel realizations. In order to reduce the complexity of BER computation, we propose a semi-analytic approach. Assuming symmetric users, with the same channel statistics and average received power, the average BER (averaged with respect to both the channel statistics and over the user population) can be bounded by

$$\begin{aligned} \mathrm{BER}^{(m)} &= \mathrm{E}\Big[f_b(\eta^{(m)}\gamma\, \mathrm{SNR}^{\mathrm{MFB}})\Big] \\ &= \frac{1}{U}\sum_{u=1}^{U} \mathrm{E}\Big[f_b(\eta_u^{(m)}\gamma_u\, \mathrm{SNR}^{\mathrm{MFB}})\Big] \\ &\stackrel{\mathrm{a}}{\geq} \mathrm{E}\left[f_b\left(\gamma \mathrm{SNR}^{\mathrm{MFB}}\frac{1}{U}\sum_{u=1}^{U}\eta_u^{(m)}\right)\right] \\ &= \mathrm{E}\Big[f_b\left(\gamma \mathrm{SNR}^{\mathrm{MFB}}\eta^{(m)}\right)\Big] \end{aligned} \tag{42}$$

where $f_b(\cdot)$ is the bit error probability vs. decoder input SNR function of the employed convolutional code, where (a) follows from Jensen's inequality applied to the convex function $f_b(\cdot)$ and where we define the average ME $\eta^{(m)}$ as the arithmetic mean of the ME of all users. Notice that, under mild conditions, for randomly spread CDMA in the large-system limit the ME converges to a constant independent of the user index, and inequality in (42) holds with equality. However, for a finite-dimensional system and given spreading sequences the above provides only a lower bound.

The expectation in the last line of (42) is with respect to the normalized channel energy γ (assumed identically distributed for all users). The evaluation of the above lower bound is much less computationally intensive than full Monte Carlo simulation of the whole system. In fact, the average ME $\eta^{(m)}$ can be obtained by running short simulations, since it requires a much smaller statistical sample to converge than the average BER. Then, the expectation with respect to γ can be obtained by either another Monte Carlo simulation or, if the distribution of γ is known, by numerical integration.

Figure 12 shows the comparison between the full Monte Carlo simulation and the semi-analytic approach for a system with $U = 32$, $L = 16$, and channel CH3. Solid lines show the BER obtained by Monte Carlo simulation for $m = 1, 2, 3, 6$ iterations respectively, plotted versus E_b/N_0. The dashed lines, show the results obtained using the semi-analytic approach. The thick solid line corresponds to the single-user MFB. For low SNR the system does not converge to the single user MFB even for a large number of iterations while for high SNR the convergence is obtained in a few iterations. We notice that the semi-analytic approach yields fairly accurate results already for such a small system.

6 Conclusions

We proposed a low complexity iterative turbo equalizer and multiuser decoder/detector for TD-CDMA systems, characterized by convolutional coding, hard-output Viterbi decoding and weighted feedback. From a rigorous large-system analysis of synchronous users and flat channels we gained the rationale for the optimization of the feedback weights. In the proposed iterative scheme, however, the presence of Viterbi decoders (not providing extrinsic information) produces biased statistics after the IC stage. Hence, we proposed a method for estimating the bias and improving the performance of the basic iterative decoder. Moreover, in order to cope with the ping-pong effect, we proposed a modified algorithm where a weighted sum of the decisions made in the two previous iterations is used for interference cancellation. The modified algorithm outperforms the the basic PIC receiver and even the receiver based on SISO decoding and soft feedback, with lower complexity.

The proposed receiver was validated in a realistic scenario, including asynchronous users, frequency selective fading channels and power control. Simulation results show that the receiver is robust to severe ISI conditions, even though its performance is degraded by uncompensated fading. However, this degradation is not very evident in the presence of sufficient multipath diversity. Eventually, a simple and fast semi-analytic approach to compute the BER was proposed and its behavior compared with the results obtained via Monte Carlo simulation.

As a concluding remark, we would like to point out that the proposed receiver structure is fully suited to be implemented in a UMTS-TDD base station, as an alternative to conventional single user decoding and high-complexity convolutional or turbo codes proposed in the current standard.

A Definition of $f(\cdot)$, $f_b(\cdot)$ and $\Gamma(\cdot)$

Consider a transmitter that maps the information bit sequence $\{b[j]\}$ onto a sequence of coded binary BPSK symbols $\{a[n]\}$ and sends over the AWGN channel defined by

$$y[n] = a[n] + \nu[n]$$

where $\nu[n]$ is Gaussian noise with distribution $\mathcal{N}(0, \sigma^2)$. The signal-to-noise ratio is $\mathrm{SNR} = \frac{1}{\sigma^2}$. The receiver decodes the signal $y[n]$ providing hard decisions $\hat{a}[n]$ on the transmitted symbols $a[n]$ and hard decisions $\hat{b}[j]$ on the information bits $b[j]$. We define

1. The SER vs. SNR characteristic

$$f(\mathrm{SNR}) = \frac{1 - \mathrm{E}\left[\hat{a}[n]a[n]\right]}{2} \tag{43}$$

2. The BER vs. SNR characteristic

$$f_b(\mathrm{SNR}) = \frac{1 - \mathrm{E}\left[\hat{b}[j]b[j]\right]}{2} \tag{44}$$

3. The correlation characteristic between decisions and noise

$$\Gamma(\mathrm{SNR}) = \mathrm{E}[\hat{a}[n]\nu[n]] \tag{45}$$

The above functions depend uniquely on the code considered (i.e., they are "characteristics" of the code) and, although they are generally unknown in closed form, they can be pre-computed by Monte Carlo simulation and stored as look-up tables in the receiver memory. Therefore, the impact of real-time evaluation of these functions on the overall receiver complexity is negligible.

References

[1] R. Prasad, W. Mohr, and E. W. Konhäuser, *Third Generation Mobile Communication Systems*. Boston: Artech House, 2000.

[2] S. Verdu, *Multiuser detection*. Cambridge, UK: Cambridge University Press, 1998.

[3] 3GPP, "ETSI TS 125 222, Multiplexing and channel coding TDD 3GPP TS 25.222 Version 4.0.0 Release 4," tech. rep., March 2001.

[4] S. Verdu and S. Shamai, "Spectral efficiency of CDMA with random spreading," *IEEE Trans. on Inform. Theory*, vol. 45, pp. 622–640, March 1999.

[5] S. Shamai and S. Verdu, "The impact of frequency-flat fading on the spectral efficiency of CDMA," *IEEE Trans. on Inform. Theory*, vol. 47, pp. 1302–1327, May 2001.

[6] M. Varanasi and T. Guess, "Optimum decision feedback multiuser equalization with successive decoding achieves the total capacity of the Gaussian multiple access channel," in *Proc. Asilomar Conference*, (Pacific Groove, CA), November 1997.

[7] R. R. Müller and S. Verdu, "Design and analysis of low-complexity interference mitigation on vector channels," *IEEE Journal on Selected Areas in Communications*, vol. 19, pp. 1429–1441, August 2001.

[8] P. Alexander, A. Grant, and M. Reed, "Iterative detection in code-division multiple-access with error control coding," *European Trans. on Telecomm.*, vol. 9, pp. 419–425, September 1999.

[9] X. Wang and V. Poor, "Iterative (Turbo) soft interference cancellation and decoding for coded CDMA," *IEEE Trans. on Commun.*, vol. 47, pp. 1047–1061, July 1999.

[10] H. ElGamal and E. Geraniotis, “Iterative multiuser detection for coded CDMA signals in AWGN and fading channels,” *IEEE J. Select. Areas Commun.*, vol. 18, pp. 30–41, January 2000.

[11] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, “Optimal decoding of linear codes for minimizing symbol error rate,” *IEEE Trans. on Inform. Theory*, vol. 20, pp. 284–287, March 1974.

[12] L. K. Rasmussen, “On ping-pong effects in linear interference cancellation for CDMA,” *IEEE 6th int. Symp. on Spread-Spectrum Tech. & Appli.*, September 2000.

[13] D. Tse and S. Hanly, “Linear multiuser receivers: Effective interference, effective bandwidth and capacity,” *IEEE Trans. on Inform. Theory*, vol. 45, pp. 641–675, March 1999.

[14] J. Boutros and G. Caire, “Iterative multiuser decoding: unified framework and asymptotic performance analysis.” submitted to IEEE Trans. on Inform. Theory, August 2000.

[15] S. Marinkovic, B. Vucetic, and J. Evans, “Improved iterative parallel interference cancellation,” in *Proc. ISIT 2001*, (Washington DC, USA), p. 34, June 2001.

[16] E. G. Ström and S. L. Miller, “Iterative demodulation of othogonal signaling formats in asynchronous DS-CDMA systems,” *Proceedings of the ISSSE*, pp. 184–187, July 2001.

[17] T. Richardson and R. Urbanke, “An introduction to the analysis of iterative coding systems.” IMA Proceedings, 2000.

[18] S. Benedetto, D. Divsalar, G. Montorsi, and F. Pollara, “Soft-Input Soft-Output building blocks for the construction of distributed iterative decoding of code networks,” *European Trans. on Commun.*, April 1998.

[19] F. Kschischang, B. Frey, and H. Loeliger, “Factor graphs and the sum-product algorithm,” *IEEE Trans. on Inform. Theory*, vol. 47, pp. 498–519, February 2001.

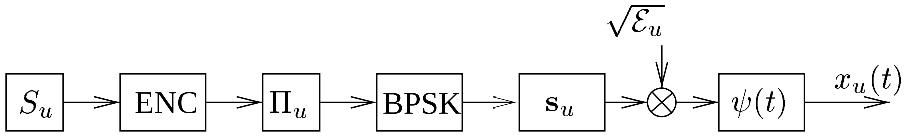
[20] J. Zhang, E. Chong, and D. Tse, “Output MAI distribution of linear MMSE multiuser receivers in DS-CDMA systems.” submitted to IEEE Trans. on Inform. Theory, May 2000.

[21] M. Kobayashi, J. Boutros, and G. Caire, "Successive interference cancellation with SISO decoding and EM channel estimation." to appear on JSAC (special issue on multiuser detection), 2001.

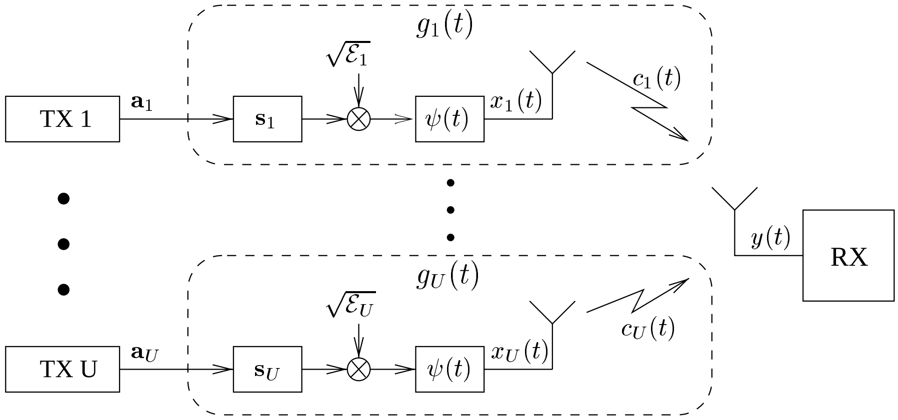
[22] C. Berrou and A. Glavieux, "Near optimum error-correcting coding and decoding: Turbo codes," *IEEE Trans. on Commun.*, vol. 44, October 1996.

[23] J. G. Proakis, *Digital Communications, 4th Edition*. McGraw-Hill, 2000.

[24] 3GPP-TSG-RAN-WG4, "TS-25.105v3.1.0 UTRA (BS) TDD Radio transmission and Reception," tech. rep., January 2000.



(a) Transmitter scheme



(b) Transmission channel

Figure 1: Transmitter and channel schemes

Table 1: MIP of the considered channels

Channel	*Path delays (in multiples of T_c)*	*Relative path powers (dB)*
CH1	0, 1.15, 34.18, 49.54, 65.66, 76.80	-2.5, 0, -12.8, -10.0, -25.2, -16.0
CH2	0, 16	0, 0
CH3	0, 16, 32	0, 0, 0

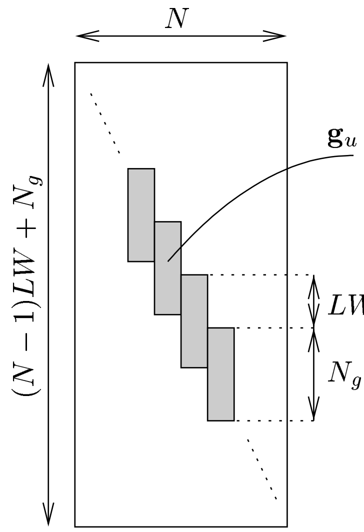


Figure 2: Structure of the matrix $\mathbf{G}_u$

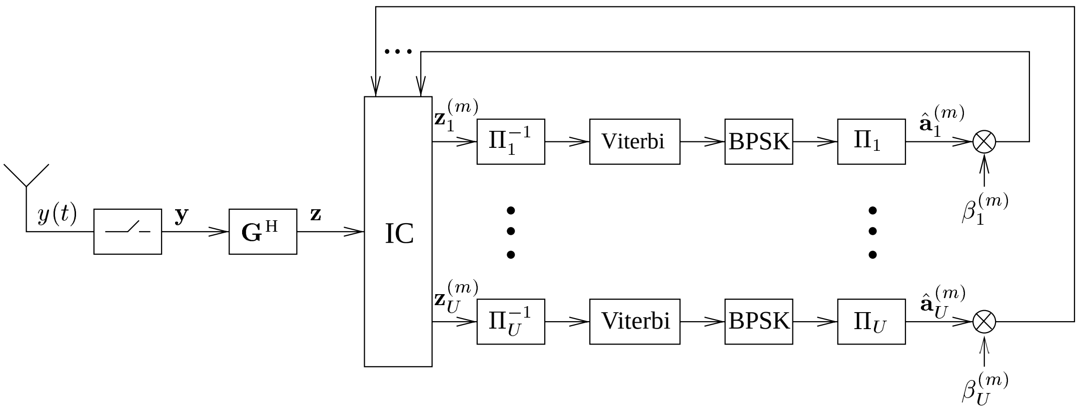


Figure 3: Scheme of the proposed turbo multiuser receiver

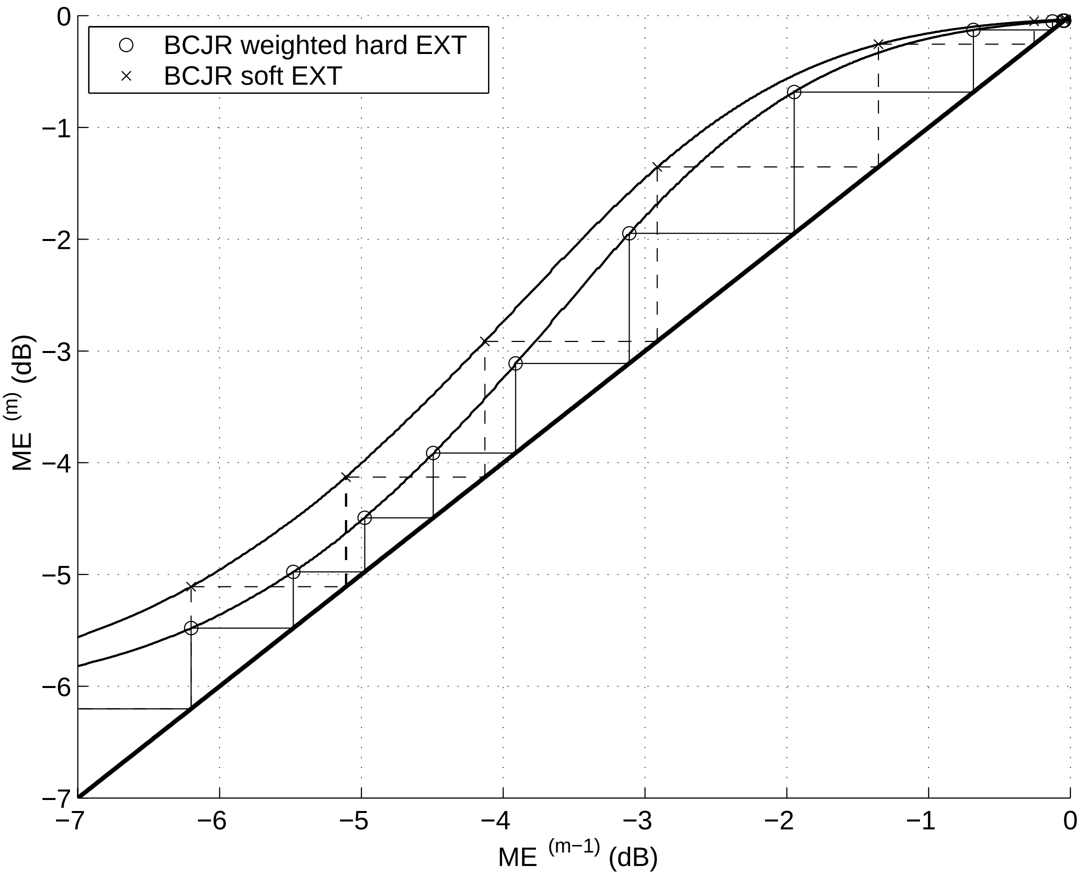


Figure 4: Evolution of the ME given by asymptotic analysis for CC$(5, 7)$, $\alpha = 2$, and $E_b/N_0 =$ 5dB

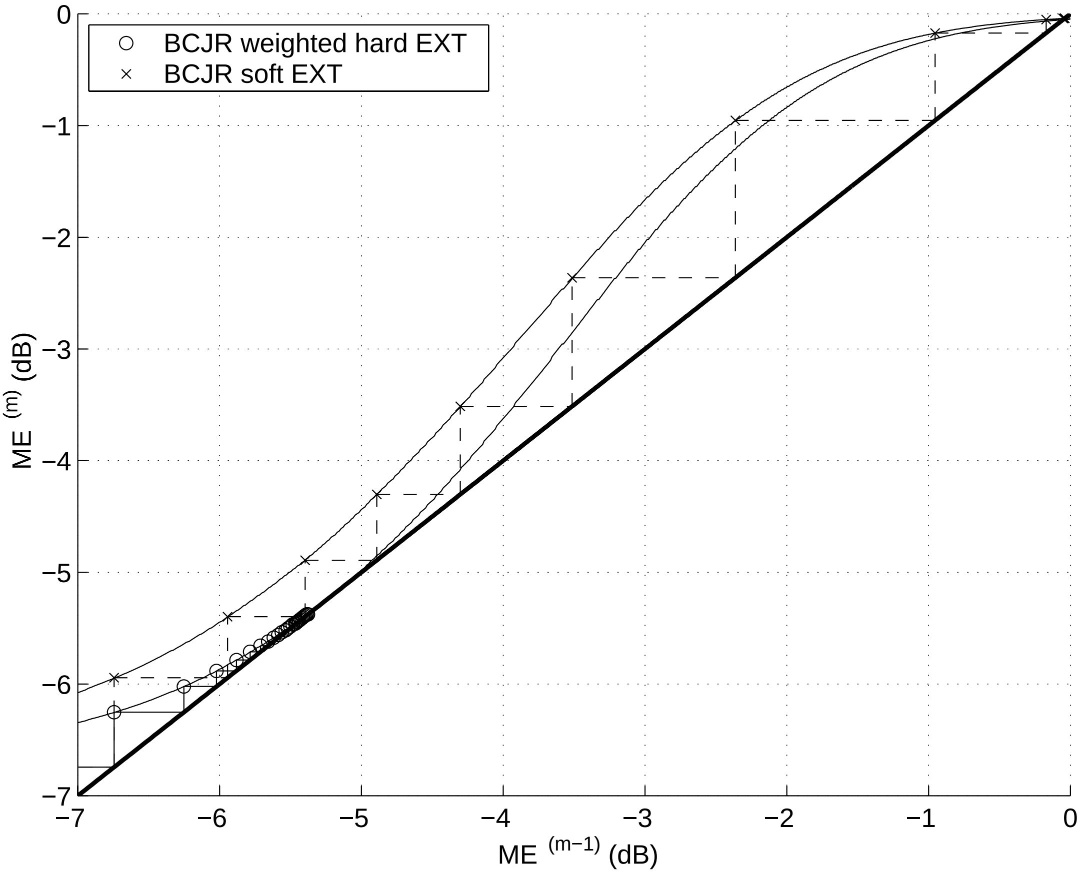


Figure 5: Evolution of the ME given by asymptotic analysis for CC$(5,7)$, $\alpha = 2.35$, and $E_b/N_0 = 5$dB

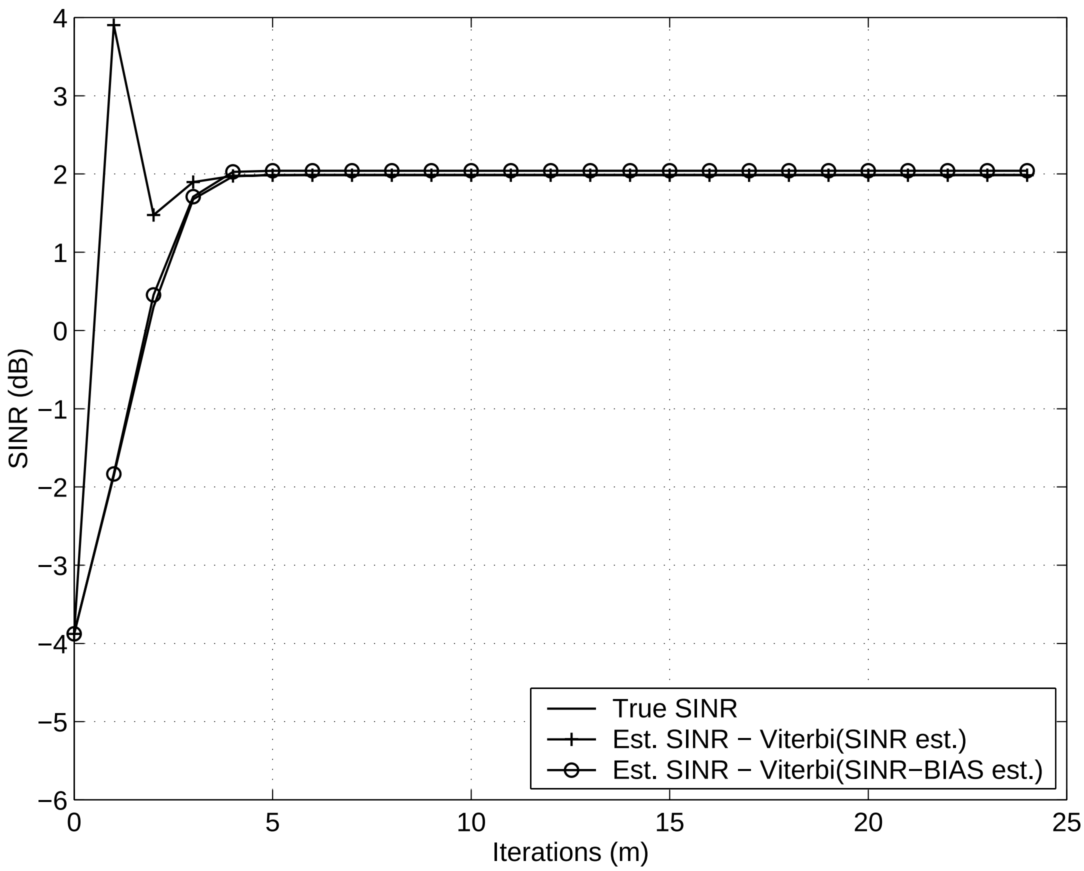


Figure 6: True and estimated SINR for “Viterbi (SINR est.)” and “Viterbi (SINR-BIAS est.)” decoders, using CC$(5, 7)$, $U = 32$, $L = 16$, and $E_b/N_0 = 5$dB

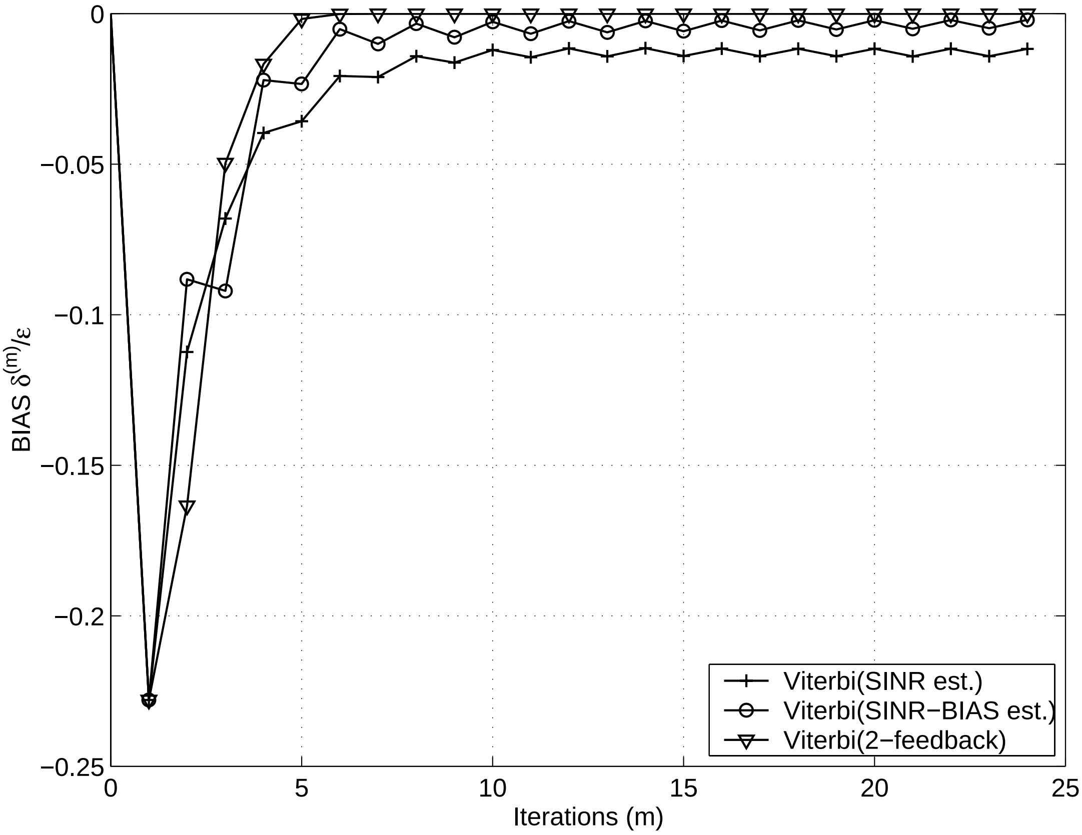


Figure 7: Bias in the statistic of $z_u^{(m)}$ using CC$(5,7)$, $U = 40$, $L = 16$, and $E_b/N_0 = 5$dB. The figure shows the oscillations due to the ping-pong effect

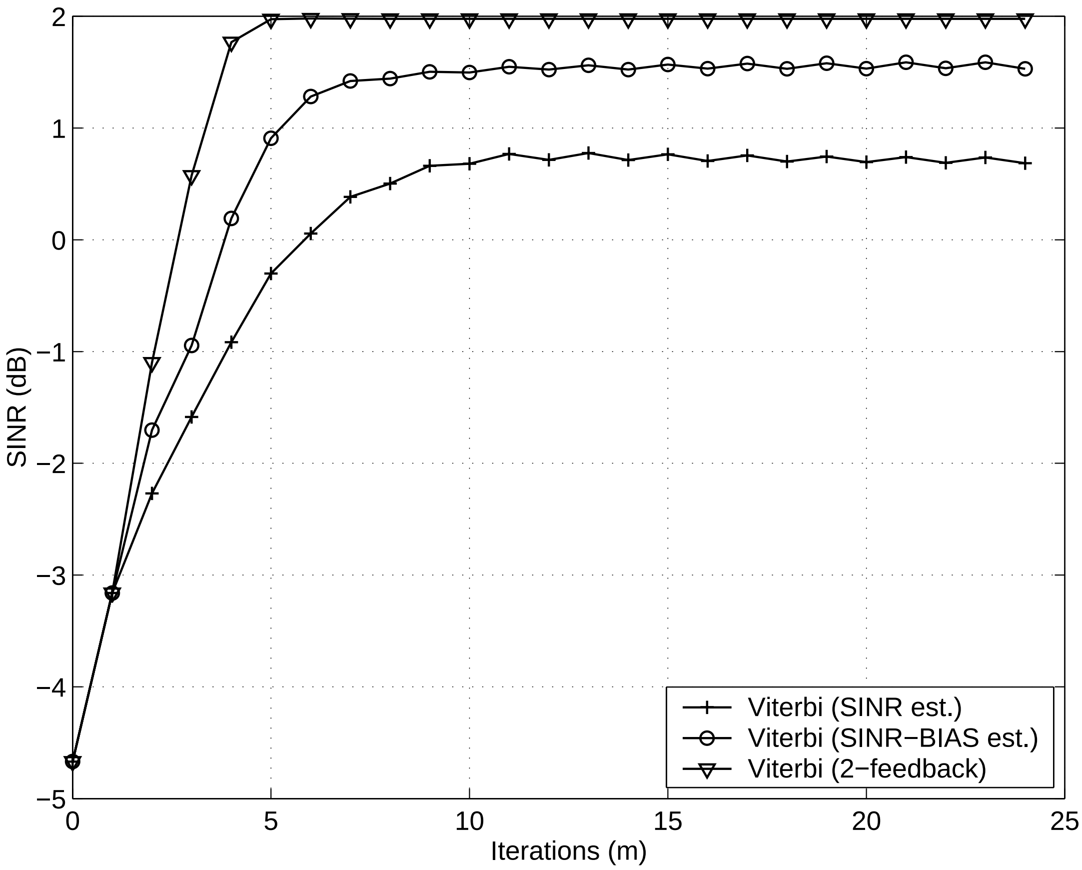


Figure 8: True SINR of several multiuser receivers using CC$(5, 7)$, $U = 40$, $L = 16$, and $E_b/N_0 = 5$dB

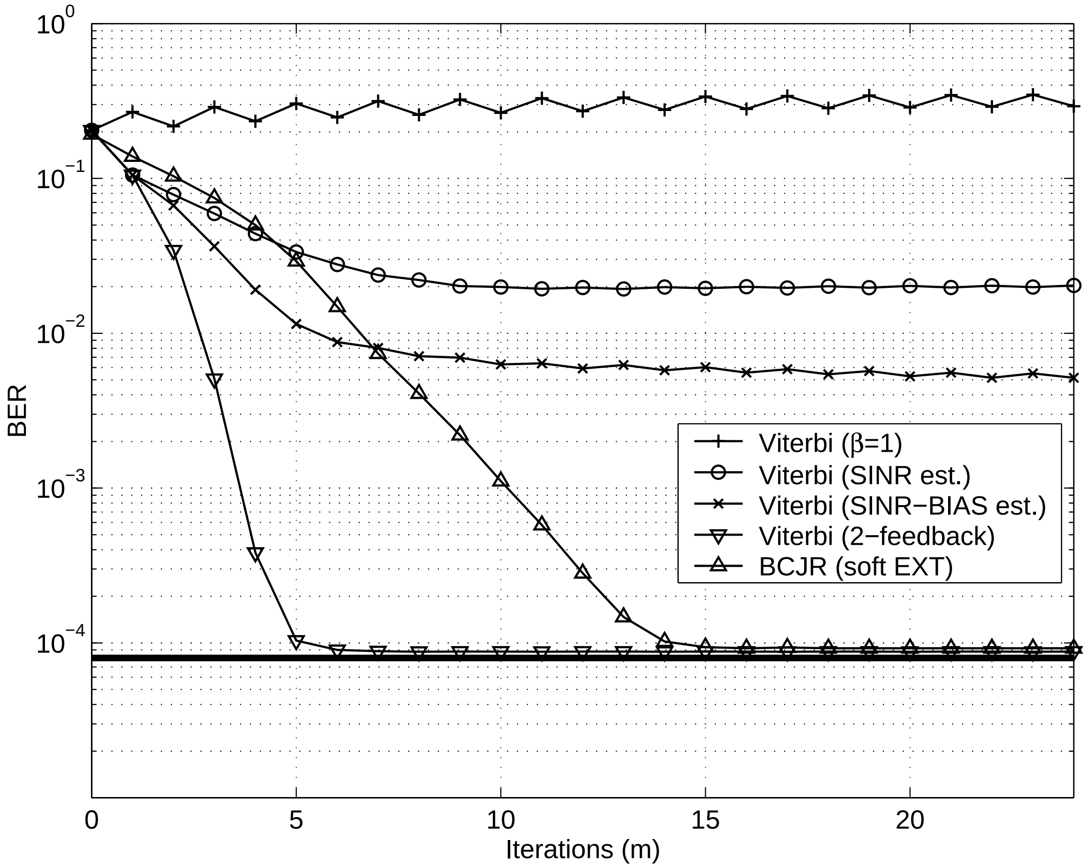


Figure 9: BER of several multiuser receivers using CC$(5, 7)$, $U = 40$, $L = 16$, and $E_b/N_0 = 5$dB

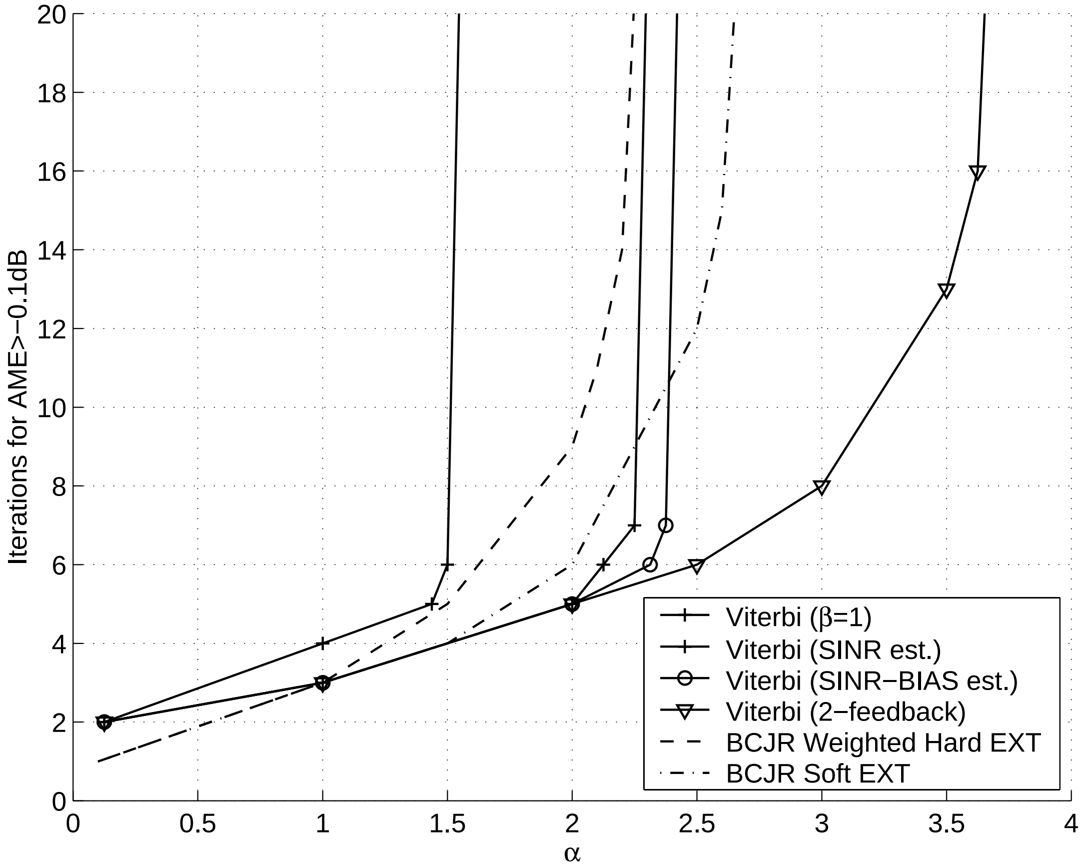


Figure 10: Number of iterations required to attain an ME> -0.1dB for several multiuser decoders using CC$(5,7)$, $L = 16$, and $E_b/N_0 = 5$dB

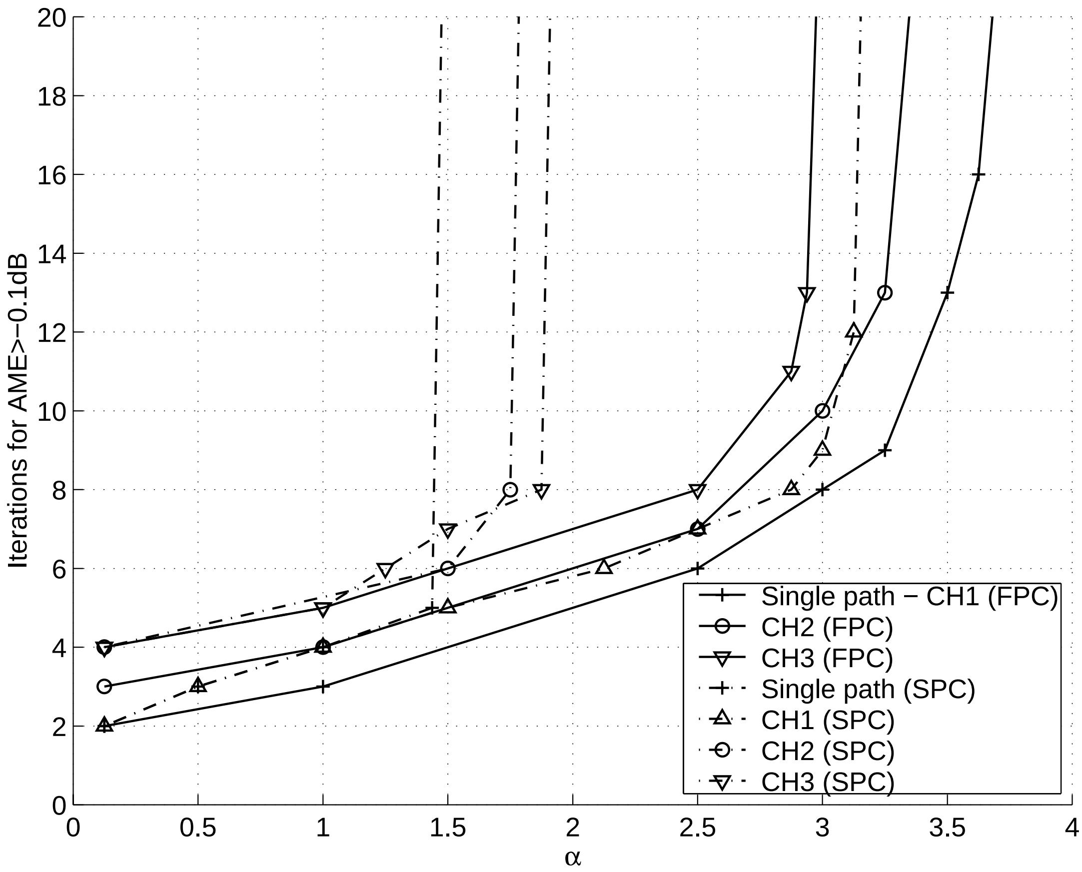


Figure 11: Performance of the "Viterbi (2-feedbacks)" multiuser decoder in the presence of multipath fading channels using CC$(5,7)$, $L = 16$, and $E_b/N_0 = 5$dB. FPC and SPC denote fast and slow power control conditions, respectively

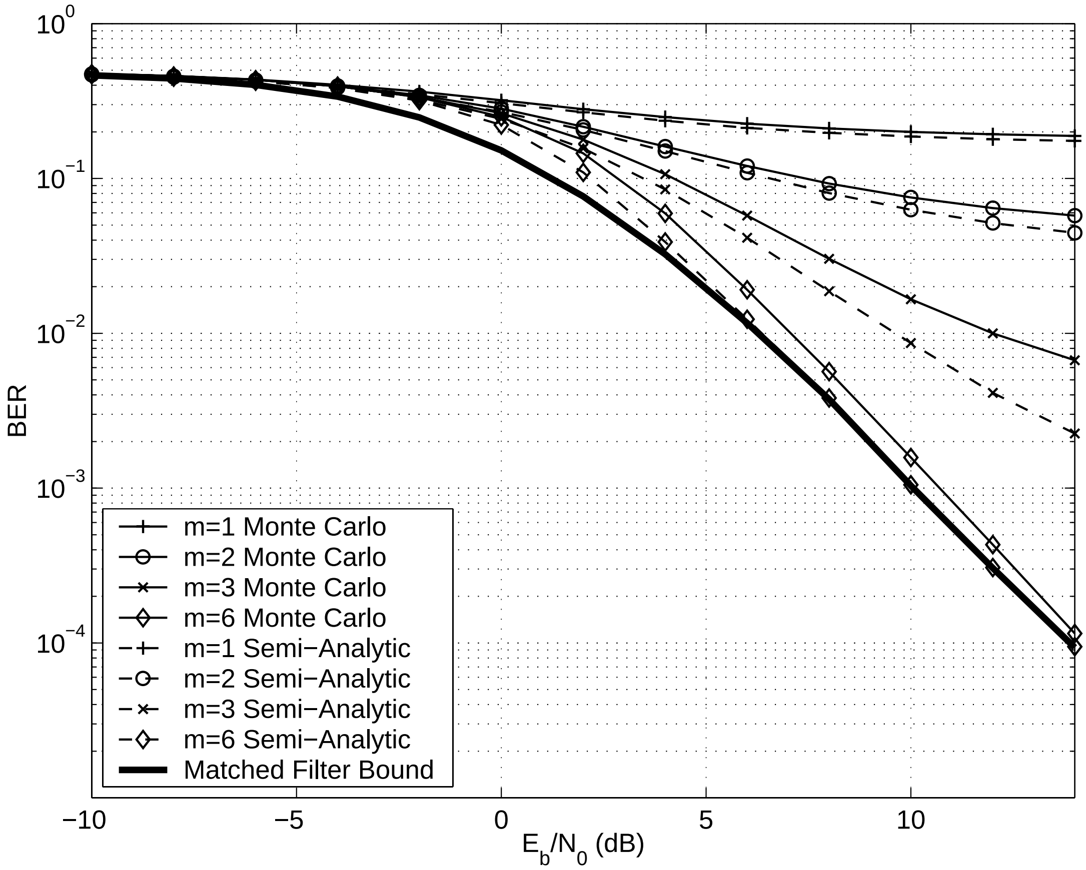


Figure 12: BER provided by the "Viterbi (2-feedbacks)" multiuser decoder for channel CH3 using CC$(5, 7)$, $U = 32$, and $L = 16$