Spatial Multiplexing by Spatiotemporal Spreading: Receiver Considerations

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Abstract— **The use of multiple transmit and receive antennas allows to transmit multiple signal streams in parallel and hence to increase communication capacity. Apart from capacity, the MIMO channel also offers potentially a large number of diversity sources. To exploit these diversity degrees, and hence enhance outage capacity, bit interleaved coded modulation is now a classical solution. In this paper we propose to exploit the diversity sources by linear precoding, to turn the fading channel into a non-fading one. Additional channel coding then only serves to enhance robustness against noise. To streamline the processing and analysis, the linear precoding considered here is convolutional instead of blockwise. We particularly focus in this paper on two non-iterative receiver strategies. Performance improvements are shown over conventional VBLAST.**

I. Introduction

Spatial multiplexing has been introduced independently in a 1994 Stanford University patent by A. Paulraj and by Foschini [1] at Bell Labs. Spatial multiplexing can be viewed as a limiting case of Spatial Division Multiple Access (SDMA) in which the various mobile users are colocated in one single user multi antenna mobile terminal. In that case, the various users are no longer distinguishable on the basis of their (main) direction (DOA) since all antennas are essentially colocated. Nevertheless, if the scattering environment is sufficiently rich, the antenna arrays at TX and RX can see the different DOAs of the multiple paths. One can then imagine transmitting multiple data streams, one stream per path. For this, the set of paths to be used should be resolvable in angle at both TX and RX. Without channel knowledge at the TX, the multiple streams to be transmitted just get mixed over the multiple paths in the matrix channel. They can generally be linearly recovered at the RX if the channel matrix rank equals or exceeds the number of streams. This rank equals the number of paths that are simultaneously resolvable at TX and RX. The assumptions we shall adopt for the proposed approach are no channel knowledge at TX, perfect channel knowledge at RX, frequency-flat channels for the initial part of the paper.

II. Linear prefiltering approach

We shall call here rate the number N_s of symbol sequences (streams, layers) at symbol rate. A general ST coding setup is sketched in Fig. 1. The incoming stream of bits gets transformed to N_s symbol streams through a combination of channel coding, interleaving, symbol mapping and demultiplexing. The result is a vector stream of symbols $\boldsymbol{b}_k$ containing N_s symbols per symbol period. The N_s streams then get mapped linearly to the

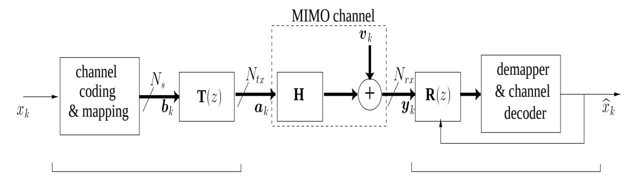

Fig. 1. General ST coding setup.

N_{tx} transmit antennas and this part of the transmission is called linear ST precoding. The output is a vector stream of symbols $\boldsymbol{a}_k$ containing N_{tx} symbols per symbol period. The linear precoding is spatiotemporal since an element of $\boldsymbol{b}_k$ may appear in multiple components (space) and multiple time instances (time) of $\boldsymbol{a}_k$. The vector sequence $\boldsymbol{a}_k$ gets transmitted over a MIMO channel $\mathbf{H}$ with N_{rx} receive antennas, leading to the symbol rate vector received signal $\boldsymbol{y}_k$ after sampling. The linear precoding can be considered to be an inner code, while the nonlinear channel coding etc. can be considered to be an outer code. As the number of streams is a factor in the overall bitrate, we shall call the case $N_s = N_{tx}$ the *full rate* case, while $N_s = 1$ corresponds to the *single rate* case. Instead of multiple antennas, more general multiple channels can be considered by oversampling, by using polarization diversity or other EM component variations, by working in beamspace, or by considering in phase and in quadrature (or equivalently complex and complex conjugate) components. In the case of oversampling, some excess bandwidth should be introduced at the transmitter, possibly involving spreading which would then be part of the linear precoding. As we shall see below, channel capacity can be attained by a full rate system without precoding ($\mathbf{T}(z) = I$). In that case, the channel coding has to be fairly intense if we want to exploit all available diversity sources, since it has to spread the information contained in each transmitted bit over space (across TX antennas) and time, see the left part in Fig. 2 and [2]. The goal of introducing the linear precoding is to simplify (possibly going as far as eliminating) the channel coding part [3]. In fact the goal of the linear precoding is to exploit all diversity sources and transform the channel virtually into a non-fading channel so that possible additional channel coding can be taken from the set of non-fading channel codes. In the case of linear dispersion codes [4],[5], transmission is not continuous but packet-wise (block-wise). In that case, a packet of T vector symbols $\boldsymbol{a}_k$ (hence a $N_{tx} \times T$ matrix) gets constructed as a lin-

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ear combination of fixed matrices in which the combination coefficients are symbols b_k. A particular case is the Alamouti code which is a full diversity single rate code corresponding to block length $T = N_{tx} = 2$, $N_s = 1$. In the first part of this paper we shall focus on continuous transmission in which linear precoding corresponds to MIMO prefiltering. This *linear convolutive precoding* can be considered as a special case of linear dispersion codes (making abstraction of the packet boundaries) in which the fixed matrices are time-shifted versions of the impulse responses of the columns of $\mathbf{T}(z)$ in Fig. 1. A number of con-

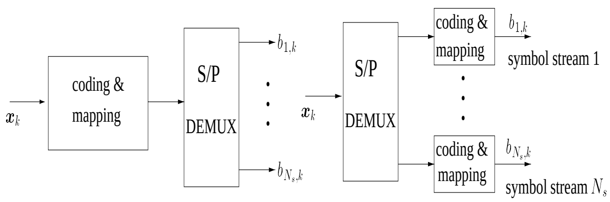

Fig. 2. Two channel coding, interleaving, symbol mapping and demultiplexing choices.

figurations are possible for the channel coding part (outer code), see Fig. 2. In the *global channel coding/mapping* case (see left part of Fig. 2), the last operation of the encoding part is spatial demultiplexing (serial-to-parallel (S/P) conversion) (mapping refers to bit interleaving and symbol constellation mapping). At the other extreme, this S/P conversion is the first operation in the case of *streamwise channel coding/mapping*, see the right part of Fig. 2. An intermediate approach consists of global channel coding followed by S/P conversion and streamwise mapping. Systems without linear precoding require at least streamwise mapping The existing BLAST systems are special cases of such approaches. VBLAST is a full rate system with $\mathbf{T}(z) = \mathbf{I}_{N_{tx}}$ which leads to quite limited diversity in the absence of outer coding. DBLAST (in a simplified version) is a single-rate system with $\mathbf{T}(z) = [1 \; z^{-1}, \ldots, z^{-(N_{tx}-1)}]^T$ which leads to full diversity (delay diversity) (on frequency-flat channels). We would like to introduce a prefiltering matrix $\mathbf{T}(z)$ without taking a hit in capacity, while achieving full diversity (in the absence of outer coding). The MIMO prefiltering will allow us to capture all diversity (spatial, and frequential for channels with delay spread) and will provide some coding gain. The optional channel coding then serves to provide additional coding gain and possibly (with interleaving) to capture temporal diversity (Doppler spread) if there is any. In its simplest form, the outer code can consist of global channel coding without interleaving. Some (multi-stream) detection schemes may require stream-wise channel coding though. Finally, though time-invariant filtering may evoke continuous transmission, the prefiltering approach is also immediately applicable to block transmission by replacing convolution by circular convolution (see below).

A. Capacity

Consider the MIMO AWGN (flat) channel

$$\boldsymbol{y}_k = \mathbf{H}\,\boldsymbol{a}_k + \boldsymbol{v}_k = \mathbf{H}\,\mathbf{T}(q)\,\boldsymbol{b}_k + \boldsymbol{v}_k \qquad (1)$$

where the noise power spectral density matrix is $S_{\boldsymbol{vv}}(z) = \sigma_v^2\, I$, $q^{-1}\,\boldsymbol{b}_k = \boldsymbol{b}_{k-1}$. The **ergodic capacity** when channel knowledge is absent at the TX and perfect at the RX is:

$$\begin{aligned} C(S_{\boldsymbol{aa}}) &= \mathrm{E}_H \frac{1}{2\pi j} \oint \frac{dz}{z} \log_2 \det(I + \tfrac{1}{\sigma_v^2}\,\mathbf{H}\, S_{\boldsymbol{aa}}(z)\,\mathbf{H}^H) \\ &= \mathrm{E}_H \frac{1}{2\pi j} \oint \frac{dz}{z} \log_2 \det(I + \tfrac{1}{\sigma_v^2}\mathbf{H}\mathbf{T}(z) S_{\boldsymbol{bb}}(z)\mathbf{T}^\dagger(z)\,\mathbf{H}^H) \\ &= \mathrm{E}_H \frac{1}{2\pi j} \oint \frac{dz}{z} \log_2 \det(I + \rho\,\mathbf{H}\,\mathbf{T}(z)\,\mathbf{T}^\dagger(z)\,\mathbf{H}^H) \end{aligned} \qquad (2)$$

where we assume that the outer coding leads to spatially and temporally white symbols: $S_{\boldsymbol{bb}}(z) = \sigma_b^2\, I$, and $\rho = \frac{\sigma_b^2}{\sigma_v^2} = \frac{SNR}{N_{tx}}$. The expectation E_H is here w.r.t. the distribution of the channel. As in [6], we assume the entries $\mathbf{H}_{i,j}$ of the channel to be mutually independent zero mean complex Gaussian variables with unit variance (Rayleigh flat fading MIMO channel model). As stated in [7], to avoid capacity loss the prefilter $\mathbf{T}(z)$ is required to be paraunitary ($\mathbf{T}(z)\mathbf{T}^\dagger(z) = \mathbf{I}$) (hence full stream TX is required). Motivated by the consideration of diversity also (see below), we propose to use the following paraunitary prefilter

$$\mathbf{T}(z) = \mathbf{D}(z)\; Q,\; \mathbf{D}(z) = diag\{1, z^{-1}, \ldots, z^{-(N_{tx}-1)}\} \qquad (3)$$

where $\mathbf{D}(z)$ introduces delay diversity and Q is a (constant) unitary matrix with equal magnitude elements, $|Q_{ij}| = \frac{1}{\sqrt{N_{tx}}}$, that performs spatial spreading (columns are spatial spreading codes). Note that for a channel with a delay spread of L symbol periods, the prefilter can be immediately adapted by replacing the elementary delay z^{-1} by z^{-L}. For the propagation channel $\mathbf{H}(z)$ (with columns $\mathbf{H}_{:,i}(z)$) combined with the prefilter $\mathbf{T}(z)$ in (3), symbol stream n ($b_{n,k}$) passes through the equivalent SIMO channel

$$\sum_{i=1}^{N_{tx}} z^{-(i-1)L}\; \mathbf{H}_{:,i}(z)\; Q_{i,n} \qquad (4)$$

which now has extended memory due to the delay diversity introduced by $\mathbf{D}(z)$. It is important that the different columns $\mathbf{H}_{:,i}$ of the channel matrix get spread out in time to get full diversity (otherwise the streams just pass through a linear combination of the columns, which would offer the same limited diversity as in VBLAST, the case without spreading). The delay diversity only becomes effective by the introduction of the mixing/rotation matrix Q, which has equal magnitude elements for uniform diversity source exploitation.

A strongly related approach with an interesting interpretation is obtained as follows. Consider grouping the symbol sequence $\boldsymbol{b}_k$ in groups of N_{tx} consecutive symbols, then one group $\boldsymbol{b}_{k-N_{tx}+1:k}$ of N_{tx} symbols forms a square matrix of size $N_{tx} \times N_{tx}$. An alternative approach is obtained by transposing the matrix $\boldsymbol{b}_{k-N_{tx}+1:k}$ before inputting its colums into $\mathbf{T}(z)$ (hence inputting the rows of $\boldsymbol{b}_{k-N_{tx}+1:k}$ into $\mathbf{T}(z)$ instead of the columns). This interleaving has no effect if no channel coding is introduced. It corresponds to spreading within streams instead of between streams. The resulting scheme can be interpreted as follows. It corresponds to the sawtooth threading approach of [12], which transforms the time-invariant (flat) MIMO channel into a periodically time-varying SIMO channel for each stream (period N_{tx}), and then we exploit the temporal fading with the $N_{tx} \times N_{tx}$ constellation rotation matrix **Q** as suggested in [8].

B. Matched Filter Bound and Diversity

The Matched Filter Bound (MFB) is the maximum attainable SNR for symbol-wise ML detection, when the interference from all other symbols has been removed. Hence the multi-stream MFB equals the MFB for a given stream. For VBLAST ($\mathbf{T}(z) = \mathbf{I}$), the MFB for stream n is

$$\mathrm{MFB}_n = \rho ||\mathbf{H}_{:,n}||_2^2 \quad (5)$$

hence, diversity is limited to N_{rx}. For the proposed $\mathbf{T}(z) = \mathbf{D}(z)\, Q$ on the other hand, stream n has MFB

$$\mathrm{MFB}_n = \rho \frac{1}{N_{tx}}\, ||\mathbf{H}||_F^2 \quad (6)$$

hence this $\mathbf{T}(z)$ provides the same full diversity $N_{tx}N_{rx}$ for all streams. Larger diversity order leads to larger outage capacity.

C. Pairwise Error Probability $\mathbf{P}_e$ (Flat Channel Case)

It was shown in [7] that the following choice for Q minimizes an upper bound to the pairwise error probability at high SNR: the Vandermonde matrix:

$$Q^s = \frac{1}{\sqrt{N_{tx}}} \begin{bmatrix} 1 & \theta_1 & \dots & {\theta_1}^{N_{tx}-1} \\ 1 & \theta_2 & \dots & {\theta_2}^{N_{tx}-1} \\ \vdots & \vdots & & \vdots \\ 1 & \theta_{N_{tx}} & \dots & {\theta_{N_{tx}}}^{N_{tx}-1} \end{bmatrix} \quad (7)$$

which is unitary (for amximum capacity) and has equal magnitude components (as appropriate for a spreading operation for maximum diversity) and where the θ_i are the roots of $\theta^{N_{tx}} - j = 0\,,\ j = \sqrt{-1}$. When $N_{tx} = 2^{n_t}$ ($n_t \in \mathbb{N}$), and for a finite QAM constellation with $(2M)^2$ points, Q^s maximizes the coding gain among all matrix Q with normalized columns, and achieves the following coding gain: $\left(\frac{6}{N_{tx}(4M^2-1)}\right)^{N_{tx}}$. The details for the case of circular convolution (wrapping) within blocks or channels with delay spread are also presented in [11]

III. ML Reception

In principle, we can perform Maximum Likelihood reception since the delay diversity transforms the (flat) channel into a channel with finite memory. However, the number of states would be the product of the constellation sizes of the N_{tx} streams to the power $LN_{tx} - 1$. Hence, if all the streams have the same constellation size $|\mathcal{A}|$, the number of states would be $|\mathcal{A}|^{N_{tx}(LN_{tx}-1)}$, which will be much too large in typical applications. Suboptimal ML reception can be performed in the form of sphere decoding [9]. The complexity of this can still be too large though. Alternatively, PIC and turbo RXs can be used as approximations to ML reception. Another suboptimal receiver structure will be considered in the next section.

IV. Stripping DFE (SIC) Reception

Let $\mathbf{G}(z) = \mathbf{H}(z)\,\mathbf{T}(z)$ be the cascade transfer function of channel and precoding. The matched filter RX is

$$\begin{aligned} \boldsymbol{x}_k = \mathbf{G}^\dagger(q)\,\boldsymbol{y}_k &= \mathbf{G}^\dagger(q)\,\mathbf{G}(q)\,\boldsymbol{b}_k + \mathbf{G}^\dagger(q)\,\boldsymbol{v}_k \\ &= \mathbf{R}(q)\,\boldsymbol{b}_k + \mathbf{G}^\dagger(q)\,\boldsymbol{v}_k \end{aligned} \quad (8)$$

where $\mathbf{R}(z) = \mathbf{G}^\dagger(z)\,\mathbf{G}(z)$, and the psdf of $\mathbf{G}^\dagger(q)\,\boldsymbol{v}_k$ is $\sigma_v^2\,\mathbf{R}(z)$. The MIMO DFE RX is then:

$$\widehat{\mathbf{b}}_k = -\underbrace{\overline{\mathbf{L}}(q)}_{\text{feedback}}\ \mathbf{b}_k + \underbrace{\mathbf{F}(q)}_{\text{feedforward}}\ \mathbf{x}_k \quad (9)$$

where feedback $\overline{\mathbf{L}}(z)$ is strictly "causal" (causal is here first between users and then in time: $\mathbf{L}(z) = I + \overline{\mathbf{L}}(z)$ is lower triangular with causal diagonal). Fig. 3 illustrates that this MIMO DFE corresponds to SIMO DFE's per stream plus cancellation of each detected stream from the RX signal (or MF output) before detection of the next stream. This scheme is hence the extension of the VBLAST "nulling (in the ZF case) and canceling" RX to the spatiotemporal case. Two design criteria for feedforward and feedback filters are possible: (MMSE) ZF and MMSE, see [7], where we indicated that triangular MIMO feedback structures allow to incorporate channel decoding before cancellation, which leads to the stripping approach of Verdu & Müller or Varanasi & Guess. Simplified RXs can be obtained by the use of a (noise) predictive DFE which allows to approximate the (LMMSE) forward filter via polynomial expansion (filtering with $\mathbf{R}(z)$) and to reduce the order of the feedback filter (predictor) to a desired complexity level.

A. Diversity Order Considerations

In VBLAST, one can easily identify the diversity orders for the various substreams in the case of a frequency-flat channel, on the basis of (MMSE) ZF RX considerations (the same diversity orders also hold for the corresponding MMSE design). In VBLAST, stream n enjoys a diversity order equal to $N_{rx} - N_{tx} + n\,,\ n = 1, \ldots, N_{tx}$, assuming $N_{rx} \geq N_{tx}$. Using similar ZF design considerations, one can obtain that for a channel with delay spread L, stream n with our convolutional precoder $\mathbf{T}(z) = \mathbf{D}(z^L)\, Q$ and the triangular MIMO DFE RX enjoys a diversity order equal to $(N_{rx} - N_{tx} + n)\, N_{tx}\, L\,,\ n = 1, \ldots, N_{tx}$.

V. MIMO DFE Reception

We now consider a classical MIMO decision feedback equalizer, in which the symbols vectors $\boldsymbol{b}_k$ are processed sequentially in time (see Fig. 4). For the approach considered here to be smoothly combined with channel coding, it is desirable that the symbols components of the vector symbol $\boldsymbol{b}_k$ belong to the same stream and that consecutive $\boldsymbol{b}_k$ belong to different streams. To this end, a third type of stream assignment (layering) should be introduced. In this case, the frame of data to be transmitted gets partitioned into consecutive blocks. Each stream has one diagonal set of symbols in any given block, and hence stream i is composed of diagonal i in every block (for a frequency-flat channel). Hence, every block contains only one vector symbol (diagonal) $\boldsymbol{b}_k$ belonging to a particular stream. The non-iterative reception gets performed by running the DFE in parallel over each block and decoding each consecutive stream sequentially, before using it in the feedback for the detection of the next stream.

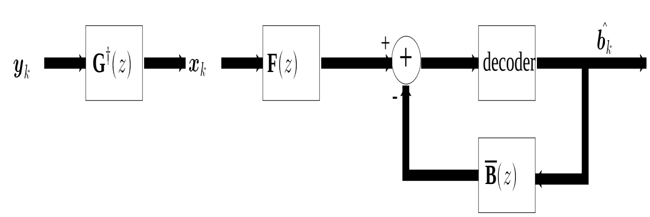

Fig. 4. MIMO DFE Receiver

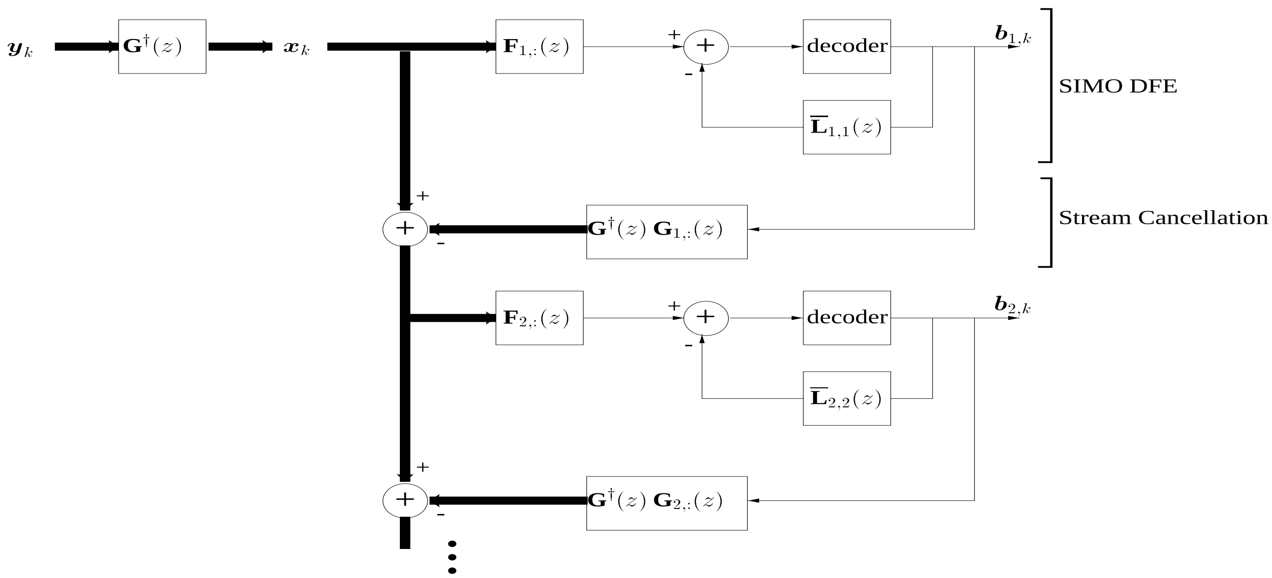

Fig. 3. Triangular MIMO DFE Receiver.

The DFE output is then:

$$\widehat{\mathbf{b}}_k \;=\; -\underbrace{\overline{\mathbf{B}}(q)}_{\text{feedback}}\;\mathbf{b}_k + \underbrace{\mathbf{F}(q)}_{\text{feedforward}}\;\mathbf{x}_k \tag{10}$$

where the feedback filter $\overline{\mathbf{B}}(z) = \sum_{i\geq 1}\mathbf{B}_i\,z^{-i}$ is such that $\mathbf{B}(z) = I + \overline{\mathbf{B}}(z)$ is causal, monic and minimum phase. We shall consider the MSE as filter design criterion.

A. MMSE MIMO DFE RX

Let's consider the backward channel model based on LMMSE [10]:

$$\mathbf{b}_k \;=\; \widehat{\mathbf{b}}_k + \widetilde{\mathbf{b}}_k \;=\; \mathbf{S}_{\mathbf{bx}}(q)\,\mathbf{S}^{-1}_{\mathbf{xx}}(q)\,\mathbf{x}_k + \widetilde{\mathbf{b}}_k \tag{11}$$

where $\mathbf{S}_{\mathbf{bx}}(z) = \mathbf{S}_{\mathbf{bb}}(z)\,\mathbf{G}^\dagger(z)\mathbf{G}(z)$ and $\mathbf{S}_{\mathbf{xx}}(z) = \mathbf{G}^\dagger(z)\mathbf{G}(z)\,\mathbf{S}_{\mathbf{bb}}(z)\,\mathbf{G}^\dagger(z)\mathbf{G}(z) + \sigma_v^2\,\mathbf{G}^\dagger(z)\mathbf{G}(z)$. Hence $\mathbf{S}_{\mathbf{bx}}(z)\,\mathbf{S}^{-1}_{\mathbf{xx}}(z) = \mathbf{R}^{-1}(z)$ with $\mathbf{R}(z) = \mathbf{G}^\dagger(z)\mathbf{G}(z) + \sigma_v^2\,\mathbf{S}^{-1}_{\mathbf{bb}}(z) = \mathbf{G}^\dagger(z)\mathbf{G}(z) + \frac{1}{\rho}\,I$. So $\mathbf{b}_k = \mathbf{R}^{-1}(q)\mathbf{x}_k + \widetilde{\mathbf{b}}_k$. We also get $\mathbf{S}_{\widetilde{\mathbf{b}}\widetilde{\mathbf{b}}}(z) = \mathbf{S}_{\mathbf{bb}}(z) - \mathbf{S}_{\mathbf{bx}}(z)\mathbf{S}^{-1}_{\mathbf{xx}}(z)\mathbf{S}_{\mathbf{xb}}(z) = \sigma_v^2\,\mathbf{R}^{-1}(z)$. Let $\mathbf{B}(z)$ be the unique causal monic minimum phase factor of $\mathbf{R}(z)$, then:

$$\mathbf{R}(z) \;=\; \mathbf{B}^\dagger(z)\,\mathbf{M}\,\mathbf{B}(z). \tag{12}$$

where $\mathbf{M}$ is a constant matrix. Then $\mathbf{b}_k = \mathbf{B}^{-1}(q)\,\mathbf{M}^{-1}\,\mathbf{B}^{-\dagger}(q)\,\mathbf{x}_k + \widetilde{\mathbf{b}}_k$. By choosing $\mathbf{F}(q) = \mathbf{M}^{-1}\mathbf{B}^{-\dagger}(q)$, we get

$$\begin{aligned}\mathbf{F}(q)\,\mathbf{x}_k &= \mathbf{M}^{-1}\,\mathbf{B}^{-\dagger}(q)\,\mathbf{x}_k \;=\; \mathbf{B}(q)\,\mathbf{b}_k - \mathbf{B}(q)\,\widetilde{\mathbf{b}}_k \\ &= \mathbf{B}(q)\,\mathbf{b}_k + \mathbf{e}_k = \mathbf{b}_k + \overline{\mathbf{B}}(z)\,\mathbf{b}_k + \mathbf{e}_k\end{aligned} \tag{13}$$

where $\mathbf{S}_{\mathbf{ee}}(z) = \mathbf{B}(z)\mathbf{R}^{-1}(z)\mathbf{B}^\dagger(z) = \sigma_v^2\,\mathbf{M}^{-1}$.

The $\overline{\mathbf{B}}(z) = \mathbf{B}(z) - I$ is tightly related to the MIMO prediction error filter $\mathbf{P}(z)$ of the spectrum $\mathbf{R}(z)$, $\mathbf{P}^\dagger(z)\mathbf{R}(z)\mathbf{P}(z) =$ Constant Matrix. Indeed, $\mathbf{P}(z) = \mathbf{B}^{-1}(z)$ obviously.

B. Unbiased MMSE MIMO DFE RX

$\mathbf{F}(q)\,\mathbf{x}_k - \overline{\mathbf{B}}(z)\,\mathbf{b}_k$ is a biased estimate of $\mathbf{b}_k$, since:

$$\begin{aligned}&\mathbf{F}(q)\,\mathbf{x}_k - \overline{\mathbf{B}}(q)\,\mathbf{b}_k \\ &= (\mathbf{M}^{-1}\,\mathbf{B}^{-\dagger}(q)\,\mathbf{G}^\dagger(q)\mathbf{G}(q) - \overline{\mathbf{B}}(z))\mathbf{b}_k \\ &\quad+\mathbf{M}^{-1}\,\mathbf{B}^{-\dagger}(q)\mathbf{G}^\dagger(q)\boldsymbol{v}_k \\ &= (I - \tfrac{1}{\rho}\,\mathbf{M}^{-1}\,\mathbf{B}^{-\dagger}(q))\,\mathbf{b}_k + \mathbf{M}^{-1}\,\mathbf{B}^{-\dagger}(q)\mathbf{G}^\dagger(q)\boldsymbol{v}_k \\ &= (I - \tfrac{1}{\rho}\,\mathbf{M}^{-1})\,\mathbf{b}_k + \mathbf{e}^U_k\end{aligned} \tag{14}$$

where $\mathbf{e}^U_k = \mathbf{M}^{-1}\,\mathbf{B}^{-\dagger}(q)\mathbf{G}^\dagger(q)\boldsymbol{v}_k - \frac{1}{\rho}\mathbf{M}^{-1}\,(\mathbf{B}^{-\dagger}(q) - I)\mathbf{b}_k$ with covariance matrix $R_{\mathbf{e}^U\mathbf{e}^U} = \sigma_v^2\,\mathbf{M}^{-1}(I - \frac{1}{\rho}\,\mathbf{M}^{-1})$. The feedforward UMMSE filter is then $\mathbf{F}^U(q) = (I - \frac{1}{\rho}\,\mathbf{M}^{-1})^{-1}\,\mathbf{M}^{-1}\mathbf{B}^{-\dagger}(q) = (\mathbf{M} - \frac{1}{\rho}\,I)^{-1}\,\mathbf{P}^\dagger(q)$, whereas the corresponding feedback filter is $\mathbf{B}^U(q) = (I - \frac{1}{\rho}\,\mathbf{M}^{-1})^{-1}(\mathbf{P}^{-1}(q) - I)$. The capacity of such a TX system with UMMSE DFE RX, assuming perfect feedback and joint decoding of the components of $\mathbf{b}_k$, is after some simple manipulations:

$$\mathbf{C} = \frac{1}{2\pi j}\oint\frac{dz}{z}\log_2\det(\rho\,\mathbf{M}) \tag{15}$$

To show that $\mathbf{C}$ is equal to the capacity of the MIMO channel, let's notice that for a minimum/maximum phase monic MIMO filter $A(z)$ ($A_0 = I$), $\frac{1}{2\pi j}\oint\frac{dz}{z}\log\det(A(z)) = 0$. This leads to:

$$\begin{aligned}\mathbf{C} &= \tfrac{1}{2\pi j}\oint\tfrac{dz}{z}\log_2\det(\rho\,\mathbf{B}^\dagger(z)\,\mathbf{M}\,\mathbf{B}(z)) \\ &= \tfrac{1}{2\pi j}\oint\tfrac{dz}{z}\log_2\det(I + \rho\,\mathbf{H}^\dagger(z)\,\mathbf{H}(z))\end{aligned} \tag{16}$$

Hence this decoding strategy conserves the capacity. Finally, for a frequency-flat channel, it can be easily shown that $\mathbf{B}(z) = \mathbf{T}(z)^\dagger\,\mathbf{L}^H\,\mathbf{T}(z)$ where $\mathbf{L}$ results from the LDU decomposition of $\mathbf{H}^H\mathbf{H} + \frac{1}{\rho}I = \mathbf{L}\,\mathbf{D}\,\mathbf{L}^H$.

C. Diversity Order Considerations

The exact analysis is somewhat involved and is omitted here for lack of space, but the following heuristic reasoning leads to the correct result. In this case we jointly detect the N_{tx} components of $\boldsymbol{b}_k$, which in the SIC design case, would each taken

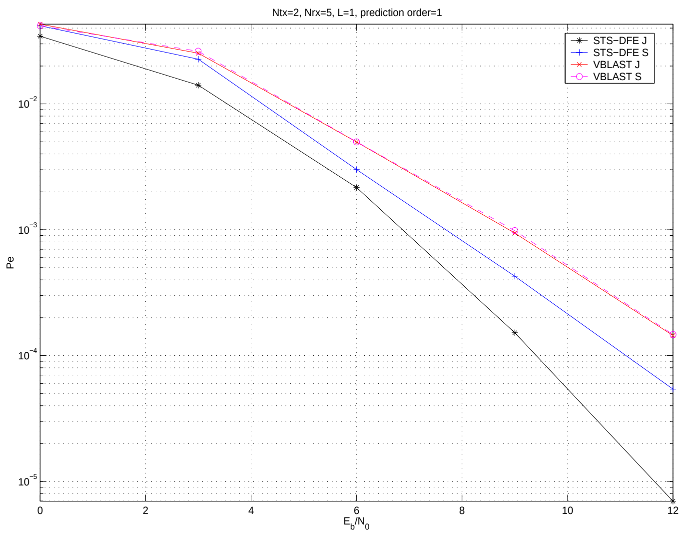

Fig. 5. VBLAST vs. STS-DFE, joint and sequential detection, for $N_{tx} = 2$, $N_{rx} = 5$, $L = 1$, pred. order 1.

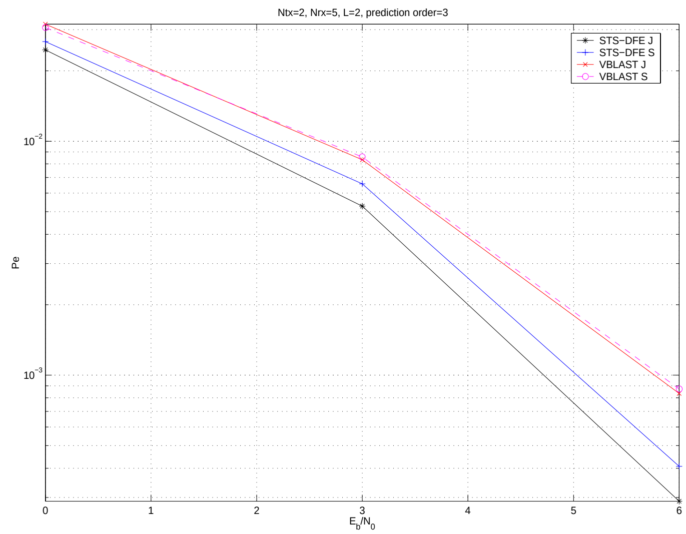

Fig. 6. VBLAST vs. STS-DFE, joint and sequential detection, for $N_{tx} = 2$, $N_{rx} = 5$, $L = 2$, pred. order 3.

separately be subject to varying degrees of diversity as stated earlier. In the different streaming design considered here, due to the joint detection of the components of $\boldsymbol{b}_k$, the resulting diversity degree becomes the average of that of its components, which is $(N_{rx} - \frac{N_{tx}-1}{2})\, N_{tx}\, L$.

VI. Simulations

We wish to evaluate here the performance attainable by the linear precoding and hence we shall consider transmission without channel coding. In this case, the organization of bits/symbols into streams becomes irrelevant (when error propagation in DFEs gets neglected). We can then also easily consider the MIMO DFE RX just considered for the case of channels with delay spread. We shall compare two transmission techniques: VBLAST, which corresponds to $T(z) = I$, and the proposed spatiotemporal spreading (STS) technique corresponding to the convolutional precoder $\mathbf{T}(z) = \mathbf{D}(z^L)\, Q$. We shall also consider two reception techniques corresponding to joint (J) or sequential (S) detection of the N_{tx} components of $\boldsymbol{b}_k$. For VBLAST with a flat channel, the joint technique corresponds to ML detection, with exhaustive evaluation of all combinations of the N_{tx} symbols involved. The sequential processing for VBLAST is the classical VBLAST receiver (triangular DFE). For STS, the receiver will be the MIMO DFE, with either joint detection of the N_{tx} components in $\boldsymbol{b}_k$ by exhaustive evaluation, or sequential detection (which corresponds to LDU factorization of $\mathbf{M}$ and absorbing the triangular factors in feedforward and feedback filters). The results of the simulations are shown in Fig. 5-7 for various configurations of N_{tx}, N_{rx}, channel delay spreads L and prediction filter $\mathbf{P}(z)$ orders. It is clear that STS works better than VBLAST, not only in terms of diversity order at higher SNR, but at all SNRs. Joint processing works better than sequential processing, especially for STS.

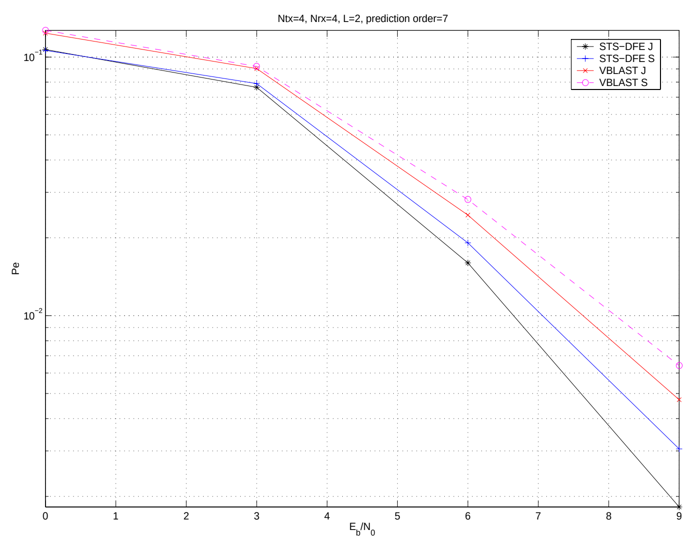

Fig. 7. VBLAST vs. STS-DFE, joint and sequential detection, for $N_{tx} = 4$, $N_{rx} = 4$, $L = 2$, pred. order 7.

References

[1] G.J. Foschini. “Layered Space-Time Architecture for Wireless Communication in a Fading Environment when Using Multi-Element Antennas”. *Bell Labs Tech. J.*, vol. 1, no. 2, pp. 41-59, 1996.

[2] G. Caire and G. Colavolpe. “On space-time coding for quasi-static multiple-antenna channels”. Subm. IEEE Trans. Inform. Theory, March 2001.

[3] V. Tarokh, N. Seshadri, and A.R. Calderbank. “Space-Time Codes for High Data Rates Wireless Comm.: Performance criterion and code construction”. *IEEE Trans. Info. Theory*, vol. 44, no. 2, pp. 744-765, March 1998.

[4] B. Hassibi and B. M. Hochwald. “High-Rate Codes that are Linear in Space and Time”. Submitted to *IEEE Trans. Info. Theory*.

[5] S. Galliou and J.C. Belfiore. “A New Familly of Linear Full-Rate Space-Time Codes Based on Galois Theory”. Draft paper.

[6] I.E. Telatar. “Capacity of Multi-antenna Gaussian Channels”. *Eur. Trans. Telecom.*, vol. 10, no. 6, pp. 585-595, Nov/Dec 1999.

[7] A. Medles and D.T.M. Slock. “Linear Convolutive Space-Time Precoding for Spatial Multiplexing MIMO Systems”. Proc. *39th Allerton Conf. on Comm., Control, and Computing*, Monticello, Illinois, USA, Oct. 2001.

[8] X. Giraud, E. Boutillon and J.C. Belfiore. “Algebraic Tools to Build Modulation Schemes for Fading Channels”. *IEEE Trans. Info. Theory*, vol. 43, no. 3, pp. 938-952, May 1997.

[9] M.O. Damen, A.Chkeif and J.C. Belfiore. “Sphere Decoding of Space-Time codes”. In *IEEE ISIT 2000*, Sorrento, Italy, June 2000.

[10] G. Ginis and J.M. Cioffi. “On the Relation Between V-BLAST and the GDFE”. *IEEE Comm. Letters*, vol. 5, no. 9, pp. 364-366, Sept. 2001.

[11] A. Medles and D. T. M. Slock, “Spatial multiplexing by spatiotemporal spreading of multiple symbol streams,” in *Proc. IEEE Symp. Wireless Personal Multimedia Communications* , Honolulu, Hawaii, Oct. 27-30 2002.

[12] H. El Gamal and A. Roger Hammons, Jr., “A new approach to layered space-time coding and signal processing,” *IEEE Trans. on Information Theory*, vol. 47, no. 6, pp. 2321–2334, Sept. 2001.