A PERFORMANCE ANALYSIS OF INTEGER-TO-INTEGER TRANSFORMS FOR LOSSLESS CODING OF VECTORIAL SIGNALS

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ABSTRACT

An efficient lossless coding procedure should take advantage of the multichannel aspect of some data standards (such as audio standards for example). In [1], the instantaneous decorrelation of several quantized scalar signals is shown to be efficiently realized by a lossless (integer-to-integer) implementation of the Karhunen-Loeve Transform (KLT, unitary transform). The implementation of this integer-to-integer transform involves a cascade of triangular matrices and truncations. We present in this paper a lossless coding procedure based on a recently introduced decorrelating scheme (Lower Diagonal Upper factorization, LDU, causal transform). We define the lossless coding gain for a transformation as the number of bits which are saved by using the corresponding lossless coding scheme. In a first step, we analyze and compare the effects of the truncations on the coding gains for the two transformations. In a second step, we analyze the effects of estimation noise uppon the coding gains : in this case, the transforms are based on an estimate $\widehat{R_{\underline{x}^q\underline{x}^q}}$ of the covariance matrix of the quantized signals $R_{\underline{x}^q\underline{x}^q}$. We find that for stationary Gaussian i.i.d. signals, the coding gains are close to their maxima after a few tens of decoded vectors. Moreover, the LDU based approach is shown to yield the highest coding gain. Theoretical assertions are confronted with simulations results.

1 INTRODUCTION

1.1 Framework of this study

Lossless coding techniques allow to reduce the number of bits required to exactly describe some data in aim of storage, or transmission. An usefull application of a lossless coding scheme is described in figure 1. The lossless coder is embedded in the core of a lossy encoder, whose performance may thereby be improved. In a first step, a very high resolution vectorial source $\underline{x}$ is quantized using a lossy source codec, represented by the box Q (Q may represent the discretization realized by any lossy coder/decoder, e.g. independent uniform scalar quantizers, independent ADPCM or MPEG Audio Codecs...). The set of quantized values $\{x_i^q\}$ obtained in the lossy coder is classically directly entropy coded using independent entropy coders $\{\gamma_i\}$. However, since these coefficients are *independently* entropy coded, it may be worth applying, after the quantization stage, a lossless transformation T. T aims to reduce the intra- and inter-signal dependencies, and hence, by further entropy coding the discrete transformed signals y_i^q, the total bitrate. Indeed, no additional degradation of the signals should occur thereby. Integer-to-integer implementations of optimal linear transforms in

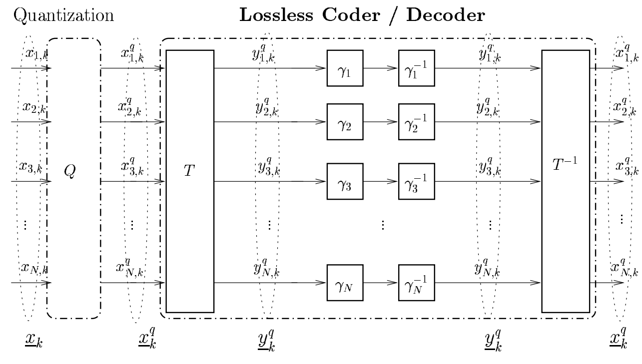

Figure 1: *Transform based lossless coding scheme embedded in a lossy codec.*

the context of transform coding can reduce intersignal dependencies, and may therefore be used in such a scheme. Two of them are reviewed in this work, the optimal causal transform (LDU) and the optimal unitary transform (KLT). The set of operations $\{Q, T, \gamma, \gamma^{-1}, T^{-1}\}$, which represents an "enhanced" lossy codec, constitutes the framework of this work. The field of audio coding appears as a natural space of application for these techniques, which may however be applied to the wide class of the vectorial sources. In the particular case of MPEG-4, MPEG members are now discussing issues in considering lossless audio coding as an extention to the MPEG-4 standard [2, 3].

1.2 Multichannel Audio

An important issue which should be taken into account in the lossless audio coding procedure is the multichannel aspect of recent audio technologies. Starting from the monophonic and stereophonic technologies, new systems (mainly due to the film industry and home entertainments systems) such as quadraphonic, 5.1 and 10.2 channels are now available. An efficient coding procedure aiming storage or transmission of these signals should, as much as possible, take advantage of the correlations between these signals.

Multichannel audio sources can be roughly classified into three categories : signals used for broadcasting, where the channels can be totally different from one to another (e.g. different audio programs in each channel, or the same program in different langages), films soundtracks (typically the format of 5.1 channels) which present a high correlation between certain channels, and finally multichannel audio sources resulting from a recording of the same scene by multiple microphones (in this case, there is indeed a great advantage to be taken from the structure of the multichannel audio signal) [4]. However, the correlations between the different channels are in most of the state of the art lossless audio coders not taken into account at all, or in a basic way only, by computing sums and differences.

1.3 Overview of this work

This work only considers transformations attempting to reduce instantaneous correlations between the scalar signals x_i^q. In the next section, we derive the expression of the ideal lossless coding gain, that is, the maximum coding gain one can expect by a transform making the transformed signal independent, though preserving the whole information about the vectorial source $\underline{x}^q$. The third part compares the gains obtained with two approaches for lossless coding based on approximation of linear transforms, a unitary (KLT) and a causal (LDU) transform. The fourth section is dedicated to estimation noise and derives the coding gains of the two approaches when the transformations are based on an estimate of the covariance matrix. The fifth section exposes and discusses several simulations results.

2 IDEAL CODING GAIN AND MUTUAL INFORMATION

Consider the coding scheme of figure (1). As stated in introduction, the sources x_i are generally not independent, and neither are indeed their quantized versions. Thus, in order to avoid to code any redundancy, one may apply a transform T before entropy coding. The resulting discrete scalar sources $\{y_i^q\}$ are further entropy coded. The transform T is chosen to be invertible so that the decoder can losslessly recover the data $\{x_i^q\}$. The lossless coding gain obtained for the transform T, expressed in bits, may then be defined as

$$G_T = \sum_i^N \left[H(x_i^q) - H(y_i^q)\right], \tag{1}$$

where H denotes (zero*th* order) discrete entropy.

2.1 Ideal Lossless Coding Gain

The question of the maximum coding gain G_T now arises, in other words: how many bits can we expect to save by making the transformed signals independent ?

Since the coding scheme must be lossless, the amount of information about the vectorial source $\underline{x}^q$ conveyed to the decoder must be at least $H(\underline{x}^q)$. Now, ideally, the several signals $\{y_i^q\}$ will be made independent by the transform, that is, the coding scheme will take advantage from a non redundant repartition of the whole information $H(\underline{x}^q)$. In this case, since an invertible transform does not change entropy, the bit rate required to independently code the y_i^q is $\sum_{i=1}^N H(y_i^q) = H(\underline{y}^q) = H(\underline{x}^q)$, which is also the minimum bit rate required to losslessly code the vectorial source $\underline{x}^q$.

Now, the relation of differential to discrete entropy of the uniformly quantized sources with stepsize Δ_i is [5]

$$H(x_i^q) + log_2\Delta_i \rightarrow h(x_i) \;\; as \;\; \Delta_i \rightarrow 0. \tag{2}$$

For the N-vectorial source $\underline{x}$, a similar relation holds (see [1], and [6] for a proof)

$$H(\underline{x}^q) + \sum_{i=1}^N log_2\Delta_i \rightarrow h(\underline{x}) \;\; as \;\; \Delta_i \rightarrow 0\;, i=1,...,N. \tag{3}$$

For Gaussian random variables (r.v.s) x_i, the differential entropy $h(x_i)$ equals $\frac{1}{2}\log_2 2\pi e\sigma_{x_i}^2$. It can then easily be shown that for sufficiently small quantization stepsizes Δ_i, we obtain

$$H(\underline{x}^q) \approx \frac{1}{2}\log_2 \frac{(2\pi e)^N \det R_{\underline{x}\underline{x}}}{\prod_{i=1}^N \Delta_i^2}, \tag{4}$$

where $R_{\underline{x}\underline{x}} = E\; \underline{x}\underline{x}^T$. The maximum coding gain is then

$$\begin{aligned} G_{max} &= \textstyle\sum_{i=1}^N H(x_i^q) - H(\underline{x}^q) \\ &= \textstyle\sum_{i=1}^N h(x_i) - h(\underline{x}) \\ &= \frac{1}{2}\log_2 \frac{\det diag\{R_{\underline{x}\underline{x}}\}}{\det R_{\underline{x}\underline{x}}}, \end{aligned} \tag{5}$$

where $diag\{.\}$ denotes the diagonal matrix made with the diagonal elements of $\{.\}$.

It is now shown that G_{max} is ideal because it corresponds in the Gaussian case to the gain obtained with a linear decorrelating transform placed before the quantizers. By writing $\det R_{\underline{x}\underline{x}} = \prod_{i=1}^N \sigma_{y_i}^2 = \prod_{i=1}^N \lambda_i$, where $\{\sigma_{y_i}^2\}$ and $\{\lambda_i\}$ are respectively the optimal prediction error variance of x_i based on $\underline{x}_{1:i-1}$, and the eigenvalues of $R_{\underline{x}\underline{x}}$, we can write equation (4) as

$$\begin{aligned} H(\underline{x}^q) &\approx \textstyle\sum_{i=1}^N \frac{1}{2}\log_2 2\pi e\sigma_{y_i}^2 - \log_2\Delta_i \\ &\approx \textstyle\sum_{i=1}^N \frac{1}{2}\log_2 2\pi e\lambda_i - \log_2\Delta_i. \end{aligned} \tag{6}$$

Equation (6) shows that the entropy of the vector $\underline{x}^q$ may be written as the sum of the entropies of N independent r.v.s of variances $\{\sigma_{y_i}^2\}$ (or $\{\lambda_i\}$), quantized with quantization stepsizes Δ_i. The required bitrate for independently coding these quantized variables is $H(\underline{x}^q)$: if we apply first a KLT or an LDU to the unquantized source $\underline{x}$, and then quantize the tranformed signals with stepsizes Δ_i , then the minimum bit rate required to entropy code these transformed signals is given by (6). Hence, the gain (5) would be obtained by a classical transform coding scheme by inverting the transformation and quantization operations. As will be illustrated in the next section however, the performance of realizable lossless coding schemes based on approximations of linear transforms must be expected to be lower than the expression (5): indeed, since the transform is placed after the quantizers and just before scalar entropy coders, its output should be discrete valued, which is not the case for optimal linear decorrelating transforms. Thus, truncations are necessary which increase the entropy of the transform signals : $\sum_i H(y_i^q)$ is generally greater than $H(\underline{x}^q)$.

2.2 Lossless Coding Gain and Mutual Information

We now show that the ideal lossless coding gain (5) can be related to the mutual information between quantized r.v.s $\{x_i^q\}$ always under Gaussian assumption.

Suppose we dispose of a set of $i-1$ quantized scalar sources x_j^q, $j = 1...i-1$, and that we wish to code an $i-th$ source x_i^q, which is not independent from the $i-1$ others. Intuitively, the best strategy would be to code the only information contained in the $i-th$ r.v. which is not shared with the $i-1$ previous variables. The mutual information $I(x_i^q;\underline{x}_{1:i-1}^q)$ allows one to quantize this idea : it represents the amount of information that the r.v. x_i^q shares with the $i-1$ others (vector $\underline{x}_{1:i-1}^q$), and is defined by

$$\begin{aligned} I(x_i^q;\underline{x}_{1:i-1}^q) &= H(x_i^q) + H(\underline{x}_{1:i-1}^q) - H(x_i^q,\underline{x}_{1:i-1}^q) \\ &= H(x_i^q) + H(\underline{x}_{1:i-1}^q) - H(\underline{x}_{1:i}^q). \end{aligned} \tag{7}$$

By writing and summing the expressions of the mutual information between x_i^q and $\underline{x}_{1:i-1}^q$ for $i = 2, ..., N$, we obtain

$$\begin{aligned}\sum_{i=2}^N I(x_i^q; \underline{x}_{1:i-1}^q) &= \sum_{i=1}^N H(x_i^q) - H(\underline{x}^q) \\ &= \sum_{i=1}^N h(x) - h(\underline{x}) \\ &= G_{max}.\end{aligned} \tag{8}$$

Thus, the maximum bitrate that can be saved using a lossless coding scheme corresponds to the sum of the mutual information shared between each new random variable and the previous ones.[1]

3 INTEGER-TO-INTEGER TRANSFORMS

Suppose as in Figure (1) that we dispose of N quantized scalar signals x_i^q. Each one of this source is a quantized version of x_i to the nearest multiple of Δ_i (denoted by $[.]_{\Delta_i}$), and takes values in the set $\Delta_i\mathbb{Z}$: $\underline{x}_{i,k}^q = [x_{1,k}^q, x_{2,k}^q, ..., x_{n,k}^q]^T = \left[[x_{1,k}]_{\Delta_1}, [x_{2,k}^q]_{\Delta_2}, ..., [x_{n,k}^q]_{\Delta_N}\right]^T$. An integer-to-integer transform T: $\Delta_1\mathbb{Z} \times \Delta_2\mathbb{Z} ... \times \Delta_N\mathbb{Z} \rightarrow \Delta_1\mathbb{Z} \times \Delta_2\mathbb{Z} ... \times \Delta_N\mathbb{Z}$ associates to each quantized $N-$vector $\underline{x}_{i,k}^q$ an $N-$vector $\underline{y}_{i,k}^q = T\underline{x}_{i,k}^q$. The transformation is chosen to be invertible so that the decoder can losslessly compute the original data by $\underline{x}_{i,k}^q = T^{-1}\underline{y}_{i,k}^q$. Since the aim of the transform T is to make the transform signals independent, the problem of its design is very similar to that of designing the best transformation in a transform coding framework. The transform T can be chosen to approximate linear decorrelating transforms such as the LDU or the KLT, which are optimal for Gaussian signals in the classical transform coding case [7, 8]. However, since the transform must be integer-to-integer, T is only an approximation of the chosen linear decorrelating transform. Although both integer-to-integer implementations tend to the maximum gain of expression (5) for quantization stepsizes arbitrarily small, a quantifiable loss in performance occurs in practical coding situations whose effects are analyzed in the following.

3.1 Integer-to-Integer implementation of the LDU

3.1.1 LDU Transform Coding

Consider a stationary Gaussian vectorial source $\{\underline{x}\}$. This source may be composed of any scalar sources $\{x_i\}$. In the classical transform coding framework, a linear transformation T is applied to each N-vector $\underline{x}_k$ to produce an N-vector $\underline{y}_k = T\underline{x}_k$ whose components are independently quantized using scalar quantizers Q_i. A number of bits r_i is attributed to each Q_i under the constraint $\sum_i r_i = Nr$. In the case of the LDU transform, the transform vector $\underline{y}_k$ is chosen to be $\underline{y}_k = \underline{x}_k - \widehat{\underline{x}_k} = L\underline{x}_k = \underline{x}_k - \overline{L}\underline{x}_k$, where $\overline{L}\underline{x}_k$ is the reference vector. The output $\underline{x}_k^q$ is $\underline{y}_k^q + \overline{L}\underline{x}_k$. This scheme appears as a generalization to the vectorial case of the classical scalar DPCM coding scheme. As detailed in [8, 7], the optimal L in terms of transform coding gain is such that $LR_{\underline{x}\underline{x}}L^T = diag\{\sigma_{y_1}^2, ...\sigma_{y_N}^2\}$, where $diag\{...\}$ represents a diagonal matrix whose elements are $\sigma_{y_i}^2$. In other words, the components $y_{i,k}$ are the prediction errors of $x_{i,k}$ with respect to the past values of $\underline{x}_k$, the $\underline{x}_{1:i-1,k}$, and the optimal coefficients $-L_{i,1:i-1}$ are the optimal prediction coefficients. Since each prediction error $y_{i,k}$ is orthogonal to the subspaces generated by the $\underline{x}_{1:i-1,k}$, the $y_{i,k}$ are orthogonal. It follows that $R_{\underline{x}\underline{x}} = L^{-1}R_{\underline{y}\underline{y}}L^{-T}$, which represents the LDU factorization of $R_{\underline{x}\underline{x}}$.

[1] Note also from (8) that high resolution quantizing does not change mutual information, which comes from the whiteness and independence of the quantization noises.

3.1.2 Integer-to-Integer Implementation of the LDU

In a first step, the linear transform $L^q = I - \overline{L^q}$ is optimized to decorrelate the quantized data x_i^q. Thus, we look for

$$\min_{L_{i,1:i-1}^q} L_i^q(R_{\underline{x}^q\underline{x}^q})L_i^{qT}, \tag{9}$$

which leads to the normal equations

$$\begin{bmatrix} & & \\ & R_{\underline{x}^q\underline{x}^q\,1:i,1:i} & \\ & & \end{bmatrix} \begin{bmatrix} L_{i,i-1}^q \\ \vdots \\ L_{i,1}^q \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \sigma_{y_i'}^2 \end{bmatrix},$$

where $\sigma_{y_i'}^2$ is the optimal prediction error variance corresponding to the optimal (continuous valued) prediction error $y_{i,k}' = x_{i,k}^q - L_{i,1:i-1}^q\underline{x}_{1:i-1,k}^q = x_{i,k}^q - \widehat{x_{i,q}^q}$. The optimal transform vector is then $\underline{y}'_k = \underline{x}_k^q - \overline{L^q}\underline{x}^q$, and the optimal transform L^q corresponds in this case to the LDU factorization of the covariance matrix of quantized data $R_{\underline{x}^q\underline{x}^q}$

$$R_{\underline{x}^q\underline{x}^q} = L^{q-1}R_{\underline{y}'\underline{y}'}L^{q-T}. \tag{10}$$

The second step is now to design an integer-to-integer approximation L_{int}^q of L^q which allows one to keep the transform structure lossless. This can easily be realized by rounding each estimate $\widehat{x_{i,q}^q}$ of $x_{i,k}^q$. Each transform coefficient is computed by

$$y_{i,k}^q = x_{i,k}^q - [\widehat{x_{i,k}^q}]_{\Delta_i} = x_{i,k}^q - [L_{i,1:i-1}^q\underline{x}_{1:i-1,k}^q]_{\Delta_i}, \tag{11}$$

see Figure(2). Let us denote by L^{q_i}, $i = 2, ..., N$, the matrix

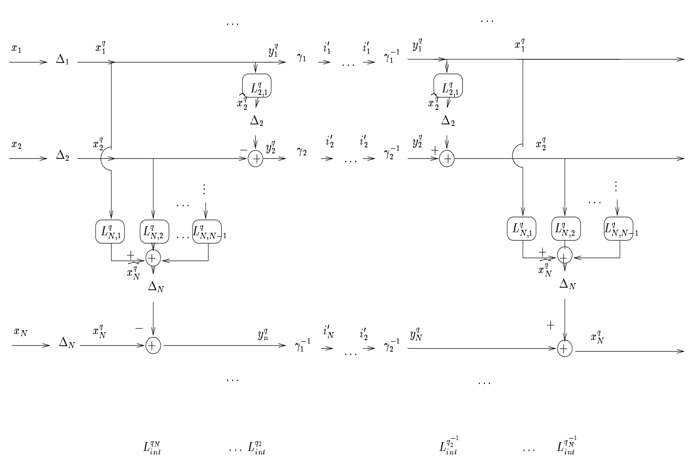

Figure 2: Lossless implementation of the LDU transform.

whose non zeros off diagonal elements correspond to the $i-th$

optimal predictor

$$L^{q_i} = I - \overline{L^{q_i}} = \begin{bmatrix} 1 & & & & & & & \\ 0 & \ddots & & & & \mathbf{0} & & \\ \vdots & \ddots & \ddots & & & & & \\ 0 & \cdots & 0 & \ddots & & & & \\ L^q_{i,1} & \cdots & \cdots & L^q_{i,i-1} & \ddots & & & \\ 0 & \cdots & & \cdots & 0 & \ddots & & \\ \vdots & & & & & \ddots & \ddots & \\ 0 & \cdots & & & & \cdots & 0 & 1 \end{bmatrix}. \tag{12}$$

Then a lossless implementation $L^{q_i}_{int}$ of L^{q_i} is obtained by $\underline{y}^q_k = L^{q_i}_{int}\underline{x}^q_k = I - [\overline{L^{q_i}}\underline{x}^q_k]_{\Delta_i}$. The inverse operation is simply $\underline{x}^q_k = L^{q_i^{-1}}_{int}\,\underline{y}^q_k = I + [\overline{L^{q_i}}\underline{x}^q_k]_{\Delta_i}$. Now, the global transform vector $\underline{y}^q_k$ can then be computed using a cascade of $N-1$ elementary transforms (see figure (2))

$$\begin{aligned} \underline{y}^q_k &= \left[L^{q_2}\left[L^{q_3}\ldots[L^{q_N}\,\underline{x}^q_k]_{\Delta_N}\cdots\right]_{\Delta_3}\right]_{\Delta_2}. \\ &= L^{q_2}_{int}\,L^{q_3}_{int}\ldots L^{q_N}_{int}\,\underline{x}^q_k \\ &= L^q_{int}\underline{x}^q_k. \end{aligned} \tag{13}$$

At the decoder, the inversion is realized by

$$\underline{x}^q_k = L^{q^{-1}}_{int}\,\underline{y}^q_k = L^{q_N^{-1}}_{int}\,L^{q_{N-1}^{-1}}_{int}\ldots L^{q_2^{-1}}_{int}\,\underline{y}^q_k. \tag{14}$$

In order to analyze the effects of the truncations (quantization $[.]_{\Delta_i}$ of the $\{\widehat{x^q_{i,k}}\}$) on the coding gain, let us compute the entropy of the variables y^q_i. Since the source x^q_i is discrete, we have

$$\begin{aligned} y^q_{i,k} &= x^q_{i,k} - \left[\widehat{x^q_{i,k}}\right]_{\Delta_i} \\ &= \left[x^q_{i,k} - \widehat{x^q_{i,k}}\right]_{\Delta_i} \\ &= \left[y'_{i,k}\right]_{\Delta_i}. \end{aligned} \tag{15}$$

Thus, the entropy $H(y^q_i)$ may be written as

$$H(y^q_i) \approx h(y'_i) - \log_2\Delta_i, \tag{16}$$

which assumes a quantization noise uniformly distributed over $[-\frac{\Delta_i}{2}, \frac{\Delta_i}{2}[$ (small quantization stepsizes). The continuous r.v.s $\{y'_i\}$ are not strictly Gaussian since each y'_i is a linear combination of i Gaussian r.v.s and $i-1$ uniform r.v.s. However, since the probability density function of a sum of uniform r.v.s tends quickly to a Gaussian p.d.f, we assume that this is the case, and

$$H(y^q_i) \approx \frac{1}{2}\log_2 2\pi e\sigma^2_{y'_i} - \log_2\Delta_i. \tag{17}$$

Note that in the integer-to-integer implementation of the LDU, the first scalar signal remains unchanged, and only $N-1$ rounding operations are involved in the lossless transformation. The bit rate required to entropy code the discrete r.v.s $\{y^q_i\}$ is then

$$\begin{aligned} \textstyle\sum_{i=1}^N H(y^q_i) \approx\ & \tfrac{1}{2}\log_2(2\pi e)\sigma^2_{x_1} - \log_2\Delta_1 \\ & + \textstyle\sum_{i=2}^N \tfrac{1}{2}\log_2(2\pi e)^{N-1}\sigma^2_{y'_i} - \log_2\Delta_i. \end{aligned} \tag{18}$$

The lossless coding gain for the integer-to-integer LDU may then be written as

$$\begin{aligned} G_{L^q_{int}} &\approx \textstyle\sum_{i=1}^N H(x^q_i) - H(y^q_i) \\ &= \tfrac{1}{2}\log_2 \frac{\prod_{i=2}^N \sigma^2_{x_i}}{\prod_{i=2}^N \sigma^2_{y'_i}} \\ G_{L^q_{int}} &\approx \tfrac{1}{2}\log_2 \frac{\det diag\{R_{\underline{x}\underline{x}}\}}{\sigma^2_{x_1}\prod_{i=2}^N \sigma^2_{y'_i}}, \end{aligned} \tag{19}$$

where subscript L^q_{int} refers to the integer-to-integer implementation of L^q. The last equality shows that $G_{L^q_{int}}$ is indeed inferior to G_{max} of (5) since the denominator involves the optimal prediction error variances obtained from $R_{\underline{x}^q\underline{x}^q} = R_{\underline{x}\underline{x}} + D$ (where D is the diagonal of the variances of the quantization noises whose i-th entry is $D(i,i) = \Delta_i^2/12$) instead of those of $R_{\underline{x}\underline{x}}$.
Moreover, since L^q diagonalizes $R_{\underline{x}^q\underline{x}^q}$, we have $\prod_{i=1}^N \sigma^2_{y'_i} = det R_{\underline{x}^q\underline{x}^q}$, where $\sigma^2_{y'_1} = \sigma^2_{x^q_1}$. Thus the coding gain $G_{L^q_{int}}$ may alternatively be approximated as

$$\begin{aligned} G_{L^q_{int}} &\approx \tfrac{1}{2}\log_2 \frac{\det diag\{R_{\underline{x}\underline{x}}\}}{\left(\sigma^2_{x^q_1} - \frac{\Delta_1^2}{12}\right)\prod_{i=2}^N \sigma^2_{y'_i}} \\ &= \tfrac{1}{2}\log_2 \frac{\det diag\{R_{\underline{x}\underline{x}}\}}{\det R_{\underline{x}^q\underline{x}^q}} + \tfrac{1}{2}\log_2\left(1 + \frac{\Delta_1^2}{12\sigma^2_{x^q_1}}\right) \\ G_{L^q_{int}} &\approx \tfrac{1}{2}\log_2 \frac{\det diag\{R_{\underline{x}\underline{x}}\}}{\det R_{\underline{x}^q\underline{x}^q}} + \frac{\Delta_1^2}{24\ln 2\sigma^2_{x_1}}. \end{aligned} \tag{20}$$

This last expression shows that we should position the most coarsely quantized signal (strongest $\frac{\Delta_i}{\sigma_{x_i}}$) in first position in order to maximize $G_{L^q_{int}}$. Moreover, one can check in the two previous expressions that $G_{L^q_{int}}$ indeed tends to G_{max} as $\Delta_i \to 0$, $i = 1, ..., N$, which means that the transform is optimal in terms of lossless coding gains in the case of negligible rounding effects.
We should here underline the similarity between the integer-to-integer implementation of the LDU and the lossless matrixing described in [9], in which however the diagonalizing aspect of the transform (and thus its optimality for Gaussian signals in the case of negligible perturbation effects) was not established. Moreover, the pertubation effects due to truncations and estimation noise are not, to our knowledge, analyzed in their published related work.

3.2 Integer-to-Integer implementation of the KLT

Concerning the KLT (unitary case), the integer-to-integer approximation is based on the factorization of a unimodular matrix cascaded with rounding operations ensuring the inversibility of the global transform. In [1], this transform was shown to be equivalent to the original KLT for arbitrarily small Δ_i. The loss in the bitrate saving which is due to the rounding operations occuring in actual coding situations was however neglected in [1], and is analyzed here for $N = 2$.
Let us denote by V^q the KLT computed on $R_{\underline{x}^q\underline{x}^q}$. We then have

$$\Lambda' = V^q R_{\underline{x}^q\underline{x}^q} V^{qT}. \tag{21}$$

We denote by λ'_i the variances of the (continuous) transform signals.
We now briefly recall the construction of the integer-to-integer transform based on V^q. As any unimodular transform, V^q can be factored into at most three lower- and upper-triangular matrices with unit diagonal as

$$V^q = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = V_1^q V_2^q V_3^q,$$
$$V_1^q = \begin{bmatrix} 1 & \frac{a-1}{c} \\ 0 & 1 \end{bmatrix}, \; V_2^q = \begin{bmatrix} 1 & 0 \\ c & 1 \end{bmatrix}, \; V_3^q = \begin{bmatrix} 1 & \frac{d-1}{c} \\ 0 & 1 \end{bmatrix}. \tag{22}$$

The transform vector $\underline{y}_k$ is then losslessly obtained by using the integer-to-integer transform V_{int}^q

$$\underline{y}_k^q = V_{int}^q \underline{x}_k^q = \left[\underbrace{V_1^q \left[\underbrace{V_2^q \left[\underbrace{V_3^q \underline{x}_k^q}_{\underline{y}_k^1} \right]_{\Delta_1}}_{\underline{y}_k^2} \right]_{\Delta_2}}_{\underline{y}_k^3} \right]_{\Delta_1}. \tag{23}$$

Since the matrices are triangular, their inverses are simply computed by changing the signs of the off-diagonal elements. In the $N = 2$ case, one can analyze the effects of the truncations at each step of the cascade (23). Denoting by $\delta_{i,j}$ the error due to truncation of the $i-th$ component of the vector $\underline{y}_k^j$, it can easily be shown that the final (discrete valued) transform vector $\underline{y}_k^q$ is obtained by

$$\underline{y}_k^q = \begin{bmatrix} y_{1,k} = \left[x_1^q + \frac{d-1}{c} x_2^q + \delta_{1,1} + \frac{a-1}{c}(cx_1^q + c\delta_{1,1} + dx_2^q + \delta_{2,2})\right]_{\Delta_1} \\ y_{2,k} = [cx_1^q + c\delta_{1,1} + dx_2^q]_{\Delta_2} \end{bmatrix}. \tag{24}$$

Assuming small quantization stepsizes (i.e. the independence of the quantization noises $\delta_{i,j}$, and the Gaussianity of the transformed signals), the discrete entropy of each transformed random variable may be approximated as

$$H(y_1^q) \approx \tfrac{1}{2}\log_2 2\pi e(\underbrace{\lambda_1' + \frac{a^2\Delta_1^2}{12} + \frac{(a-1)^2}{c}\frac{\Delta_1^2}{12}}_{\lambda_1'' = \lambda_1' + e_1}) - \log_2 \Delta_1$$
$$H(y_2^q) \approx \tfrac{1}{2}\log_2 2\pi e(\underbrace{\lambda_2' + \frac{c^2\Delta_1^2}{12}}_{\lambda_2'' = \lambda_2' + e_2}) - \log_2 \Delta_2. \tag{25}$$

Thus, y_i^q may be seen as a continuous r.v. of variance $\lambda_i'' = \lambda_i' + e_i$, quantized with stepsize Δ_i. The terms e_i are the increase in the variance of the transform signals due to the truncations. The corresponding expression for the lossless coding gain in the $N = 2$ case is then

$$\begin{aligned} G_{V_{int}^q} &= \textstyle\sum_{i=1}^2 H(x_i^q) - H(y_i^q) \\ &\approx \tfrac{1}{2}\log_2 \frac{\prod_{i=1}^2 \sigma_{x_i}^2}{\prod_{i=1}^2 \lambda_i''}, \end{aligned} \tag{26}$$

where subscript V_{int}^q refers to the integer-to-integer implementation of V^q. Comparing with the gain obtained for the lossless implementation of the LDU (19) we have $G_{V^q,int} \le G_{L^q,int}$ (this follows from the following series of inequalities $\prod_{i=1}^2 \lambda_i'' \ge \prod_{i=1}^2 \lambda_i' = \prod_{i=1}^2 \sigma_{y_i'}^2 > \sigma_{x_1}^2 \sigma_{y_2'}^2$). Thus the gain for the integer-to-integer KLT is clearly inferior to that of the integer-to-integer LDU for the $N = 2$ case. Indeed, only one triangular transform/truncation is involved in the LDU case whereas three triangular transforms/truncations are generally necessary to losslessly implement the KLT. In the general N-case, the triangular structure of the prediction matrix allows one to implement the lossless causal transform using $N - 1$ truncations (see (13)), which is most probably less than the number required in the unitary case, where the transform matrix has not a triangular structure.

Expression (26) holds for $N = 2$, since the perturbation terms e_i in (25) have been analytically derived in this case only. However, the truncation effects can be similarly analyzed for a general N, and the expression (26) would hold more generally by plugging in the corresponding e_i.

Note also that, as expected, $G_{V_{int}^q}$ tends to G_{max} as Δ_i tends to 0, $i = 1, ..., N$.

4 EFFECTS OF THE ESTIMATION NOISE ON THE LOSSLESS CODING GAINS

We analyze in this section the coding gains of an adaptive lossless coding scheme based on an estimate of the covariance matrix $\widehat{R_{\underline{x}^q\underline{x}^q}} = R_{\underline{x}^q\underline{x}^q} + \Delta R = \frac{1}{K}\sum_{i=1}^K \underline{x}_i^q \underline{x}_i^{qT}$, where K is the number of previously decoded vectors available at the decoder. We suppose independent identically distributed Gaussian real vectors $\underline{x}_i^q$, which is for example the case if the sampling period of the scalar signals is high in comparison with their typical correlation time. (Again, the r.v.s are not strictly Gaussian because of the contribution of the uniform quantization noise. This contribution is however small for a high resolution quantization). Thus, the first and second order statistics of ΔR are known [10] : $(\Delta R)_{ii}$ is, for suficiently high K, a zero mean Gaussian random variable with covariance matrix such that $Evec(\Delta R)\,(vec(\Delta R))^T = \frac{2}{K} R_{\underline{x}^q\underline{x}^q} \otimes R_{\underline{x}^q\underline{x}^q}$, where $\otimes$ denotes the Kronecker product. For each realization of ΔR, the coder computes in a first step the linear transformation $\widehat{T^q}$ ($\widehat{T^q} = \widehat{L^q}$ or $\widehat{V^q}$) which diagonalizes $\widehat{R_{\underline{x}^q\underline{x}^q}}$: $\widehat{T^q}\widehat{R_{\underline{x}^q\underline{x}^q}}\widehat{T^q}^T = \widehat{\Sigma}$. Then, by using the previously exposed factorizations, the coder computes the corresponding integer-to-integer transform $\widehat{T_{int}^q}$.

In order to derive the coding gains for the two approaches in presence of estimation noise, we need the following result [11].

Result Suppose that the transformation $\widehat{T^q}$ ($\widehat{T^q} = \widehat{L^q}$ or $\widehat{V^q}$) is based on an estimate of the covariance matrix $\frac{1}{K}\sum_{i=1}^K \underline{x}_i^q \underline{x}_i^{qT}$. Without estimation noise, the variance of the transform signals obtained by applying the transform T to the quantized signals would be $(T^q R_{\underline{x}^q\underline{x}^q} T^{qT})_{ii} = (\Sigma)_{ii}$. (For the LDU, $(\Sigma)_{ii} = R_{\underline{y}'\underline{y}'}$, and $(\Sigma)_{ii} = \Lambda'$ for the KLT, as in equations (10) and (21)). Now, the actual variances of the signals obtained by applying $\widehat{T^q}$ to $\underline{x}^q$ are $E(\widehat{T^q} R_{\underline{x}^q\underline{x}^q} \widehat{T^q}^T)_{ii} = (\Sigma + \Delta\Sigma)_{ii}$. Then for $\widehat{T^q} = \widehat{L^q}$, $\widehat{V^q}$, it can be shown [11] that for sufficiently high K

$$E \sum_{i=1}^N \frac{(\Delta\Sigma)_{ii}}{(\Sigma)_{ii}} \approx \frac{N(N-1)}{2K}, \tag{27}$$

where E denotes the expectation. We can now derive the gains obtained when the transformations are based on an estimate of the covariance matrix by means of K vectors.

4.1 Coding Gain for the integer-to-integer LDU

One has to compute the difference

$$G_{\widehat{L_{int}^q}}(K) = E \sum_{i=1}^N H(x_i^q) - H(y_i^q, K), \tag{28}$$

where only the entropies $\{H(y_i^q, K)\}$ of the discrete variables $\{y_i^q\}$, obtained by applying $\widehat{L_{int}^q}$ to $\underline{x}^q$, depend on K. Since the variance of the first variable y_1^q is not affected by the transformation, we have

$$E\ H(y_1^q, K) = H(x_1^q) \approx \frac{1}{2}\log_2(2\pi e)\sigma_{x_1}^2 - \log_2 \Delta_1. \quad (29)$$

Concerning the $N-1$ remaining r.v.s $\{y_i^q\}$, they may be seen as r.v.s obtained by applying $\widehat{L^q}$ to $\underline{x}^q$, and then by quantizing the resulting continuous valued r.v.s with stepsize Δ_i. Thus, by denoting $(\widehat{L^q} R_{\underline{x}^q\underline{x}^q}\widehat{L^q})_{ii} = (R_{\underline{y}'\underline{y}'})_{ii} + \Delta(R_{\underline{y}'\underline{y}'})_{ii}$, we have

$$\begin{aligned}
&E\ H(y_i^q, K) \approx E\ \tfrac{1}{2}\log_2(2\pi e)(\widehat{L^q} R_{\underline{x}^q\underline{x}^q}\widehat{L^q})_{ii} - \log_2 \Delta_i \\
&= \tfrac{1}{2}\log_2\left((2\pi e(R_{\underline{y}'\underline{y}'})_{ii}\left(1 + \tfrac{\Delta(R_{\underline{y}'\underline{y}'})_{ii}}{(R_{\underline{y}'\underline{y}'})_{ii}}\right)\right) - \log_2 \Delta_i \\
&\approx \tfrac{1}{2}\log_2 2\pi e(R_{\underline{y}'\underline{y}'})_{ii} - \log_2 \Delta_i + \tfrac{1}{2\ln 2}E\ \tfrac{\Delta(R_{\underline{y}'\underline{y}'})_{ii}}{(R_{\underline{y}'\underline{y}'})_{ii}}.
\end{aligned} \quad (30)$$

Thus, we have

$$\begin{aligned}
&E\textstyle\sum_{i=1}^N H(y_i^q) \approx \tfrac{1}{2}\log_2(2\pi e)\sigma_{x_1}^2 - \log_2 \Delta_1 \\
&+\textstyle\sum_{i=2}^N \tfrac{1}{2}\log_2(2\pi e)^{N-1}\sigma_{y'_i}^2 - \log_2 \Delta_i + \sum_{i=2}^N \tfrac{1}{2\ln 2}E\ \tfrac{\Delta(R_{\underline{y}'\underline{y}'})_{ii}}{(R_{\underline{y}'\underline{y}'})_{ii}}.
\end{aligned} \quad (31)$$

Comparing with the bit rate required to code the y_i^q when the transformation is not perturbed (18), the last term corresponds to an excess bit rate due to estimation noise. Using the result (27), and the fact that $E\ \Delta(R_{\underline{y}'\underline{y}'})_{11} = 0$, this term may be written as

$$\sum_{i=2}^N \frac{1}{2\ln 2}E\ \frac{\Delta(R_{\underline{y}'\underline{y}'})_{ii}}{(R_{\underline{y}'\underline{y}'})_{ii}} = \frac{1}{2\ln 2}E\sum_{i=1}^N \frac{\Delta(R_{\underline{y}'\underline{y}'})_{ii}}{(R_{\underline{y}'\underline{y}'})_{ii}} \approx \frac{N(N-1)}{4\ln 2K}. \quad (32)$$

Finally, the lossless coding gain for an integer-to-integer implementation of the LDU when the transform is based on K observed vectors may be approximated as

$$\begin{aligned}
G_{\widehat{L^q_{int}}}(K) &= E\ \textstyle\sum_{i=1}^N H(x_i^q) - H(y_i^q, K) \\
&\approx G_{L^q_{int}} - \tfrac{N(N-1)}{4\ln 2K}.
\end{aligned} \quad (33)$$

for high K and under high resolution assumption.

4.2 Lossless Coding Gain for the integer-to-integer KLT

In this case, one has to compute the difference

$$G_{\widehat{V^q_{int}}}(K) = E\ \sum_{i=1}^N H(x_i^q) - H(y_i^q, K), \quad (34)$$

where only the entropies $H(y_i^q, K)$ of the discrete variables y_i^q, obtained by applying $\widehat{V^q_{int}}$ to $\underline{x}^q$, depend on K.
Leading a similar analysis as in previous subsection (see [6] for details), the lossless coding gain with estimation noise for the integer-to-integer KLT may be approximated as

$$\begin{aligned}
G_{\widehat{V^q_{int}}}(K) &\approx \tfrac{1}{2}\log_2 \tfrac{\det diag\{R_{\underline{x}\underline{x}}\}}{\det R_{\underline{x}^q\underline{x}^q}} - \tfrac{1}{2\ln 2}\textstyle\sum_{i=1}^N \tfrac{e_i}{\lambda_i^q} - \tfrac{N(N-1)}{4\ln 2K} \\
&\approx G_{V^q_{int}} - \tfrac{N(N-1)}{4\ln 2K},
\end{aligned} \quad (35)$$

under high resolution assumption and for sufficiently high K. As in section 3.2, this expression holds for $N = 2$ (in which case we have derived analytically the gain $G_{V^q_{int}}$), but would hold more generally with the corresponding $G_{V^q_{int}}$.

5 SIMULATIONS

5.1 Lossless Coding Gains without estimation noise

5.1.1 Results for $N = 2$.

In order to check the accuracy of the theoretical results, we generated real Gaussian vectors with covariance matrix $R_{\underline{x}\underline{x}}$ (covariance matrix of a first order autoregressive process with normalized crosscorrelation coefficient ρ). These vectors were quantized using the same normalized quantization stepsize $\frac{\Delta}{\sigma_x}$. This experiment was made for several values of $\frac{\Delta}{\sigma_x}$, and the optimal decorrelating transformations L^q and V^q were computed using the corresponding covariance matrix $R_{\underline{x}^q\underline{x}^q}$. The integer-to-integer transforms L^q_{int} and V^q_{int}, based on the transforms L^q and V^q were then implemented and used to compute the transformed data $\underline{y}_i^q$. We realized this experience ten times for each $\frac{\Delta}{\sigma_x}$ and averaged the different obtained gains. These gains are plotted in figure (3) versus $\frac{\Delta}{\sigma_x}$. The theoretic maximum coding gain is related

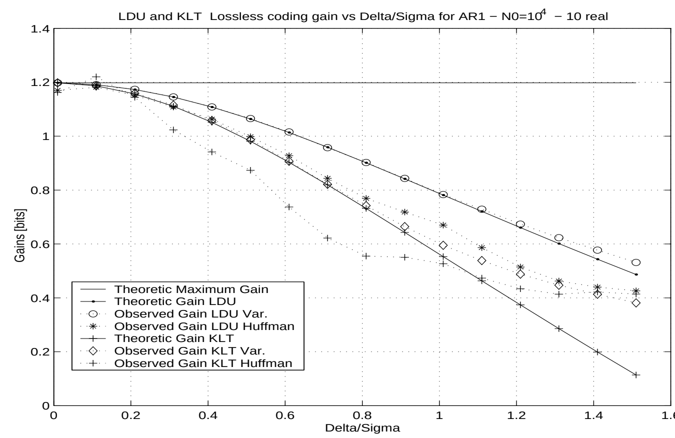

Figure 3: Lossless coding gains for the integer-to-integer implementations of the LDU and KLT vs quantization stepsize. $N = 2$, $\rho = 0.9$.

to the mutual information between the unquantized variables as defined in (8). The theoretic gains for LDU and KLT are then given by (19) and (26) respectively. The observed lossless coding gains were then computed in two different ways. The first one is to measure the actual variances of the transformed signals, (averaged estimates) and computing the corresponding gain by (19) and (26) These gains are referred to by "Observed Gain $\{KLT, LDU\}$ Var." in figure (3). The second way is obviously to design Huffman codes for the quantized signals $\{x_i^q\}$, and then Huffman codes for the signals $\{y_i^q\}$ [12]. Since the average length of the codewords will be close to the entropy of each scalar source, the difference of the average codelengths gives a precise insight of how many bits are saved by using an integer-to-integer transform. These gains are referred to by "Observed Gain $\{KLT, LDU\}$ Huffman" in figure (3).
It first can be seen that for high resolution (small values of $\frac{\Delta}{\sigma_x}$), the predicted gains correspond well to the observed ones. The rounding effects due to the lossless implementation of the transforms can indeed be seen to increase as the quantization gets coarser. The observed coding gains based on the estimates of the variances of the transformed signals correspond well to the predicted ones for a wide range of coding situations (up to $\frac{\Delta}{\sigma_x} \approx 1$). When the quantization

becomes even coarser, the quantization noises are not independent anymore, and the mutual information between the quantized variables $\{x_i^q\}$ is superior to the theoretical one. (Simulations [6] indicate that the assumption of independence of the quantization noises is reasonable up to $\frac{\Delta}{\sigma_x} \approx 1$). When these dependencies become not negligible, the transforms take more advantage of the information shared by the quantized variables, and the gains are slightly superior to the predicted ones.
Figure (3) shows also that the gains based on a Huffman coding of the losslessly transformed signals are slightly lower than those given by the theory, and than the gains based on variance estimates. This may be explained as follows. We supposed theoretically that the relation of differential to discrete entropy is given by (2), which is a high resolution approximation. [6] shows that the actual entropy is greater than the theoretical one, and that this mismatch grows with $\frac{\Delta}{\sigma}$. Now, our theory predicts the same relationship between discrete and differential entropies for the variables x_2^q and y_2^q. The latter, however, may be seen (see Section 3) as the optimal prediction y_2' of x_2^q quantized with stepsize Δ_2. Thus, relatively to its own standard deviation, y_2' is more coarsely quantized than x_2, and the actual entropy of the quantized r.v. y_2^q is greater than the predicted one. This mismatch is greater than the mismatch between predicted and observed entropies for x_2^q. A similar analysis can be made for the transform signals obtained with a lossless implementation of the KLT (where the mismatch is even greater since the lowest variance is generally lower than $\sigma_{y_2'}^2$). As a conclusion,

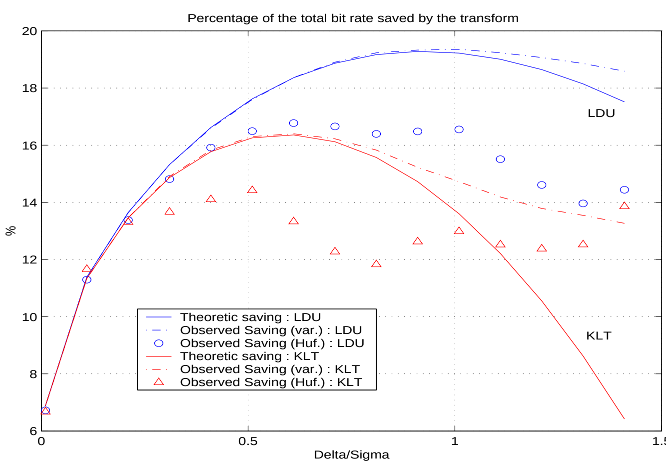

Figure 4: Percentage of the total bit rate saved by using integer-to-integer transform. $N = 2$ $\rho = 0.9$

the curves obtained for $N = 2$ correspond well to the predicted results for a high resolution quantization. Figure (4) illustrates the percentage of the total bit rate which can be saved in this case ($N = 2$) by using these integer-to-integer transforms. It shows that a non negligible part of the bit rate (until $\approx 15 - 17\%$ in this case) can be saved. Finally, the lossless implementation of the LDU yields higher coding gains than that of the KLT.

5.1.2 Position of the first signal

Figure (5) illustrates the codings gains obtained for the integer-to-integer LDU applied to two scalar sources of unit variance, versus their crosscorrelation coefficient ρ. In the first case, denoted by "1" in the legend, the first signal x_1 is

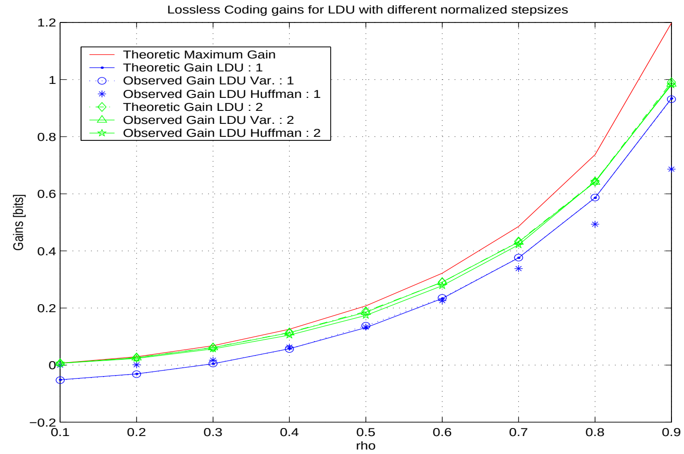

Figure 5: Importance of the position of the first signal.

quantized with stepsize $\Delta_1 = 0.1$ and the second signal x_2 with stepsize $\Delta_1 = 1$. In the second case, denoted by "2" in the legend, the stepsizes are 1 for x_1 and 0.1 for x_2. The curves show as expected that the most coarsely quantized signal (case 2) must be placed in first position in order to maximize the lossless coding gain.

5.1.3 Results for $N > 2$.

The coding gains obtained for the integer-to-integer LDU with $N = 5, \Delta = 0.51$ are presented in figure (6). In this

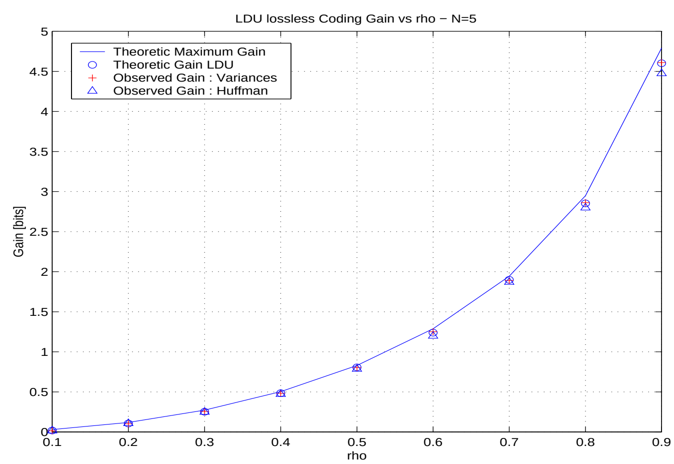

Figure 6: Lossless Coding Gain for integer-to-integer LDU with $N = 5$. $\Delta = 0.51$.

case, the data are real Gaussian i.i.d. vectors with covariance matrix $R = HR_{\underline{xx}}H^T$. H is a diagonal matrix whose ith entry is $(i)^{2/3}$ (the coarseness of the quantization decreases as i increases). It can be seen that the predicted gains match well the observed ones. In particular, the previously exposed mismatch between the predicted gains and the observed gains based on Huffman coding concerns only the few first y_i^q because the stepsize Δ becomes, as i grows, relatively smaller comparatively to the standard deviation of the prediction error y_i'. Thus, this mismatch becomes negligible relatively to the total gain.

5.2 Coding Gains with estimation noise

In a first experiment (figure (7)), the data were generated as in section 5.1.1 ($N=2$, $\frac{\Delta_i}{\sigma_{x_i}}=0.51$). The coding gain G_{max} refers to the mutual information given by (8). The theoretic gains for LDU and KLT are given by (33) and (35) respectively (gains referred to as "G(K) Transform Asymptotic"). The observed coding gains are, as in the previous section, whether based on the estimates of the variances of the transform signals (gains referred to as "G(K) observed variances"), whether based on the actual gain computed by Huffman coding. In this case, a Huffman code is designed for the signals obtained with integer-to-integer transforms based on an estimate of the covariance matrix of quantized data $\widehat{R_{\underline{x}^q\underline{x}^q}}$ with K vectors. The theoretic curves correspond

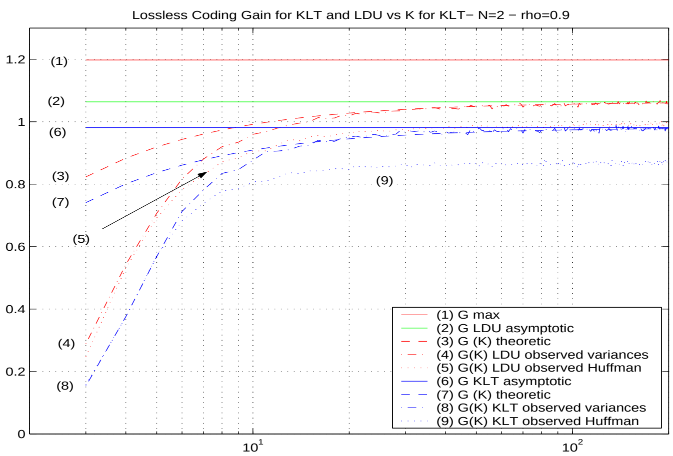

Figure 7: Coding Gains with estimation noise versus K for N=2, $\frac{\Delta_i}{\sigma_{x_i}}=0.51$.

well to the observed ones for the observed gains based on variances estimates for $K \approx$ a few tens. The slight mismatch between Huffman based and variance based observed gains is due to the overestimation explained in the previous subsection. Huffman based and variance based observed gains reach 90% of their maximal value for $K \approx 10$ decoded vectors. That is, basing our conclusions on averaged codewords lengths obtained with a Huffman code, 90% of 16% of the total bit rate can be saved for entropy coding each 2-vector as soon as $K \approx 10$ in the case of the integer-to-integer LDU. In the case of the integer-to-integer KLT, 90% of 15% of the total bit rate can be saved for a comparable estimation noise. Figure (8) plots the lossless coding gain with estimation noise versus K for the same type of data as in the previous subsection ($5.1.3: N=5,\ \Delta=0.51,\ \rho=0.9$). Theoretic and observed gains correspond well for $K \approx$ a few tens.

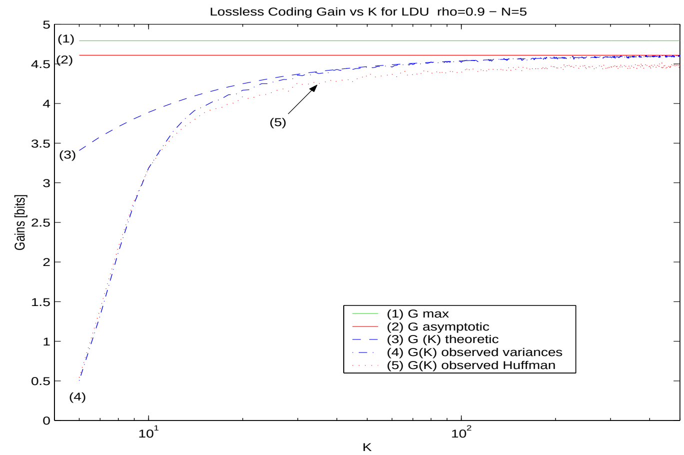

Figure 8: Coding Gains with estimation noise versus K for N=5. $\Delta = 0.51$.

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