

# On the Spatial and Temporal Degrees of Freedom of UWB Communications

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**Abstract:** Ultra-Wide Band (UWB) was presented as a promising radio technology. Because of their large bandwidth, UWB networks are supposed able to deliver high data rates at short ranges. But after analyzing measurements many works observed saturation of the number of channel Degrees of Freedom (DoF) versus bandwidth. This motivates the work presented in this paper where we focus on sub space behavior to analyze the relationship between spatial and temporal resolution for UWB channel. This work presents also a study of the UWB propagation mechanism and its impact on the limitation of the maximum number of channel degrees of freedom.

## 1. Introduction

The large bandwidth facilitated in Ultra Wide Band systems promises high potentials regarding capacity and flexibility. To exploit these potentials, reliable models of the UWB radio channel are necessary. When a communication system utilizes a sufficiently narrow bandwidth around a carrier frequency, the propagation of electromagnetic waves is commonly approximated by a monochromatic wave, i.e., a wave with a constant wavelength equal to the carrier frequency. Most of the previously published channel models are based on the constant wave-length approximation. However, in UWB systems, the fractional bandwidth is typically more than 25 %, making the constant wavelength approximation invalid when such systems are considered. Thus the channel models need to be revised to describe the different propagation effects encountered in the UWB radio channel.

Several authors have studied the effect of the non-constant wavelength on wave propagation phenomena [1, 2, 4, 5]. In these works, the behavior of the Fresnel zone for (ultra) wideband communication was investigated. The Fresnel zone defines a volume in which the most significant portion of the transmitted energy is carried. Berkhout in 1984 [1], Knapp in 1991 [4] and Brühl in 1996 [2] showed that the concept of the first Fresnel zone can be extended to broadband signals, which can be classified, as ultra wide band signals. According to Brühl's definition, the wide-band Fresnel zone is defined as the area around a specular point which leads to maximum (reflected) energy. It is shown in [2] that the energy reaching the receiver via a reflector saturates at some point when the bandwidth of the signal is increased. This saturation starts from a certain critical wavelength corresponding to a combination of all the wideband signal wavelengths. Following [5], this critical wavelength is difficult to calculate.

For ultra wide band signals, these results mean that there is some critical bandwidth that allows to capture

the maximum energy at the receiver and there is no need to increase the bandwidth more because there is no gain. To understand this idea, let us consider a channel impulse response with  $N$  resolvable independents paths, the energy is given by:

$$E_N = \sum_{i=1}^N \|h_i\|^2 \quad (1)$$

where  $h_i$  is the path gain. So if the captured energy at the receiver saturates, then the critical bandwidth that ensures the minimum temporal resolution to capture the total energy was reached. In other words, it means that the additional paths that will be extracted if we increase the bandwidth more than this critical value will be dependent on the previously extracted ones. We can then conclude, based on [5] results, that for wideband communication, one good approximation is to consider the Fresnel zone corresponding to the critical wavelength. Motivated by previous works, we present in this paper a study on the available Degrees of Freedom (DoF) in spatial and temporal domains for ultra wide band communications, to explain the saturation of the DoF versus the bandwidth observed in many works [7, 8].

## 2. System model

Figure 1 illustrates the underlying model of UWB radio SISO system operating at bandwidth  $B = [f_{min}, f_{max}]$  with  $f_{min}$  and  $f_{max}$  corresponding respectively to  $\lambda_{max}$  and  $\lambda_{min}$ . The wave propagates from

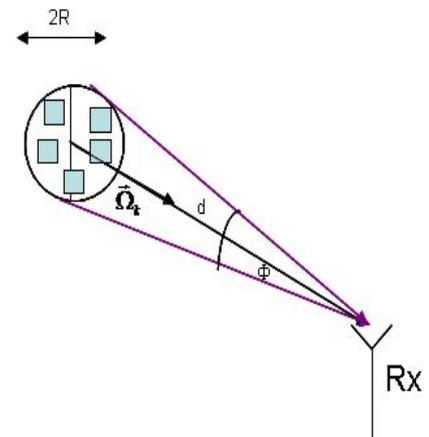


Figure 1: Cluster definition

transmitter to receiver. Along its path a wave interacts with a certain number of scatterers. Following [9], we assume that the far field conditions holds and a group

of close scatterers could be seen as confined in a region called cluster and denoted  $\mathcal{C}_k$ , with  $k = 1, \dots, L$  for  $L$  clusters involved in the whole propagation process for some considered environment. Let us focus on the radio propagation process between one  $\mathcal{C}_k$  region and the receiver element. Using a spherical coordinate system at an arbitrary origin  $\mathcal{O}_k$  located in a  $\mathcal{C}_k$  region, the position of an arbitrary spatial point  $\mathcal{P}m$  is determined by a normalized position vector,  $\vec{r}_{k,m} \in \mathcal{R}^3$ , to the critical wavelength  $\lambda_c$ . We describe a wave direction as a unit vector  $\vec{\Omega}_k$  with its initial point located on a sphere of unit radius centered at the reference point [3]. This unit vector  $\vec{\Omega}_k$  is uniquely determined by its spherical coordinates  $(\theta, \phi) \in [0, \pi] \times [0, 2\pi]$  according to  $\vec{\Omega}_k = [\cos(\phi) \sin(\theta), \sin(\phi) \sin(\theta), \cos(\theta)]^T$ . The angles  $\theta$  and  $\phi$  refer respectively to the azimuth and the co-elevation of  $\vec{\Omega}_k$ . The response of this system is defined by the vector  $c_k(\vec{\Omega}_k) \propto \exp(-j2\pi\vec{\Omega}_k \cdot \vec{r}_{k,m})$ . Here  $(\cdot)$  denotes a scalar product. For unpolarized antenna and following [10], the study of degrees of freedom of one side radio communication (transmitter to cluster or cluster to receiver) is equivalent to studying the kernel:

$$a(\vec{\Omega}_k, \vec{r}_{k,m}) = \exp(-j2\pi\vec{\Omega}_k \cdot \vec{r}_{k,m}), \vec{\Omega}_k \in \Omega_k \times \mathcal{V}_k \quad (2)$$

with  $\Omega_k$  and  $\mathcal{V}_k$  representing respectively the solid angle corresponding to all paths coming from cluster  $\mathcal{C}_k$  and the neighborhood describing all positions  $\mathcal{P}m$ . In the following, we consider that only the circular surface orthogonal to the unit vector  $\vec{\Omega}_k$  and corresponding to the sphere representing the cluster is involved in the propagation process.

### 3. Spatial UWB channel Degrees of freedom

Based on previous explanations, we will focus only on independent paths reaching the receiver. We define UWB Fresnel zone as the area around a specular point, which leads to maximum (reflected) energy. Each such UWB Fresnel zone represents one independent path or one DoF and each independent path can be seen as a coherent combination of unresolvable sub-paths belong to the same UWB Fresnel zone. The radius  $R_F$  of UWB Fresnel zone corresponding to the critical wavelength  $\lambda_c$  is given by next equation [5]:

$$R_F = \sqrt{\frac{z\lambda_c}{2}} \quad (3)$$

where  $z$  is the half distance between the transmitter and the receiver with  $z \gg \lambda_c$ . For uwb channels, wavelengths are very small and first Fresnel zones represent small areas. Then sub-paths have a close amplitudes and phases and are dependent. One path can then be represented by a profile or a shape generated by dependent sub-paths coming from the same Fresnel zone. Two independent paths represent two different Fresnel zones. We can see the surface representing the cluster as a circular pattern generating  $N$  independent paths as shown in Figure 2. In real channels, the small circular zones are not disjoint and there is a continuity at their borders.

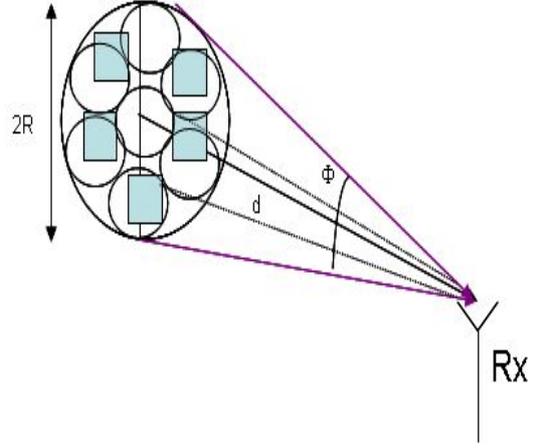


Figure 2: Maximum spatial resolution due to limited Fresnel zone at cluster  $\mathcal{C}_k$

Consider the xy-plane of radius  $R_n$  normalized to the wavelength. Expressing the position vector  $\vec{r}_{k,m}$  in spherical coordinates [10]

$$\vec{r}_{k,m} = r[\cos(\phi') \sin(\theta'), \sin(\phi') \sin(\theta'), \cos(\theta')]^T \quad (4)$$

with  $r$  the normalized distance to a position  $\mathcal{P}m$  in the circular surface, with respect to the wavelength, and  $(\theta', \phi') \in [0, \pi] \times [0, 2\pi]$ . Tse [10] referring to [11] show that for a circular pattern with a normalized radius  $R_n$  (in his case, pattern represent one MIMO circular array), the kernel of eq. (2) could be expressed as:

$$a(\vec{\Omega}_k, \vec{r}_{k,m}) = a(\phi, \phi'), \quad (\phi, \phi') \in \Phi_k \times [0, 2\pi], \quad (5)$$

$$a(\phi, \phi') = \sum_{n=-2\pi R_n}^{2\pi R_n} j^n J_n(2\pi n R_n) \exp(jn\phi + jn\phi'),$$

where  $J_n(\cdot)$  is the Bessel function of the first kind and  $n$ th order, and  $\Phi_k$  represents the azimuth range corresponding to the cluster solid angle  $\Omega_k$ . He concluded that the resolution of such spatial limited circular array is  $1/2R_n$  and the corresponding sub-space dimension or  $N$  the number of degrees of freedom (DoF) is  $2R_n\Phi_k$  for  $R_n\Phi_k \gg 1$ . As  $R_n = \frac{R}{\lambda}$  and  $R \approx \frac{d_k \cdot \Phi_k}{2}$ , this number of DoF can be expressed as

$$2R_n\Phi_k \approx \frac{d_k \Phi_k^2}{\lambda} \quad (6)$$

In our case, and using Fresnel zone concept, we can resolve at the receiver side  $N$  independent paths. This maximum number is giving by, the ratio between the surface of the cluster  $\mathcal{C}_k$  involved in the propagation process,  $S_C$  with radius  $R$ , and the surface of Fresnel zone,  $S_F$  with radius  $R_F = \sqrt{d_k \lambda_c / 2}$  with  $d_k / \lambda_c \gg 1$ , corresponding to critical wavelength  $\lambda_c$ :

$$N = \frac{2S_C}{S_F} \quad (7)$$

$$= \frac{d_k \Phi_k^2}{\lambda_c} \quad (8)$$

The factor 2 in eq. (6) is due to the fact that each spatial degree of freedom is complex and corresponds to two real DoF. So, we can conclude from the above derivations, that the maximum number of spatial DoF corresponds to the maximum number of separated subset of waves coming from the cluster. The term subset here means a group of non separable waves travelling in the same Fresnel zone. To face this limitation of spatial resolution and so the limitation of the total number of temporal DoF, one has to follow what is used in imaging community based on seismic techniques and called migration techniques. This approach is analogous to Synthetic Aperture Radar (SAR) and is applied in high spatial resolution applications. For wireless mobile communications, this can be seen as a MIMO technique.

#### 4. Relationship between temporal and spatial UWB channel Degrees of freedom

Following previous explanations, we can say that for UWB channels, there is a critical wavelength  $\lambda_c$  that limits the spatial resolution and then limits the channel temporal resolution  $\Delta t$ . In other words, if we consider a channel impulse response of UWB channel defined within the interval  $[\lambda_{min}, \lambda_{max}]$ , there is a critical wavelength  $\lambda_c$ , that will limit the maximum number of DoF (independent paths) reaching the receiver from a given cluster  $\mathcal{C}_k$ . As mentioned in Section I, the exact expression of  $\lambda_c$  is difficult to derive, [5] used the geometrical mean ( $\lambda_g = \sqrt{\lambda_{min} \cdot \lambda_{max}}$ ) of  $\lambda_{min}$  and  $\lambda_{max}$  and mentioned that this overestimate  $\lambda_c$ . In his case this over estimation was about 8%. Focusing on the study of path loss exponent for UWB channel modeling, [12] derived the expression of breakpoint distance versus an equivalent wavelength. The breakpoint was defined in [13] as the distance at which the path length difference between the waves coming from the center and the edge of the first Fresnel zone is equal to half a wavelength. [12] found that the evolution of the breakpoint is directly linked to the average wavelength given by

$$\lambda_c = \frac{\lambda_{min} + \lambda_{max}}{2} \quad (9)$$

#### 5. Numerical Results

For numerical simulations, we used the database measurements performed at Eurecom and described in [7]. In Figure 3, we plot the evolution of number of DoF versus bandwidth for an LOS indoor UWB channel. As we can see from this figure, the number of DoF does not scale linearly with the bandwidth. We plot also in Figure 4 the evolution of the critical wavelength computed using equation (9). This figure shows that the average wavelength tends to saturate with the bandwidth but to understand clearly the saturation of DoF observed in Figure 3, we need to analyze also the evolution of cluster solid angle for UWB frequency range. In [14], the authors showed in Figure 5, for indoor channels, the evolution of the angle spread versus frequency ranging from  $2GHz$  to  $8GHz$ . We can conclude from this work that the angle spread decreases when we increase the frequency but starts to saturate from frequency equal to  $4GHz$ . As,

in eq. (8), the number of DoF is directly related to the square of the solid angle, this will have a strong effect for the lowest frequencies but for highest frequencies, the effect of the critical wavelength will be more crucial.

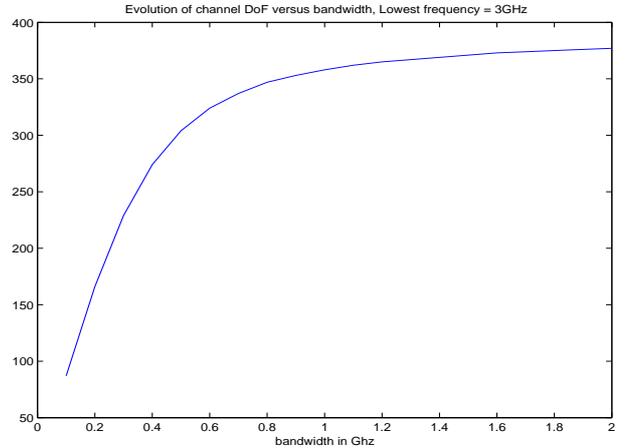


Figure 3: Evolution of number of DoF versus bandwidth

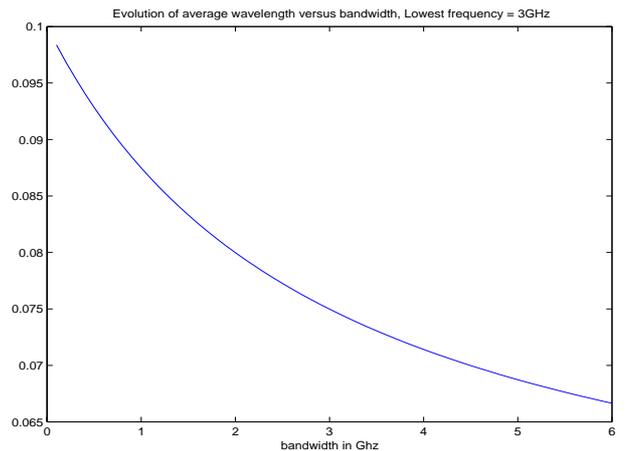


Figure 4: Evolution of average wavelength versus bandwidth

#### 6. Conclusion

In this paper, we analyzed the specific propagation of UWB channel and derived a relationship between the maximum number of channel degrees of freedom, the cluster solid angle and the critical wavelength. We also showed, based on numerical analysis, the saturation of the critical wavelength versus the bandwidth which has a critical impact on the limitation of spatial resolution. Furthermore, we proposed to investigate MIMO capabilities to improve the resolution so that to increase the number of channel degrees of freedom.

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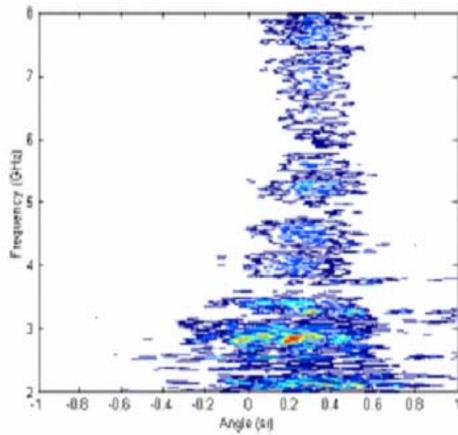


Figure 5: Evolution of cluster angle spread versus bandwidth [14]

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