Brief Announcement: A Dynamic Exchange Game

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ABSTRACT

Our work aims to study a game based on an extended variant of the stable fixtures problem where multiple matches can be established between pairs of players, moreover preference orders are subject to alteration due to player strategies.

Categories and Subject Descriptors: G.2.1: Matching Theory

General Terms: Algorithms, Economics.

Keywords: Stable Matching, Game Theory, Peer-to-Peer.

1. THE STABLE EXCHANGE PROBLEM

We consider a game where $\mathcal{I}$ denotes the player set ($|\mathcal{I}| = n$ is the number of players), $\mathcal{S}$ depicts the collection of strategy sets ($\mathcal{S} = (\mathcal{S}_i)$ for $\forall i \in \mathcal{I}$), $\mathcal{P}$ function gives the player consequences $P = (P_i)$ for $\forall i \in \mathcal{I}$ on the combination of strategy sets ($\mathcal{P} : \mathcal{S}_1 \times \cdots \times \mathcal{S}_n \to \mathcal{R}^n$).

We extend the stable fixtures (SF) problem (presented by Irving and Scott in [2] as a generalization of the stable roommates problem, investigated in [1]) with the possibility of multiple matches between two given players. Therefore player i's strategy is $s_i \subseteq \mathcal{S}_i = \{\{i, j, c_{ij}\} : j \in \{\mathcal{I} \setminus i\}, 0 \leq c_{ij} \leq \min(c_i, c_j)\}$, where c_{ij} (resp. c_i) denotes player i's number of matches towards player j (resp. the maximum number of i's matches). A matching $\mathcal{M}$ is a set of matches $\{i, j, c_{ij}\}$ such that $\{i, j, c_{ij}\} \in s_i$, $\{j, i, c_{ij}\} \in s_j$ and $\sum_{j:\{i,j,c_{ij}\}\in\mathcal{M}} c_{ij} \leq c_i$ holds for $\forall i, j \in \mathcal{I}$; moreover $\mathcal{M}$ is stable if, likewise in the SF problem's case, there is no blocking match, i.e. no match $\{i, j, c'\} \notin \mathcal{M}$, thus $c' > c_{ij}$ for $\forall i, j : (i, j, c_{ij}) \in \mathcal{M}$, such that either i has fewer matches than c_i or $P_i(i, j, c')$ is greater than P_i of at least one of his matches in $\mathcal{M}$; *and* either j has fewer matches than c_j or $P_j(j, i, c')$ is greater than P_j of at least one of his matches in $\mathcal{M}$; where we denote players i and j's c'th *pairwise* match's consequence for i by $P_i(i, j, c')$. To avoid inconsistency in the consequence order of consecutive matches between given players, we assume that $P_i(i, j, c') > P_i(i, j, c'')$ holds for any pair of matches between players i and j if $c' < c''$ for $\forall i, j$.

ASSUMPTION 1. *We investigate the capacity-uniform case, i.e.* $c_i = C$ *for* $\forall i \in \mathcal{I}$*, of the stable exchange problem.*

Let us suppose that the consequence function $\mathcal{P}$ and thus the preference order on $\mathcal{S}$ are defined based on a player parameter set denoted by $\alpha = (\alpha_i)$, such that α_i for $\forall i \in \mathcal{I}$ is a positive scalar of $[0, 1]$. The implications of the α parameter vector on $\mathcal{P}$ are compacted in the following assumption for the capacity-uniform case, i.e. when Assumption 1 holds.

ASSUMPTION 2. *For* $\forall i \in \mathcal{I}$, $\mathcal{P}_i(i, j, c') > \mathcal{P}_i(i, k, c')$ *holds for a given* $c' \leq C$ *for any given pair* $j, k \in \{\mathcal{I}\setminus i\}$ *if, and only if* $\alpha_j > \alpha_k$*. For the case* $\alpha_j = \alpha_k$, $\mathcal{P}_i(i, j, c') = \mathcal{P}_i(i, k, c')$ *for any* $c' \leq C$.

PROPOSITION 1. *At least one stable matching exists for a given exchange problem instance corresponding to Assumptions 1 and 2, and a slightly extended version of Irving's algorithm (presented in [2]) finds it in polynomial time.*

2. THE DYNAMIC EXCHANGE GAME

Let α be a *strategy* variable vector the players can decide on, hence influence the $\mathcal{P}$ function. The setting relates to the uniform exchange problem with Assumptions 1 and 2.[1] A *joint* strategy for player i is a scalar value α_i of $[0, 1]$ *and* an instance $s_i \subseteq \mathcal{S}_i = \{(i, j, c_{ij}) : j \in \{\mathcal{I} \setminus i\}, 0 \leq c_{ij} \leq \min(c_i, c_j)\}$. Every player $i \in \mathcal{I}$ selfishly maximizes her payoff P_i (the optimal player strategy tuple is $(\alpha_i^*, s_i^*) = \arg_i(\max(P_i(\alpha, \mathcal{S})))$ for $\forall i \in \mathcal{I}$), given by $\mathcal{P}$ on α *and* s, i.e. $\mathcal{P} : \alpha \times \mathcal{S} \to \mathcal{R}^n$. In equilibrium $P_i(\{\alpha_i^*, \alpha_{-i}^*\}, \{s_i^*, s_{-i}^*\}) \geq P_i(\{\alpha_i, \alpha_{-i}^*\}, \{s_i, s_{-i}^*\})$ holds for any α_i, s_i and for $\forall i \in \mathcal{I}$, where α_{-i}^* and s_{-i}^* depict the best response counter strategy sets.

The motivation behind this work [3] comes from peer-to-peer backup and storage applications: players are peers characterized by their storage capacities (c) that they share with other peers willing to reciprocate. A globally known peer profile (α) indicates the peer reliability: it is assumed to be observed, maintained and advertised by the peer set for all participants. Here, matching represents the peer selection overlay.

3. REFERENCES

[1] R. W. Irving. An efficient algorithm for the "stable roommates" problem. *Journal of Algorithms*, 6(4):577–595, 1985.

[2] R. W. Irving and S. Scott. The stable fixtures problem - a many-to-many extension of stable roommates. *Discrete Appl. Math.*, 155(16):2118–2129, 2007.

[3] L. Toka and P. Michiardi. A Dynamic Exchange Game. *www.eurecom.fr/~toka/pub/Podc'08.pdf*, 2008.

[1] In an extended version of the model we plan to relax the assumption on uniform C, moreover we plan to consider capacities as strategy variables (along with α) by defining a joint payoff function on the whole strategy set.

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