Newsvendor Model Of Capacity Sharing

Vijay G Subramanian

EECS Dept., Northwestern University

Joint work with R. Berry, M. Honig, T. Nguyen, H. Zhou & R. Vohra

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Facing A Spectrum Crunch?

Spectrum much in the news at present:

• Providers complain about “spectrum crunch”
  Smartphone “clogging” networks
  Reason AT&T tried acquiring T-Mobile?

• Lot of good spectrum not used commercially

• FCC opening TV white-space
  Incentive auctions proposed

Challenge: What is a good policy solution for future?
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Possible Solutions

- Unlicensed/open access
  - “Driving” innovation\(^1\), e.g. WiFi
  - Can lead to tragedy of the commons\(^2\)

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\(^2\)“The impact of additional unlicensed spectrum on wireless services competition” Nguyen, et al., Dyspan 2011
\(^3\)NYTimes article
\(^4\)“Cooperative profit sharing in coalition-based resource allocation in wireless networks” Singh, et al., TON’12
\(^5\)“Do international roaming alliances harm consumers?” Bühler, Feb’09, working paper
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• Cognitive radio as answer$^3$?
  Can improve efficiency
  Issues remain: Interference, Sensing, etc.

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• Liberal licenses to increase competition?
  Let providers re-sell/lease spectrum/assets: contracts & tariffs
  Structure contracts/mechanisms to achieve social goals
  Allow third-party scavengers to aggregate spectrum
  Flexible contracts for end-users

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Normal operation
Markets operate separately
Longer-term competition for users
Roaming allows some sharing
Sharing at times of congestion?
Problem Set-up

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"Since I can bank on your investment, I’ll invest less ..."
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Sharing at times of congestion?

Concerns: Tacit collusion; **Under investment**

“Since I can bank on your investment, I’ll invest less ... ... maybe not if I make money from your traffic?”
Sharing Scenario

- Allow sharing at times of congestion
- Demand variable
- Providers pay to transfer load
- Customers see no extra cost
Sharing Scenario

Allow sharing at times of congestion
Demand variable
Providers pay to transfer load
Customers see no extra cost

How to structure contracts?
Want to incentivize sharing
Want to serve more customers
More capacity to be provisioned
Newsvendor Model

Single firm determining inventory in face of uncertain demand

Long history in operations management
Edgeworth1888: Cash balance with withdrawals
ArrowHarrisMarschak1951: Formally developed model
Newsvendor Model

Single firm determining inventory in face of uncertain demand

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$p_i$: per unit reward for service, $c_i$: per unit cost of capacity
$D_i$: random demand with cdf $F_i$, density $f_i$, $q_i$: Amount of spectrum bought

Profit $\pi_i = p_i \mathbb{E}[\min(q_i, D_i)] - c_i q_i$
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\[ D_i \text{: random demand with cdf } F_i, \text{ density } f_i, \text{ } q_i \text{: Amount of spectrum bought} \]

Profit \( \pi_i = p_i \mathbb{E}[\min(q_i, D_i)] - c_i q_i \)

Optimal purchase \( q_i^{NV} = F_i^{-1} \left( 1 - \frac{c_i}{p_i} \right) \)
Application To Spectrum Sharing

Scenarios: Two providers with separate markets

- Both under or over: no sharing
- SP1 more demand, SP2 more capacity
  SP2 lets SP1’s traffic use network
  Gets $(1 - \alpha)$ fraction of revenue
- SP2 more demand, SP1 more capacity
  SP1 lets SP2’s traffic use network
  Gets $(1 - \beta)$ fraction of revenue
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Set-up: Contract, prices given; spectrum bought; demands revealed
Modeled as a game with non-cooperative agents
Profits depend on other provider’s spectrum purchase
What is the equilibrium strategy?

Note: This model also applies to long-term purchase of electricity, when real-time reselling is allowed
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Model A Of Sharing

Provider prioritizes self-traffic
Remainder capacity used for competitor
Profit = Newsvendor profit + Extra

Theorem
The spectrum game outlined has a unique pure sub-game perfect equilibrium if $p_1 \geq (1 - \beta) p_2$ and $p_2 \geq (1 - \alpha) p_1$. In addition, the equilibrium can be obtained by iterating the best-response correspondences.
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Model B Of Sharing

Provider treats all traffic same
Need to drop some self-traffic!
Owing to neutrality, commonly used
Profit = Newsvendor profit + $\Delta$

\[ \text{Theorem} \]

The spectrum game outlined has a unique pure sub-game perfect equilibrium if $p_1 = p_2$ and when $\alpha = \beta = 0$.

Provider gets all revenue of traffic she serves
Provider treats all traffic same
Need to drop some self-traffic!
Owing to neutrality, commonly used
Profit = Newsvendor profit + $\Delta$

**Theorem**

*The spectrum game outlined has a unique pure sub-game perfect equilibrium if $p_1 = p_2$ and when $\alpha = \beta = 0$.*

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Numerical Examples

Set-up:

- General dependent demands
  Co-monotone, independent & counter-monotone
  Extremes approached with Frank copulas
- Model A sharing only
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Set-up:

- General dependent demands
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  Extremes approached with Frank copulas
- Model A sharing only

In all cases: **Sharing is incentive-comptabile**

Expected profit is greater than no sharing case

What about spectrum/capacity procurement?
Not just spectrum but includes infrastructure
Note: $\alpha, \beta < 0.5$, spectrum owner gets more of extra revenue
Numerical Example 1

Demands: Weibull, scale 0.5, shape 0.5, mean 1

Heavy-tailed

Heavy-tailed $\Rightarrow$ more spectrum bought even for $\alpha > 0.5$
Numerical Example 2

Demands: Uniform $[0, 2]$, mean 1
Bounded demand

$\text{Purchased spectrum}$

Counter Monotone
Independent
Co–Monotone
No Sharing

Bounded $\Rightarrow$ more spectrum only when $\alpha < 0.5$
Numerical Example 3

Demands:

SP1 - Uniform [0, 2], mean 1
SP2 - Weibull, scale 0.5, shape 0.5, mean 1

Asymmetric demand

Equilibrium purchase is asymmetric
Conclusions & Future Work

Well-designed sharing schemes can be beneficial
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Model A:

1. Proposition

   Co-monotone case equals no sharing.
   Therefore, sharing is incentive compatible.

2. Contract structure determines when more demand is served
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Well-designed sharing schemes can be beneficial

Model A:

1. **Proposition**
   
   *Co-monotone case equals no sharing.*
   
   *Therefore, sharing is incentive compatible.*

2. Contract structure determines when more demand is served

Model B:

1. *To be shown that this is incentive compatible*

2. *Types of contracts that lead to more purchase not known*
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Model A:

1. Proposition
   
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Model B:

1. *To be shown that this is incentive compatible*
2. *Types of contracts that lead to more purchase not known*

*Can contract also be part of decision process?*
Thank You For Your Attention