Dynamic Exchange of Communication Services

[Poster]

Leonidas Georgiadis
Aristotle University of Thessaloniki, Greece

George Iosifidis
University of Thessaly, and CERTH, Greece

Leandros Tassiulas
University of Thessaly, and CERTH, Greece

1. ABSTRACT

Motivation.The increasing mobile data demand and the proliferation of advanced user-owned network equipment have given rise to collaborative schemes, where users satisfy each other's communication needs whenever they have spare network resources [2]. A prerequisite for the success of these models is to ensure that users will share their resources in a fair fashion, and hence will agree to cooperate. Ideally, from a system design point of view, each user should receive resources (or, service) commensurate to his contribution. When this is not possible, e.g., due to asymmetries in resource availability, we would prefer to have a *lexicographically optimal* (or, lex-optimal) outcome that balances the resource exchanges as much as possible.

Nevertheless, achieving such an allocation is an intricate task since: (i) the service exchange is constrained by an underlying graph that prescribes, for each node, the subset of the nodes he can serve and receive services from, (ii) each user takes servicing decisions independently whenever he has idle network resources, aiming to maximize the total service he receives in exchange, (iii) each user is not aware of the resource availability of other users, nor he is aware of their current service allocation decisions (towards the other nodes). In this totally dynamic, fully decentralized and graph-constrained market setting, the following question arises: *how the lex-optimal allocation can be achieved in an asynchronous fashion by the users*?

Model. We consider a service exchange market that is modeled as an undirected connected graph $G = (\mathcal{N}, \mathcal{E})$ with node and edge set $\mathcal{N}$ and $\mathcal{E}$, respectively. Let $\mathcal{N}_i = \{j : (i,j) \in \mathcal{E}\}$ be the set of neighbors of node $i \in \mathcal{N}$. The market operation is continuous in the time domain and each node i creates service tokens according to a Poisson process with rate $\lambda_i > 0$. Let $d_{ij}(t_i) = \{1, 0\}$ denote the decision of node i to allocate or not, respectively, the token created in time instance t_i, to his neighbor $j \in \mathcal{N}_i$. Also, we denote with $D_{ij}(t) \in \mathcal{Z}^+$ the number of tokens that node i has allocated to node $j \in \mathcal{N}_i$ until time t. The users do not reserve tokens, and the tokens are not splittable. Therefore, it holds

$$\sum_{j \in \mathcal{N}_i} d_{ij}(t_i) = 1, \ \forall\, i \in \mathcal{N}, \ \forall\, t_i \,. \qquad (1)$$

We denote with $R_i(t)$ and $D_i(t)$ the total service tokens that node $i \in \mathcal{N}$ has received and allocated respectively, until time t, by (to) all his neighbors. We characterize the performance of each node i by the ratio $\rho_{ij}(t) = D_{ji}(t)/D_{ij}(t)$, $j \in \mathcal{N}_i$. This ratio admits a natural interpretation as the token exchange *price* among nodes $(i,j) \in \mathcal{E}$. Moreover, we define the price per node $i \in \mathcal{N}$ as $\rho_i(t) = R_i(t)/D_i(t)$, and the vector $\boldsymbol{\rho}(t) = \big(\rho_i(t) : i \in \mathcal{N}\big)$. We are interested in the asymptotic behavior of the market, i.e., the average rate of total exchanged tokens.

Dynamic Exchange Policy. Every rational node i who creates a token in time instance t_i, is expected to allocate it to node $j \in \mathcal{N}_i$ for which the current price $\rho_{ij}(t_i)$ is the largest among all his neighbors $k \in \mathcal{N}_i$. We can interpret this strategy as each node i "selling" his token to his neighbor offering the highest token exchange price. This is a basic user-rationality condition for this type of competitive markets. The steady state of the market is represented by a competitive equilibrium $\bar{\boldsymbol{\rho}}$ which satisfies the following properties: for every pair of nodes $(i,j) \in \mathcal{E}$: (i) If $\bar{D}_{ij} = 0$, then $\bar{D}_{ji} = 0$, (ii) If $\bar{D}_{ij} > 0$, then $\bar{D}_{ji}/\bar{D}_{ij} = \bar{\rho}_i$, (ii) If j_1, j_2 are neighbors of i, $\bar{D}_{ij_1} > 0$ and $\bar{\rho}_{j_2} > \bar{\rho}_{j_1}$ then $\bar{D}_{ij_2} = 0$. The main question is if this nodes' behavior leads to the competitive equilibrium, and whether the latter is related to the lex-optimal point.

Conclusions. Interestingly, the answer to both questions is affirmative. Moreover this equilibrium has a rich structure as we have proved for the static respective model in [1]. This surprising results reveals that such myopic and selfish best-response allocation strategies, converge to the maxmin fair solution such that it would be imposed by a central market coordinator if there was one, with global information and full control over the nodes decisions. Finally, similar models arise in graphical economies which extend the classical Arrow - Debreu economies by imposing graph constraints on the subsets of buyers and sellers that can trade. Our analysis is also applicable to this setting, without requiring the existence of money (credit) as a trading mechanism.

Acknowledgements. This work was was supported by the project SOFON which is implemented under the ARISTEIA Action of the OPERATIONAL PROGRAMME EDUCATION AND LIFELONG LEARNING, and is co-funded by the ESF and National Resources.

2. REFERENCES

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