Harmonic influence in large-scale networks

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Harmonic influence has been recently introduced as a measure of the relative influence of two nodes in a network that naturally emerges in models of opinion dynamics and social influence [2]. Given two nodes s_0 and s_1 in a connected network, that are assigned values $x_{s_0} = 0$ and $x_{s_1} = 1$, respectively, the harmonic influence vector x measures the relative influence of s_1 with respect to s_0 on the different nodes in the network. It is characterized by the property that the harmonic influence value x_v in any node $v \neq s_0, s_1$ coincides with the weighted average of the values of its neighbors. In other words, the harmonic influence vector is the solution of the Laplace equation on the network with boundary conditions on s_0 and s_1.

Harmonic influence can be given interpretations both in terms of random walks and electrical networks. More precisely, the value x_v coincides with the probability that a random walk on the network started in node v hits node s_1 before node s_0; on the other hand, x_v coincides with the voltage of node v in an electrical network where links' weights correspond to conductances and the voltages in s_0 and s_1 are fixed to the values 0 and 1, respectively.

In [2], sufficient conditions for the harmonic influence vector to be almost constant throughout a large-scale network (a phenomenon referred to as *homogeneous influence*) were investigated. It was shown that harmonic influence is homogeneous in *highly fluid* networks, characterized by the property that the product between the mixing time of the associated stochastic matrix and the relative degree of the nodes s_0 and s_1 be vanishing in the large size limit.

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In this work, we first study conditions under which harmonic influence *polarizes* in a large-scale network. Here, polarization refers to the existence of a cut in the network such that most of the nodes on the one side of it have harmonic influence value close to 0, and most of the nodes on the other side have value close to 1. In particular, we prove that, when the total size of the links between the two sides of a cut is negligible with respect to the degrees of s_0 and s_1, then the harmonic influence vector polarizes across this cut.

Then, we consider random interconnections between two highly fluid networks, one containing node s_0 and the other one containing node s_1 and prove the existence of a phase-transition. When the expected value of the total weight of the links interconnecting the two networks is negligible with respect to weight of s_0 and s_1, then harmonic influence polarizes across this cut. Conversely, when the weights of the interconnecting links are sufficiently concentrated around their expected value and their total expected value is much larger than the degree of s_0 and s_1, then harmonic influence is homogeneous. Detailed statements of our results can be found online at [1]. Their proofs are based on techniques from electrical networks and random walks theory [4, 3, 5].

1. REFERENCES

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