

Cifre PhD Proposal: “Learning in Blotto games and applications to modeling attention in social networks”

Keywords: Game theory, sequential learning, Blotto game, social networks, modeling

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Background and PhD topic description:

The Colonel Blotto game is a fundamental model of strategic resource allocation: two players allocate a fixed amount of resources to a fixed number of battlefields with given values, each battlefield is then won by the player who allocated more resources to it, and each player maximizes the aggregate value of battlefields he wins. It recently gained a very high interest in theoretical and applied research communities because of its potential to model many important problems of resource allocation in strategic settings ranging from international war to computer security. In particular, it provides a good model of competition for attention of users in social networks. There, battlefields correspond to users and their values correspond to the value of convincing the users (e.g., in advertisement, it would be the value of the product bought by the user). Theoretical solutions of the Colonel Blotto game could therefore enable important progresses in designing strategies to allocate resources optimally to capture the attention of users in a social network, a topic of high importance in the online world with applications for instance to advertisement campaigns or information propagation.

Applications of the Colonel Blotto game, however, have remained limited so far mostly due to the lack of solutions of the game in realistic cases. Indeed, although it was originally proposed by Borel in 1921 [2], the first Nash equilibrium solution of the game was given in 1950 [6] in a simple case (2 or 3 battlefields). In 2006, a Nash equilibrium solution was given for an arbitrary number of battlefields [9] (see also a survey in [10]), but only if all battlefields have the same value, which is not realistic in applications. In our recent work, we proposed first ideas towards a Nash equilibrium solution for arbitrary battlefields values [11], and towards a Nash equilibrium solution of the Blotto

game on a graph [8]; but those ideas need to be developed to reach a general Nash equilibrium solution useful in the application to competition for attention of users in social networks.

Another barrier for applications is that the Nash equilibrium solution assumes complete information on the players payoffs, which is not always appropriate. The machine learning community has been very active in recent year to develop sequential learning methods in order to adjust the strategies while learning the unknown payoff parameters, in particular in the classical setting of the multi-armed bandit problem [3, 5]. These methods, however, are not adapted in a competitive environment such as the one modeled by the Colonel Blotto game and developing learning algorithms in fully game-theoretic settings is currently an open problem.

The overall goal of this thesis will be to develop solutions of the Blotto games in order to use it to model competition for attention of users in social networks. Specifically, we will look to address the two key barriers mentioned above, that is:

- (i) First, we will look for a general Nash equilibrium solution. In particular, we will include arbitrary battlefield values, more than two players (in order to be able to model more than two competitors) and to take into account externalities on a graph (i.e., the fact that winning a battlefield has an effect on the value of neighboring battlefields in the social network). We will leverage preliminary ideas mentioned above and propose and analyze heuristics to compute approximate Nash equilibria in cases where the exact solution is not possible.
- (ii) Second, we will develop sequential learning methods adapted to the game-theoretic setting where several competitors are concurrently performing a learning task whose outcome depends on each other; and apply those to design strategies for the competitors to dynamically adjust their resource allocation while learning the users value. To this end, we will combine ideas from the multi-armed bandit literature [3, 5] with game theoretic ideas from repeated games [1, 4, 12] (see also [7]).

Further information and application procedure

Candidates should have a strong background in mathematics (probability and preferentially either learning or game theory/optimization or both) and an interest in the application to modeling social networks.

Interested candidates are invited to send the following documents to patrick.loiseau@eurecom.fr and alonso.silva@nokia.com:

- a detailed CV,
- a list of courses and grades in the last two years (at least),
- the name of 2-3 references willing to provide a recommendation letter for their application,
- a short statement of interest and any other information useful to evaluate the application.

The position will be open until filled but the screening of application will start on May 16, so **interested candidates are invited to send their application material by May 16, 2016**. The start of the PhD is expected in Fall 2016 (or after a minimum delay of 2 months due to administrative procedures).

The PhD is fully funded. The PhD student will be mainly based in the Alcatel-Lucent Bell Labs France research center (in the Paris region) and will spend short visits at EURECOM (in Sophia-Antipolis).

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