Two-way Wireless Communication via a Relay Station

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Consider two-way radio networks with a single relay-node (2M’s!)

Examples:
1. Rapidly deployable backbone (Satellite, HAPS, WiMAX) for civil protection networks
2. Wireless interconnection backbone for large fixed/wireless networks

Key elements:
1. Relay is for traffic residing within the same network (i.e. it is not a gateway to other networks
2. No direct connection between the nodes → all traffic between nodes passes through the relay
3. Duplexing (Nodes → Relay and Relay → Nodes) is used so that signal space dimensions are disjoint (i.e. TDD or FDD)
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Problem Scope

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Network Topology

Active Relay

Communicating Nodes
Network Topology

Active Relay

No Direct Link

Communicating Nodes
Network Topology

Traffic Flow

Active Relay

Communicating Nodes
Similar problem to the distributed source-coding (lossless) and reconstruction problem of Wyner, Wolf and Willems [Wyner2002]

They considered a three-node network with a satellite relay

Important ingredient: the relay does not need to completely reconstruct the set of variables, only to pass on sufficient information on the downlink for the nodes to do so. This results in saved bandwidth due to correlation between the sources.
Ultimate channel model

\[ y_R = \sqrt{P_1 Rh_1 Rx_1} + \sqrt{P_2 Rh_2 Rx_2} + z_R \]
\[ y_1 = \sqrt{P_{R1} h_{R1} x_R} + z_1 \]
\[ y_2 = \sqrt{P_{R2} h_{R2} x_R} + z_2 \]

• quasi-static line-of-sight (LOS) channels
  – motivated by the propagation environments associated with the target applications that are envisaged for this type of system.

• complex channel gains (amplitude and phase) are deterministic and known to the transmitters.
  – justified given that we have a two-way channel and that bandwidth for channel-state information (CSI) on the downlink will be negligible
  – required since we will be performing multiuser coding at the waveform level
  – in a real system, the fact that the propagation channel’s phase is quasi-static is not enough. Phase shifts due to electronics must also be accounted for, which will ultimately determine the feasibility of the results presented here.
Strategies for Relaying

• (Amplify)-and Forward [Rankov05,Knopp06,Katabi06]: If the output alphabet of the uplink channel is the same as the output alphabet of the relay encoder, we can pass the received sequence directly to the destinations (accumulation of noise). The advantage here is that the relay does not have to decode the multiuser uplink signal and thus does not incur a multiplexing loss. This will clearly be sub-optimal because of noise accumulation. Katabi referred to this as Analog Network Coding.

• Decode and Forward: Here the relay decodes the uplink signal and re-encodes the two streams on the downlink. The disadvantage of this scheme is that by imposing complete decoding of the uplink signal a multiplexing loss is unavoidable (due to the MAC sum-rate limitation). This will also clearly be sub-optimal for high-capacity downlink (i.e. uplink limits performance). It is an issue of downlink SNR.

• Compress and Forward: More later

• Partial Decode-and-Forward [Zhang et al 06]: Here we do not decode completely at the relay but rely on the fact that the users have side information (i.e. their own signals sent on the uplink). The key is to design the uplink codes so that partial decoding can be performed. This type of approach will lead to an optimal solution.
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\[ X_1 \sim N(0, P_1) \]
\[ Z_R \sim N(0, \sigma^2) \]
\[ Y_R \rightarrow X_R = \sqrt{\frac{P_R}{P_1 + P_2 + \sigma^2}} Y_R \]
\[ X_2 \sim N(0, P_2) \]

\[ Z_1 \sim N(0, \sigma^2) \]
\[ -\sqrt{\frac{P_R}{P_1 + P_2 + \sigma^2}} X_1 \rightarrow Y_1 = \sqrt{\frac{P_R}{P_1 + P_2 + \sigma^2}} (X_2 + Z_R) + Z_1 \]

\[ Z_2 \sim N(0, \sigma^2) \]
\[ -\sqrt{\frac{P_R}{P_1 + P_2 + \sigma^2}} X_2 \rightarrow Y_2 = \sqrt{\frac{P_R}{P_1 + P_2 + \sigma^2}} (X_1 + Z_R) + Z_2 \]
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\( Y_R \)

\( X_2 \sim N(0, P_2) \)
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Zhang et al. "Physical Layer Network Coding" [2006]

- perfect phase alignment so signals "add up in the air"
- distance properties maintained
- detect difference symbol, instead of individual symbols

Symbols are different | Symbols are same | Symbols are different
Two-Way Relay Channel (2 users)

\[ W_{12} \rightarrow X_{12} \rightarrow p(y_R|x_{12}, x_{21}) \rightarrow X_{21} \rightarrow W_{21} \]

\[ \hat{W}_{21} \rightarrow Y_1 \leftrightarrow Y_2 \rightarrow \hat{W}_{12} \]

\[ X_{ab,i} = f_{ab,i} \left( W_{ab}, Y_{a}^{i-1} \right) \in \mathcal{X}_a \]

\[ X_{R,i} = f_{R,i} \left( Y_{R}^{i-1} \right) \in \mathcal{X}_R \]

\[ \hat{W}_{ab} = g_{ab} \left( W_{ba}, Y_{b}^{n} \right) \]
Outer-bound (cut-set)

\[ R_{12} \leq \min (I (X_{12}; Y_{R} | X_{21}), I (X_{R}; Y_{2})) \]
\[ R_{21} \leq \min (I (X_{21}; Y_{R} | X_{12}), I (X_{R}; Y_{1})) \]

for some \( p(x_{12}, x_{21})p(y | x_{12}, x_{21}) \) and \( p(x_{R})p(y_{1} | x_{R})p(y_{2} | x_{R}) \)

- Intuitive explanation
  - First term in minimization: imagine an ideal downlink where \( Y_{R} \) could be made available without distortion to both nodes. This would amount to a two-way channel with a common output (Shannon61,Hekstra89)
  - Second term: Imagine an ideal uplink where the relay could decode both messages perfectly before relaying them to the destination. This would be an outer-bound on achievable rates since it is the maximum information rate between the relay and each node. We might be tempted to restrict ourselves to the broadcast channel rates, but this would not exploit the fact that the nodes know their own signals, which can clearly only increase rates.
Outer-bound (cut-set)

From [Cover91], we have that

\[ R_{a \rightarrow b} \leq I (X_{a \rightarrow b}, X_R; Y_b | X_{b \rightarrow a}) \]

\[ = I (X_R; Y_b | X_{b \rightarrow a}) + I (X_{a \rightarrow b}; Y_b | X_{b \rightarrow a}; X_R) \]

\[ = H (Y_b | X_{b \rightarrow a}) - H (Y_b | X_R, X_{b \rightarrow a}) + \]

\[ \underbrace{I (X_{a \rightarrow b}; Y_b | X_{b \rightarrow a}, X_R)}_{=0} \]

\[ \overset{(a)}{=} H (Y_b) - H (Y_b | X_R) \]

\[ = I (X_R; Y_b) \]

where (a) follows from the fact that conditioning reduces the entropy of the first term, and that conditioned on \( X_R, Y_b \) and \( X_{b \rightarrow a} \) are independent.
In addition, we have the second inequality

\[ R_{a \rightarrow b} \leq I(X_{a \rightarrow b}; Y_R, Y_b | X_R, X_{b \rightarrow a}) \]

\[ \overset{(a)}{=} I(X_{a \rightarrow b}; Y_R | X_{b \rightarrow a}, X_R) \]

\[ = H(Y_R | X_{b \rightarrow a}, X_R) - H(Y_R | X_{a \rightarrow b}, X_{b \rightarrow a}, X_R) \]

\[ \overset{(b)}{=} H(Y_R | X_{b \rightarrow a}) - H(Y_R | X_{a \rightarrow b}, X_{b \rightarrow a}) \]

\[ = I(X_{a \rightarrow b}; Y_R | X_{b \rightarrow a}) \] (1)

- (a) follows from the fact that conditioned on \( X_R \), \( Y_b \) is independent of \( X_{a \rightarrow b} \),

- (b) follows from the fact that conditioning reduces the entropy of the first term in the sum and that conditioned on \( X_{a \rightarrow b} \) and \( X_{b \rightarrow a} \), \( Y_R \) is independent of \( X_R \) (since it depends only on past values of \( Y_R \)).

- Simplification with respect to the general cut-set bound is due to the fact that the relay has no information of its own to send or receive and that no direct channel exists between the two communicating nodes.
Discrete-memoryless Binary Adder Channel (Outer bound)

\( Z_R \in \{0, 1\} : \Pr(Z_R = 1) = \epsilon_R \)

\( Z_1 \in \{0, 1\} \quad \Pr(Z_1 = 1) = \epsilon_1 \)

\( Z_2 \in \{0, 1\} \quad \Pr(Z_2 = 1) = \epsilon_2 \)

\[
I(X_{12}; Y_R|X_{21}) \leq H(Y_R|X_{21}) - H(Y_R|X_{12}, X_{21}) \leq 1 - H(\epsilon_R)
\]

\[
I(X_{12}; Y_R|X_{21}) \leq 1 - H(\epsilon_R)
\]

\[
I(X_R; Y_1) \leq 1 - H(\epsilon_1)
\]

\[
I(X_R; Y_2) \leq 1 - H(\epsilon_2)
\]

\( R_{12} \leq 1 - H(\max(\epsilon_R, \epsilon_2)) \)

\( R_{21} \leq 1 - H(\max(\epsilon_R, \epsilon_1)) \)

**ACHIEVABLE!**
Discrete-memoryless Binary Adder Channel (Achievability)

• Suppose $R_{12} \geq R_{21}$. Consider two random codebooks $C_{12}$ with rate $R_{12} - R_{21}$ and $C_c$ with rate $R_{21}$. $C_c$ is the common codebook.

• Time-share between both codebooks. During the first time-slot of duration $(1 - \alpha)n$ dimensions user 1 transmits alone to the relay using $C_{12}$. Call the information sequence $X_{12}^{(a)}$. Arbitrarily small error probability for detection of $X_{12}^{(a)}$ is achievable if $R_{12} - R_{21} < (1 - \alpha) I(X_{12}; Y_R) \leq (1 - \alpha) (1 - H(\epsilon_R))$

• During the second time-slot of duration $\alpha n$ dimensions, both users transmit their information sequences $X_{12}^{(b)}$ and $X_{21}$ which are codewords of the same linear code over GF(2), $C_c$. As a result the relay receives the modulo-2 sum of the two codewords. Linear codes achieve the capacity of the BSC and thus arbitrarily small error probability for the detection of $X_{12}^{(b)} \oplus X_{21}$ is possible if $R_{21} \leq \alpha (1 - H(\epsilon_R))$. 
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$\alpha n \quad (1 - \alpha)n$

$X^{(a)}_{12} \in C_{12} \quad X^{(a)}_{12} \quad X^{(b)}_{12}$

$X_{s} = X^{(b)}_{12} \oplus X_{21} \in C_{c}$
Discrete-memoryless Binary Adder Channel (Achievability)

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Discrete-memoryless Binary Adder Channel (Achievability)

- The relay encodes with a two-dimensional codebook indexed by \((i, j), i = 1, \ldots, 2^{n(R_{12} - R_{21})}, j = 1, \ldots, 2^{nR_{21}}\). The \(i\) dimension is used to encode \(X_{12}^{(a)}\) and the \(j\) dimension to encode \(X_{12}^{(b)} \oplus X_{21}\).

- At receiver 1 (weak receiver), \(X_{12}\) is known and so dimension \(i\) of the codeword is known. Arbitrarily small error probability is achievable for detection of \(j\) or \(X_{12}^{(b)} \oplus X_{21}\) and consequently \(X_{21}\) if \(R_{21} < I(X_R; Y_1) \leq 1 - \mathcal{H}(\epsilon_1)\).

- At receiver 2 (strong receiver) \(X_{21}\) is known. Arbitrarily small error probability for detection of \((i, j)\) and consequently \(X_{12}\) is achievable if \(R_{12} < I(X_R; Y_2) \leq 1 - \mathcal{H}(\epsilon_2)\).

- In summary, three cases:
  1. “strong relay” \(\rightarrow \epsilon_R \geq \max(\epsilon_1, \epsilon_2)\). Set \(\alpha = 1\) and \(R_{12} = R_{21} = 1 - \mathcal{H}(\epsilon_R)\).
  2. “medium relay” \(\rightarrow \epsilon_a \geq \epsilon_R \geq \epsilon_b\). Set \(\alpha = \frac{1 - \mathcal{H}(\epsilon_a)}{1 - \mathcal{H}(\epsilon_R)}\) and \(R_{ab} = 1 - \mathcal{H}(\epsilon_R), R_{ba} = 1 - \mathcal{H}(\epsilon_a)\).
  3. ”weak relay” \(\rightarrow \epsilon_a \geq \epsilon_b \geq \epsilon_R\). Set \(\alpha = \frac{1 - \mathcal{H}(\epsilon_a)}{1 - \mathcal{H}(\epsilon_b)}\) and \(R_{ba} = 1 - \mathcal{H}(\epsilon_a), R_{ab} = 1 - \mathcal{H}(\epsilon_b)\).
AWGN Channels

\[ Z_R \in \mathbb{R} : f_{Z_R}(u) = \frac{1}{\sqrt{2\pi\sigma_R^2}} \exp \left( -\frac{u^2}{2\sigma_R^2} \right) \]

\[ f_{Z_1}(u) = \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp \left( -\frac{u^2}{2\sigma_1^2} \right) \]

\[ f_{Z_2}(u) = \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp \left( -\frac{u^2}{2\sigma_2^2} \right) \]

I(\(X_{12}; Y_R|X_{21}\)) \leq \log_2 \left( 1 + \frac{P_{1R}}{\sigma_R^2} \right)

\[ R_{12} \leq \min \left( \log_2 \left( 1 + \frac{P_{1R}}{\sigma_R^2} \right), \log_2 \left( 1 + \frac{P_{2R}}{\sigma_2^2} \right) \right) \]

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I(\(X_R; Y_2\)) \leq \log_2 \left( 1 + \frac{P_R}{\sigma_2^2} \right)

\[ \text{ACHIEVABLE (sometimes)!} \]
AWGN Channel (Achievability)

- Suppose $R_{12} \geq R_{21}$. Consider one random Gaussian codebook $C_{12}$ with rate $R_{12} - R_{21}$ and one nested Lattice code, $C_c$, with rate $R_{21}$. $C_c$ is again the common codebook.

- Superimpose $X^{(a)}_{12}$ (Gaussian) and $X^{(b)}_{21}$ (NLC) as

$$X_{12} = \sqrt{(1 - \alpha)P_1R}X^{(a)}_{12} + \sqrt{\alpha P_1R}X^{(b)}_{21}$$

where $\alpha P_1R = P_{2R}$.

- Decoding at relay: Relay decodes $X^{(b)}_{12} + X_{21} \mod \Lambda$ by turning the received signal into a modulo-$\Lambda$ additive noise channel [Erez2004] at rate

$$R_{21} = R^{(b)}_{12} = \frac{1}{2} \log_2 \left( 1 + \frac{\alpha P_1R}{\sigma^2_R + (1 - \alpha)P_1R} \right)$$

The sum signal is then stripped out and the extra part of user 1 remains at rate

$$R^{(a)}_{12} = \frac{1}{2} \log_2 \left( 1 + \frac{(1 - \alpha)P_{2R}}{\sigma^2_R} \right)$$
**AWGN Channel (Achievability)**

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  $$R_{12}^{(a)} = \frac{1}{2} \log_2 \left(1 + \frac{(1 - \alpha)P_{2R}}{\sigma_R^2}\right)$$
\[ \mathcal{C} = \{ \Lambda_1 \mod \Lambda \} = \{ \Lambda_1 \cap \mathcal{V} \} \]
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Coarse Lattice $\Lambda \subseteq \Lambda_1$

Fine Lattice $\Lambda_1$
$\mathcal{C} = \{\Lambda_1 \mod \Lambda\} = \{\Lambda_1 \cap V\}$

Coarse Lattice $\Lambda \subseteq \Lambda_1$

Fine Lattice $\Lambda_1$

$X_1 + X_2$
$\mathcal{C} = \{\Lambda_1 \mod \Lambda\} = \{\Lambda_1 \cap \mathcal{V}\}$
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Superimpose $X_{12}^{(a)}$ (Gaussian) and $X_{21}^{(b)}$ (NLC) as

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The sum signal is then stripped out and the extra part of user 1 remains at rate

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AWGN Channel (Achievability)

- Suppose $R_{12} \geq R_{21}$. Consider one random Gaussian codebook $C_{12}$ with rate $R_{12} - R_{21}$ and one nested Lattice code, $C_c$, with rate $R_{21}$. $C_c$ is again the common codebook.

- Superimpose $X_{12}^{(a)}$ (Gaussian) and $X_{21}^{(b)}$ (NLC) as

$$X_{12} = \sqrt{(1 - \alpha)P_{1R}}X_{12}^{(a)} + \sqrt{\alpha P_{1R}}X_{21}^{(b)}$$

where $\alpha P_{1R} = P_{2R}$.

- Decoding at relay: Relay decodes $X_{12}^{(b)} + X_{21}$ mod $\Lambda$ by turning the received signal into a modulo-$\Lambda$ additive noise channel [Erez2004] at rate

$$R_{21} = R_{12}^{(b)} = \frac{1}{2} \log_2 \left( 1 + \frac{\alpha P_{1R}}{\sigma^2_R + (1 - \alpha)P_{1R}} \right)$$

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- On the downlink, relay and nodes proceed as in the case of the binary adder channel
- choice of $\alpha$ depends on power scenarios, in order to match UL/DL rates for weaker user.
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Weak relay

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\frac{1}{2} \log_2 \left(1 + \frac{P_{R1}}{\sigma^2}\right) + \frac{1}{2} \log_2 \left(1 + \frac{P_{R2}}{\sigma^2}\right) \leq \frac{1}{2} \log_2 \left(1 + \frac{P_1+P_2}{\sigma^2}\right)
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• Optimality for weak relay and strong relay with equal UL powers, otherwise weak user is favoured.
Practical Issues

• Lattice codes only for feasibility to approach limits. Practical codes would be QAM with bit-interleaving and linear codes. The only difference is the decoding metric which accounts for the additive structure of the channel and detection of differences.

• The main problem is ensuring phase coherence at relay on uplink channel

• Alternate solution is compression with side-information (Wyner-Ziv)

\[ \hat{y}_R = \sqrt{P_1}x_1 + \sqrt{P_2}x_2 + z_R + \hat{z}_R \]
distortion

• Take case \( P_1 = P_2 = P \) and \( P_R = kP \). Rate-distortion with side-information at receiver (1 or 2) is

\[ R_1(D) = \frac{1}{2} \log_2 \left( \frac{\sigma^2 + 2P}{D} \right) - I(X_1; \hat{Y}_R) \approx \frac{1}{2} \log_2 \left( 1 + \frac{P + \sigma^2}{D} \right) \]

• The information rate for compression comes from the DL rate

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where \(\hat{y}_R\) represents the distortion

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\]
• Resulting SNR is

$$\text{SNR}_{CF} = \frac{P}{\sigma^2} \left( \frac{kP}{(k + 1)P + \sigma^2} \right)$$

• Similarly for Amplify-and-Forward

$$\text{SNR}_{AF} = \frac{P}{\sigma^2} \left( \frac{kP}{(k + 2)P + \sigma^2} \right)$$

• for $k = 2$ (i.e. relay uses total sum power of users on DL), 3dB degradation for Amplify-and-Forward, 1.8 dB for Compress-and-Forward (big deal right?) with respect to Physical-Layer Network Coding (which requires $k = 1$!).

• Big difference comes when we consider the more practical case of half-duplex communications.

$$\alpha N\text{-dimensions } \quad (1 - \alpha)N\text{-dimensions}$$
Practical Issues

• Amplify-and-Forward is limited to

\[ \frac{1}{4} \log_2 \left( 1 + \frac{P}{\sigma^2} \left( \frac{kP}{(k + 2)P + \sigma^2} \right) \right) \]

since \( \alpha = .5 \) is necessary (same source and channel bandwidth!)

• Partial-decode and Forward is limited to

\[ \frac{\alpha}{2} \log_2 \left( 1 + \frac{\alpha P}{\sigma^2} \right) \]

with

\[ \left( 1 + \frac{\alpha P}{\sigma^2} \right)^\alpha = \left( 1 + \frac{(1 - \alpha)kP}{\sigma^2} \right)^{1-\alpha} \]

\( \alpha N \)-dimensions \( (1 - \alpha)N \)-dimensions