Distributed Space-Time codes for a variety of channels

MIMO Relay MAC

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March, 29 2007

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Outline

1 Introduction

2 MIMO

3 The Relay A.F. channel

4 Slotted Amplify-and-Forward

5 Distributed STC

6 MAC
**Decomposition of a wireless channel into elementary ones**

**Elementary channels**
- MIMO channel
- MAC (Multiple Access Channel)
- Broadcast Channel
- Relay Channel
- Interference Channel

**A 5 nodes wireless network**

*Figure:* Components of this network are elementary channels
Either each node knows its downstream channels (CSIT) or no node knows its downstream channels (no CSIT). Nevertheless, all nodes know their upstream channels (CSIR).

- **CSIT:** Depending on the channel,
  - Optimisation of the mutual information
  - Resource allocation
  - Dirty Paper Coding
  -...

- **No CSIT:** Tools are
  - Outage probability (too complex)
  - Diversity-Multiplexing gain Tradeoff (DMT)
Ergodic capacity and Outage probability

- The ergodic capacity is asymptotically equal to

\[ C(SNR) = q \log_2 SNR + O(1) \]

where \( q \) is the number of degrees of freedom of the system.
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For block fading channels, the outage probability is asymptotically equal to

\[ P_{\text{out}} = O\left(\frac{1}{\text{SNR}^d}\right) \]

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**Ergodic capacity and Outage probability**

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- We are interested in block fading channels. Can we have both the diversity and the multiplexing gain?

**YES...but [Zheng and Tse, 2003]**

We can both gain on \( q \) and \( d \) but there is a fundamental tradeoff between the two gains.
The Diversity-Multiplexing gain Tradeoff (I)

- $P_e$ can be the **outage** probability (when considering the theoretical limits) or the **word error** probability (when considering a code performance).

**Definitions**

Define the spatial multiplexing gain and the diversity gain

$$r = \lim_{\text{SNR} \to \infty} \frac{R(\text{SNR})}{\log_2 \text{SNR}} \quad d = -\lim_{\text{SNR} \to \infty} \frac{\log P_e(\text{SNR})}{\log(\text{SNR})}.$$  

Equivalently, we can write (exponential equality)

$$P_e(\text{SNR}) \propto \text{SNR}^{-d}$$
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Equivalently, we can write (exponential equality)

$$P_e(\text{SNR}) \doteq \text{SNR}^{-d}.$$ 

**Theorem**

The optimal tradeoff curve $d^*(r)$ is a piecewise-linear function connecting the points $d^*(k)$ for $k = 0, 1, \ldots, \min \{n_t, n_r\}$ where

$$d^*(k) = (n_t - k)(n_r - k).$$
The Diversity-Multiplexing gain Tradeoff (II)

\( n_t = n_r = 4 \) DMT for a MIMO channel

Remark the extremal values: Maximum diversity 16 and maximum multiplexing gain 4

Figure: The DMT for a \( n_t = n_r = 4 \) MIMO channel
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**NVD Codes**

**Definition**

A space-time block code $C$ is a **Non Vanishing Determinant code** (NVD code) (Belfiore and Rekaya, 2003) for an $n_t \times n_t$ MIMO channel if

1. $C$ is a linear dispersion code
2. Entries of the codewords depend on $n_t^2$ QAM or HEX information symbols
3. The minimum determinant of $C$ is

$$\delta_{\text{min}}(R) \equiv \min_{\mathbf{X} \in C \setminus \{0\}} |\det \mathbf{X}|^2 \geq \psi > 0$$

where $\psi$ does not depend on $R$, the spectral efficiency of the code.
**NVD Codes**

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$$\delta_{\min}(R) \triangleq \min_{X \in \mathcal{C} \setminus \{0\}} |\text{det}X|^2 \geq \psi > 0$$ (1)

where $\psi$ does not depend on $R$, the spectral efficiency of the code.

**Example**

Let consider a counterexample. The $2 \times 2$ space-time block code of (Damen et al., 2002) has a minimum determinant which tends to 0 when $R \to \infty$. So this code is not an NVD code.
**Information lossless codes**

**Definition**

A linear dispersion space-time code is information lossless if the mutual information of the equivalent channel obtained by including the encoder in the channel is equal to the mutual information of the MIMO channel.
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**Figure:** Information lossless STBC: \( I(X_1, Y_1) = I(X, Y) \)
The Golden Code

Codewords are

\[
X = \frac{1}{\sqrt{5}} \begin{pmatrix}
\alpha (z_1 + z_2 \varphi) & \alpha (z_3 + z_4 \varphi) \\
i \cdot \bar{\alpha} (z_3 + z_4 \bar{\varphi}) & \bar{\alpha} (z_1 + z_2 \bar{\varphi})
\end{pmatrix}
\]

with \( \varphi = \frac{1+\sqrt{5}}{2}, \bar{\varphi} = \frac{1-\sqrt{5}}{2}, \alpha = 1 + i - i \varphi, \bar{\alpha} = 1 + i - i \bar{\varphi} \) and \( z_j \in \text{QAM} \).
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The two layers of \( X \) (the two diagonals) can be vectorized,

\[
\text{vec}X_1 = \begin{pmatrix} x_{11} \\ x_{22} \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} \alpha & \alpha \varphi \\ \bar{\alpha} & \bar{\alpha} \bar{\varphi} \end{pmatrix} \cdot \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}
\]

\[
\text{vec}X_2 = \begin{pmatrix} x_{12} \\ x_{21} \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} \alpha & \alpha \varphi \\ i \bar{\alpha} & i \bar{\alpha} \bar{\varphi} \end{pmatrix} \cdot \begin{pmatrix} z_3 \\ z_4 \end{pmatrix}
\]
The Golden Code

Codewords are

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X = \frac{1}{\sqrt{5}} \left( \begin{array}{cc}
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\end{array} \right)
\]

Remark the transform which maps \((z_1, z_2)\) onto the first layer

\[
U = \frac{1}{\sqrt{5}} \left[ \begin{array}{cc}
\alpha & \alpha \varphi \\
\bar{\alpha} & \bar{\alpha} \bar{\varphi}
\end{array} \right]
\]

Number \( i \) isolates the first layer from the second one so that minimum determinant is not zero.
Minimum determinant

Result

We obtain

\[ \delta_{\text{min}} = \min_{X \neq 0} |\det X|^2 = \frac{1}{5} \]

We can prove that it is the best minimum determinant that one could obtain.

NVD code

The Golden code is a NVD code with rate 2 (symbols PCU). It achieves the diversity-multiplexing gain tradeoff of the MIMO $2 \times n_r$ channel. Moreover it is an information lossless code.
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The channel

- We restrict to one strategy: Amplify-and-Forward (AF).
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Figure: One-relay channel
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Figure: Two-relay channel
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**Figure:** One-relay channel

**Figure:** Two-relay channel

- NAF (Nabar et al., 2004, Azarian et al., 2005) is optimal for 1 relay as it achieves the best DM tradeoff. What happens when $N > 1$ relay?
One-Relay: Frame structure

- We have to define a **cooperation frame**. Consider the NAF (Nabar et al., 2004).
One-Relay: Frame structure

We have to define a cooperation frame. Consider the NAF (Nabar et al., 2004).

**Figure:** Cooperation frame for the one-relay channel
We have to define a **cooperation frame**. Consider the NAF *(Nabar et al., 2004)*.

**Protocol**

1. A codeword is sent by the source in both time slots.
2. The relay sends, in slot 2, an amplified version of the received signal.

**Figure:** Cooperation frame for the one-relay channel
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Protocole

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2. The relay sends, in slot 2, an amplified version of the received signal.

Received Signal

\[
\begin{align*}
y_1 &= f x_1 + v_1 \\
y_R &= g x_1 + w \\
y_2 &= h(\beta y_R) + f x_2 + v_2
\end{align*}
\]
**One-Relay: Frame structure**

- We have to define a **cooperation frame**. Consider the NAF (Nabar et al., 2004).

![Figure: Cooperation frame for the one-relay channel](image)

**Protocol**

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\mathbf{y}_2 &= h(\beta \mathbf{y}_R) + f\mathbf{x}_2 + \mathbf{v}_2
\end{align*}
\]  

**Equivalent channel**

\[
\mathbf{Y} = \mathbf{H} \cdot \mathbf{X} + \mathbf{Z} \quad \text{with} \quad \mathbf{H} = \begin{bmatrix} \frac{f}{\sqrt{1+|\beta h|^2}} & 0 \\ \frac{\beta gh}{\sqrt{1+|\beta h|^2}} & \frac{f}{\sqrt{1+|\beta h|^2}} \end{bmatrix}
\]
**N-relay NAF: Frame structure**

- **NAF** frame (Non orthogonal Amplify-and-Forward) (Azarian et al., 2005) with length $2N$
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![Frame Structure Diagram](image)

**Figure:** NAF frame for 2 relays
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![Frame structure diagram]

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**Equivalent channel matrix**

$$
H = \text{diag} \left( \mathcal{H}_i \right) \quad \text{with} \quad \mathcal{H}_i = \begin{bmatrix}
\frac{f}{\beta_i g_i h_i} & 0 \\
\frac{f}{\sqrt{1+|\beta_i h_i|^2}} & \frac{f}{\sqrt{1+|\beta_i h_i|^2}}
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\end{bmatrix}
\]

**Diversity Multiplexing gain tradeoff (DMT)**

\[
d^*_{\text{NAF}, N}(r) = (1 - r)^+ + N(1 - 2r)^+
\]
**Analysing the Diversity Multiplexing gain Tradeoff**

- **Aim:** Achieve the MISO bound

\[ d_{\text{MISO}}(r) = (N + 1)(1 - r)^+ \]
**Analyzing the Diversity Multiplexing gain Tradeoff**

- **Aim:** Achieve the MISO bound

\[ d_{\text{MISO}}(r) = (N+1)(1-r)^+ \]

- **DDF** (Azarian et al., 2005, Katz and Shamai, 2005, Mitran et al., 2005) (the best known Decode-and-Forward scheme) achieves the MISO bound in the low multiplexing gain regime. But for high multiplexing gain, both DDF and NAF are far from the optimal performance.
Analysing the *Diversity Multiplexing gain Tradeoff*

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**Figure:** Large number of relays: **DDF** vs NAF vs MISO bound
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The 2-relay 3-slot scheme

- Improve the DMT of the NAF scheme by protecting $\frac{2}{3}$ of the frame instead of $\frac{1}{2}$. 
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**Figure:** 2-relay 3-slot frame
The 2-relay 3-slot scheme

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\[ d_{2,3}^*(r) = (1 - r)^+ + 2(1 - \frac{3}{2}r)^+ \]  

Figure: 2-relay 3-slot frame
**Best performance of the Slotted Amplify-and-Forward (SAF)**

**Definition**

A $M$-slot, $N$-relay SAF frame is composed of $M$ time slots. During the first one, the source transmits $l$ symbols, during time slot $i \neq 1$, the source and the relay $R_j$ ($j = 1, \ldots, N$) transmit $l$ symbols.
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- This scheme has a linear equivalent channel model
  \[ Y = H_{M \times M} \cdot X + Z. \]

- So, there exist $M \times M$ information preserving space-time block codes achieving the DM tradeoff (Oggier et al., 2004, Elia et al., 2005).
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Theorem

The DMT of an $M$-slot, $N$-relay SAF scheme is upperbounded by

\[ d^*(r) = (1-r)^+ + N \left(1 - \frac{M}{M-1} r\right)^+ \]  \hfill (4)
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SAF: Examples and discussion

1. $N = 1, M = 2$: This is the 1-relay NAF scheme. Upperbound (4) is achieved.

2. $M = 2, N$ arbitrary: Upperbound (4) is achieved by combining the NAF scheme with the relay selection scheme of [Bletsas et al., 2005].

3. $M = 3, N = 2$ SAF: optimal with ordered relays (see 3)
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- This upperbound has been found by considering a genie-aided model: no half duplex constraint and the channels between source and relays are perfect.
- The SAF scheme can never beat the non cooperative scheme for $r > \frac{M-1}{M}$ even without the half duplex constraint (causality).
- The NAF scheme is SAF ($M = 2N$) but not optimal when $N > 1$. 
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   - The SAF scheme can never beat the non cooperative scheme for $r > \frac{M-1}{M}$ even without the half duplex constraint (causality).
   - The NAF scheme is SAF ($M = 2N$) but not optimal when $N > 1$.

**Guideline**

Protect most of the transmitted data by extra paths. **Proposal of a naive SAF.**
**Naive SAF**

*Definition*

Source transmits continuously during $M$ time slots. From time slot 2 to time slot $M$, each selected relay transmits what it received during the preceding time slot.
Naive SAF

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\[
\begin{array}{cccccc}
S & 1 & 2 & 3 & \cdots & \tilde{N} & \tilde{N} + 1 \\
\tilde{R}_1 & 1 & 1 & & & & \\
\tilde{R}_2 & 2 & 2 & & & & \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\tilde{R}_{\tilde{N}} & & & & & \tilde{N} - 1 & \tilde{N} \\
D & 1 & 2 & 3 & \cdots & \tilde{N} & \tilde{N} + 1 \\
\end{array}
\]

Figure: Naive SAF protocol. $\tilde{N}$ is the number of selected relays.
**Introduction**

MIMO \ THE RELAY A.F. CHANNEL \ SLOTTED AMPLIFY-AND-FORWARD \ DISTRIBUTED STC \ MAC \ CONCLUSION

**Naive SAF**

**Definition**

Source transmits continuously during $M$ time slots. From time slot 2 to time slot $M$, each selected relay transmits what it received during the preceding time slot.

$$S \begin{array}{ccc} 1 & 2 & 3 \end{array} \cdots \begin{array}{c} \hat{N} \ N + 1 \end{array}$$

$$\hat{R}_1 \begin{array}{c} 1 \end{array} \begin{array}{c} 1 \end{array}$$

$$\hat{R}_2 \begin{array}{c} 2 \end{array} \begin{array}{c} 2 \end{array}$$

$$\vdots$$

$$\vdots$$

$$\hat{R}_{\hat{N}}$$

$$D \begin{array}{ccc} 1 & 2 & 3 \end{array} \cdots \begin{array}{c} N \ N + 1 \end{array}$$

**Figure:** Naive SAF protocol. $\hat{N}$ is the number of selected relays

**Theorem**

*If the relays are isolated from each other (path loss between relays equal to $\infty$), then the naive SAF scheme achieves the DMT upperbound,*

$$\sigma^*(r) = (1 - r)^+ + N \left(1 - \frac{M}{M-1} r\right)^+$$
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**Distributed perfect space-time block codes**

- Use the linear equivalent channel model of the SAF scheme. At each time slot is sent a row of a perfect STBC codeword. For instance, if $M = 2$, use the Golden Code (Belfiore et al., 2005).
- Simulation results with a perfect $3 \times 3$ STBC
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![Figure: Codeword Error Rate: Naive SAF vs. NAF](image)

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**Figure:** Codeword Error Rate: Naive SAF vs. NAF

**Figure:** Symbol Error Rate: Naive SAF vs. NAF
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Outage and DMT analysis

- Error probability and outage probability behave in the same way: many error (or outage) events.
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**Example**

2-users single transmit antenna case

\[ P_{\text{out}} = P_{\text{out},1} + P_{\text{out},2} + P_{\text{out},3} \]

First user in outage  Second user in outage  Both users in outage
Outage and DMT analysis

- Error probability and outage probability behave in the same way: many error (or outage) events.

**Example**

2-users single transmit antenna case

\[
\min \{ (1-r), 2(1-2r) \}
\]

\[
P_{\text{out}} = P_{\text{out,1}} + P_{\text{out,2}} + P_{\text{out,3}}
\]

*Figure: Diversity-Multiplexing gain Tradeoff*
Outage probabilities

Outage results are given for 2 single transmit antenna users and respectively 1 and 2 receive antennas.
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**Figure:** Outage probability 4 bits pcu p.u.: 1 receive antenna.
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**Figure:** Outage probability 4 bits pcu p.u.: 2 receive antennas
Error probabilities

- Error probabilities are given for a DMT achieving code (first code ever found) compared to time-sharing QAM modulations.
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**Figure:** Error probability 4 bits pcu p.u.: 1 receive antenna.
Error probabilities

- Error probabilities are given for a DMT achieving code (first code ever found) compared to time-sharing QAM modulations.

Figure: Error probability 4 bits pcu p.u.: 1 receive antenna.

Figure: Error probability 4 bits pcu p.u.: 2 receive antennas.
Space-Time Codes (both localized and distributed) that achieve the DMT for many channels
Conclusion

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**Perspective:** Explore other channels in the same way (broadcast, interference, ...)

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ENST, Paris

March, 29 2007

Distributed ST codes

GDR ISIS, the 3 M’s

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The End
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