Throughput Guarantees for Wireless Networks with Opportunistic Scheduling

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Abstract—In this paper we analyze achievable throughput guarantees in wireless time-division multiplexing (TDM) networks. Approximations of the throughput guarantee violation probability for users communicating in time-slotted systems are obtained for any scheduling algorithm with a given mean and variance of the number of bits transmitted in a time-slot and a distribution for the number of time-slots allocated to a user within a time window. We investigate the corresponding throughput guarantees for three scheduling algorithms, (i) Round Robin Scheduling, (ii) Maximum Carrier-to-Noise Ratio Scheduling, and (iii) Opportunistic Round Robin Scheduling, when the users’ channels are independently and identically distributed.

I. INTRODUCTION

For efficient utilization of the scarce radio spectrum available for wireless communication, opportunistic scheduling has recently attracted much attention. Opportunistic scheduling increases the system spectral efficiency by selecting users with favorable channel conditions [1], [2]. However, always selecting the best user can lead to starvation of users experiencing long-lasting deep fades. It is therefore necessary to develop scheduling algorithms that take both the channel conditions and the quality-of-service (QoS) demands of the users into account.

As a first approach to fulfilling QoS, we can look at how the fairness in the rate allocation between the users evolves over time for different scheduling algorithms [3], [4]. However, in order to be able to offer more exact QoS to the users, we have to analyze different service guarantees. Previous publications considering service guarantees have often looked at scheduling algorithms that provide delay guarantees for the mobile users [5], [6], [7], [8], [9]. However, for none of these algorithms theoretical results are found regarding which throughput guarantees that can be offered for different scheduling algorithms given a set of system parameters. Being able to do such simple analytical investigations can be an advantage when choosing, designing and comparing opportunistic scheduling algorithms. In addition, some of these publications take parameters like buffer overflow probability or battery power into account when conducting scheduling. Although these are important parameters for the mobile users, the variation in these parameters are often due to the behavior of the mobile users: A user is in most cases himself responsible for large traffic loads or low battery power, and a wireless service provider in many cases only be interested in offering a throughput guarantee within a given time window.

There are two types of throughput guarantees that can be offered to the customers, namely hard or deterministic throughput guarantees, and soft or stochastic throughput guarantees. The deterministic throughput guarantees promise, with unit probability, a certain bit-rate to each of the users within a given time window. Opportunistic scheduling gives priorities to the users with the best channel conditions (subject to varying constraints), and the times between each time a user is scheduled can therefore vary significantly. For some of the opportunistic scheduling algorithms it can therefore be difficult to fulfill deterministic guarantees over a time window. For such scenarios it can be a more realistic solution to provide soft throughput guarantees to the users. The soft throughput guarantees promise that each of the users will be allocated a specified throughput over a defined time window, with a probability less than unity. In this paper we analyze soft throughput guarantees for different system parameters and different scheduling policies. These guarantees are dependent on the available bandwidth, the channel characteristics, the number of users in the system, and the chosen scheduling policy.

Contributions. Quantifying the soft throughput guarantees that can be given for a certain scheduling algorithm without conducting experimental investigations have to the best of our knowledge not been looked into before. We obtain a general expression for an approximation of the throughput guarantee violation probability (TGVP) for a given mean and variance of the number of bits transmitted in a time-slot, and a given distribution for the number of time-slots allocated to a user within a time window. We also develop closed-form expressions for the corresponding soft throughput guarantee approximations that can be obtained for a set of system parameters and three different scheduling algorithms: (i) Round Robin Scheduling (RR), (ii) Maximum CNR Scheduling (MCS) and (iii) Opportunistic Round Robin Scheduling (ORR).

Organization. The rest of this paper is organized as follows. In Section II we present the system model. We develop a general expression for the approximation of a throughput guarantee in Section III. Next, in Section IV we develop expressions for the throughput guarantees achievable for several well-known scheduling algorithms. We show plots of the obtained expressions and compare these with simulated results in Section V. In Section VI we list our conclusions.
II. SYSTEM MODEL

We consider a single base station that serves \( N \) users using time-division multiplexing (TDM). The analysis conducted in this paper is valid both for the uplink and the downlink. In any case we assume that the total available uplink for the users is \( W \). Each user measures its own CNR perfectly, and before performing scheduling the base station is assumed to receive these measurements from all the users. For each time-slot the base station takes a scheduling decision and distributes this decision to the selected user before transmission starts.

It is assumed that the channels of all users are independently and identically distributed (i.i.d.) flat Rayleigh block-fading channels with average received CNR \( \bar{\gamma} \). The block or the time-slot duration \( T_{TS} \) is assumed to be one coherence time, i.e., the time for which the channels can be regarded more or less as constant. We also assume that the CNR values from time-slot to time-slot are uncorrelated. The two latter assumptions have been obtained by using the Central Limit Theorem (CLT) [12, p. 1231].

III. HOW TO QUANTIFY THE THROUGHPUT GUARANTEES

A soft or stochastic throughput guarantee can be expressed as the probability of not fulfilling a throughput guarantee, i.e., the throughput guarantee violation probability, TGVP. Defining the throughput guarantee as \( B \) bits over a time window of \( K \) time-slots for all the \( N \) users in the system, we can analytically define the problem as finding an TGVP less than \( \epsilon \) [10]:

\[
\Pr(b < B) \leq \epsilon, \tag{1}
\]

i.e., such that the probability of the number of bits \( b \) being assigned to a user within a time window of \( K \) time-slots being below \( B \) bits, is less than or equal to \( \epsilon \).

A. Obtaining Approximate Throughput Guarantee Violation Probabilities

To be able to obtain an exact TGVP we have to find a probability density function (PDF) for the sum of bits that a user can transmit in \( k \) time-slots. From [11] and several other publications, we conclude that finding an exact closed-form expression for the value of the TGVP \( \Pr(b < B) \) is a complex problem that has not yet been solved and may very well not be solvable. We will therefore look at how we can approximate the TGVP instead.

We now formulate a proposition that can be used as a tool to specify an achievable soft throughput guarantee of \( B \) bits over a time window of \( K \) time-slots. For i.i.d. users transmitting over a time-slotted block fading channel, with \( b_j \) bits assigned to the user scheduled in time-slot \( j \), and the probability that a user gets \( k \) time-slots denoted as \( p_k(k) \), the following holds:

**Proposition:** The probability that the throughput constraint \( B \) is violated over \( K \) time-slots can be approximated as:

\[
\Pr(b < B) \approx p_k(0) + \frac{1}{2} \sum_{k=1}^{K} p_k(k) \operatorname{erfc}\left(\frac{-B/k - \mu_{\bar{b}_k}}{\sqrt{2}\sigma_{\bar{b}_k}}\right), \tag{2}
\]

where \( \bar{b}_k = \frac{1}{K} \sum_{j=1}^{K} b_j \) is the average number of bits assigned to a user that is allocated \( k \) time-slots and \( \mu_{\bar{b}_k} \) and \( \sigma_{\bar{b}_k}^2 \) is the mean and variance of \( \bar{b}_k \), respectively. Also, \( \operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^2} dt \) is the complementary error function.

**Proof:** The allocation of a different number of time-slots to a user constitute mutually exclusive events. The TGVP for an arbitrary user over \( K \) time-slots can therefore be expressed as follows using the law of total probability:

\[
\Pr(b < B) = p_k(0) + \Pr(b < B|1) \cdot p_k(1) + \Pr(b < B|2) \cdot p_k(2) + \ldots + \Pr(b < B|K) \cdot p_k(K), \tag{3}
\]

where \( \Pr(b < B|k) \) denotes the TGVP when the user is assigned \( k \) time-slots and \( p_k(k) \) denote the probability that a user gets \( k \) time-slots within the interval of \( K \) time-slots.

To be able to discuss a total throughput guarantee \( B \) within \( K \) time-slots, we first consider the number of bits allocated to a certain user within the \( j \)th time-slot, and denote this number by \( b_j \). For a system using codes which operate at the Shannon limit we will have \( b_j = 0 \) if the user is not given the time slot; if the time-slot is allocated to the user we will however have \( b_j = T_{TS}W \log(1 + \gamma_j) \), where \( \gamma_j \) is the CNR experienced in time-slot \( j \).

We can now express the probability for violating the throughput guarantee \( B \) when \( k \) of \( K \) time-slots are scheduled to a user (\( K - k \) of the \( b_j \)'s are zero) as:

\[
\Pr(b < B|k) = \Pr\left(\sum_{j=1}^{k} b_j < B\right) = \Pr\left(\frac{1}{k} \sum_{j=1}^{k} b_j < \frac{B}{k}\right) = \Pr\left(\bar{b}_k < \frac{B}{k}\right) \approx \frac{1}{2} \operatorname{erfc}\left(-\frac{B/k - \mu_{\bar{b}_k}}{\sqrt{2}\sigma_{\bar{b}_k}}\right), \tag{4}
\]

where \( \operatorname{erfc}\left(-\frac{x - \mu}{\sqrt{2}\sigma}\right) = \Pr\left(\mathcal{N}(\mu, \sigma^2) \leq x\right) \) is the cumulative distribution function (CDF) of a Gaussian distributed random variable with mean \( \mu \) and variance \( \sigma^2 \). In the expression in (4) we have \( \mu_{\bar{b}_k} = \mu_{b_j} \) and \( \sigma_{\bar{b}_k}^2 = \sigma_{b_j}^2 / k \) where \( \mu_{b_j} \) and \( \sigma_{b_j}^2 \) are the mean and variance of the number of bits allocated to the scheduled user in time-slot \( j \). The approximation above has been obtained by using the Central Limit Theorem (CLT) [12, p. 1231].
By inserting (4) into (3), we see that the expression for the total throughput guarantee can be expressed as in (2).

IV. THROUGHPUT GUARANTEES FOR DIFFERENT SCHEDULING ALGORITHMS

The proposition in the previous section gives an approximation for the TGVP for any scheduling algorithm with a given mean and variance for the number of bits assigned in \( k \) time-slots, and a given PDF for the number of time-slots allocated to a user within a time window of \( K \) time-slots. In this section we investigate the throughput guarantee behavior of three known scheduling algorithms, namely (i) Round Robin Scheduling (RR), (ii) Max CNR Scheduling (MCS), and (iii) Opportunistic Round Robin Scheduling (ORR) [4].

A. Throughput Guarantees for Round Robin Scheduling

To have a reference algorithm we start by investigating RR, where the users are assigned time-slots in a sequential fashion. A Throughput Guarantees for Round Robin Scheduling (ORR) [4], (ii) Max CNR Scheduling (MCS), and (iii) Opportunistic Round Robin Scheduling (ORR) [4].

B. Throughput Guarantees for Max CNR Scheduling

For MCS, where the user with the highest CNR is chosen in each time-slot, the number of time-slots allocated to a user within \( K \) time-slots is distributed according to the binomial distribution:

\[
p_k(k) = \binom{K}{k} \left( \frac{1}{N} \right)^k \left( 1 - \frac{1}{N} \right)^{K-k}.
\]

For this scheduling policy, the CNR of the chosen user has the following CDF [1]:

\[
P_{\gamma^*}(\gamma) = P_{\gamma^*}^{N-1}(\gamma) p_{\gamma}(\gamma),
\]

Differentiating this expression we obtain the PDF of the CNR of the selected user:

\[
p_{\gamma^*}(\gamma) = N P_{\gamma^*}^{N-1}(\gamma) p_{\gamma}(\gamma),
\]

We can now use this PDF to obtain the mean value for \( \bar{b}_k \) for MCS, using a similar derivation as for [16, Eq. (44)]:

\[
\mu_{b_j} = W T_{TS} \int_0^\infty (\log(1 + \gamma)) p_{\gamma}(\gamma) d\gamma
\]

Similarly, we can obtain the second moment of the number of bits \( b_j \) allocated to the scheduled user in time-slot \( j \):

\[
E[b_j^2] = (W T_{TS})^2 \int_0^\infty (\log(1 + \gamma))^2 p_{\gamma}(\gamma) d\gamma
\]

C. Throughput Guarantees for Opportunistic Round Robin Scheduling

To increase the short-term fairness between the users, Kulkarni and Rosenberg introduced the ORR scheduling policy [17]. The original algorithm provides the highest short-term fairness when the time-slots are allocated in rounds of
For the first competition within a round, the best user out of all the users is selected. For each new competition the winner from the last competition is taken out. Consequently, only one user participates in the last competition of a round. The advantage of this algorithm is that it opportunistically takes advantage of this algorithm is that it opportunistically takes advantage of the channel conditions of the users and at the same time ensures that the allocated time-slots are evenly distributed among the users after every complete round.

Because fairness and throughput guarantees are related to each other we choose to investigate if this algorithm is suitable for guaranteeing throughputs in a system with opportunistic scheduling. For i.i.d. users, the order of the users within each round is arbitrary, is expressed as (5). Inserting this expression together with the expressions for the mean and variance of the CNR for a user getting time-slots can be expressed as the average over all the rounds:

\[
P_{\gamma^*}(\gamma|k = [K/N]) = \frac{1}{N} \sum_{n=1}^{N} P_{\gamma^*}^n(\gamma),
\]

where \( P_{\gamma^*}(\gamma) \) is the CDF of a single user with average CNR \( \gamma \). However, the CDF of the CNR for a user getting \( k = [K/N] \) out of \( K \) time-slots can be expressed as the average over all the rounds:

\[
P_{\gamma^*}(\gamma|k = [K/N]) = \frac{(k-1) \sum_{n=1}^{N} n P_{\gamma^*}^{n-1}(\gamma)}{kN} + \frac{\sum_{n=kN-K+1}^{N} n P_{\gamma^*}^{n-1}(\gamma)}{k(K-(k-1)N)}. \tag{15}\]

Differentiating these CDFs with regard to \( \gamma \), we obtain the corresponding PDFs:

\[
p_{\gamma^*}(\gamma|k = [K/N]) = \frac{1}{N} \sum_{n=1}^{N} nP_{\gamma^*}^{n-1}(\gamma)p_{\gamma^*}(\gamma), \tag{16}\]

and,

\[
p_{\gamma^*}(\gamma|k = [K/N]) = \frac{(k-1) \sum_{n=1}^{N} n P_{\gamma^*}^{n-1}(\gamma)p_{\gamma^*}(\gamma)}{kN} + \frac{\sum_{n=kN-K+1}^{N} n P_{\gamma^*}^{n-1}(\gamma)p_{\gamma^*}(\gamma)}{k(K-(k-1)N)}. \tag{17}\]

We can now express the first moment of \( \mu_{\beta_k} = \mu_{b_j} \) as:

\[
\mu_{\beta_k} = W T_{TS} \int_0^\infty \log_2(1+\gamma)p_{\gamma^*}(\gamma)d\gamma. \tag{18}\]

For \( k = [K/N] \) we have:

\[
\mu_{\beta_k} = \frac{W T_{TS}}{N} \sum_{n=1}^{N} A(n), \tag{19}\]

and for \( k = [K/N] \) we have:

\[
\mu_{\beta_k} = W T_{TS} \left( \frac{(k-1) \sum_{n=1}^{N} A(n)}{kN} + \frac{\sum_{n=kN-K+1}^{N} A(n)}{k(K-(k-1)N)} \right), \tag{20}\]

where \( A(n) \) is given by [4, Eq. (20)]:

\[
A(n) = \frac{n}{\ln 2} \sum_{j=0}^{n-1} \left( \frac{n-1}{1+j} \right)^{-\gamma} E_1 \left( \frac{1+j}{\gamma} \right). \tag{21}\]

Similarly, we can express the second moment of the number of bits allocated to the scheduled user \( b_j \) in time-slot \( j \) as:

\[
E[b_j^2|k] = (W T_{TS})^2 \int_0^\infty (\log_2(1+\gamma))^2 p_{\gamma^*}(\gamma)d\gamma. \tag{22}\]

For \( k = [K/N] \) we have:

\[
E \left[ b_j^2 \right] = \left[ \frac{K}{N} \right] \right] = \frac{(W T_{TS})^2}{N} \sum_{n=1}^{N} B(n), \tag{23}\]

and

\[
E \left[ b_j^2 \right] = \left[ \frac{K}{N} \right] \right] = \frac{(W T_{TS})^2}{N} \sum_{n=1}^{N} B(n) + \frac{(W T_{TS})^2}{N} \sum_{n=kN-K+1}^{N} B(n), \tag{24}\]

where \( B(n) \) is given by:

\[
B(n) = \frac{n}{\gamma(ln 2)^2} \sum_{j=0}^{n-1} \left( \frac{n-1}{j} \right)^{-\gamma} E_1 \left( \frac{1+j}{\gamma} \right). \tag{25}\]

The derivation of this expression can be found in [15]. As for the previous two algorithms we can obtain the variance as \( \sigma^2_{b_k} = E[b_j^2] - \mu^2_{b_k} \). The variance of \( b_k \) can now be expressed as \( \sigma^2_{b_k} = \sigma^2_{b_k}/k \). The distribution for the number of assigned time-slots \( p_k(k) \) is the same as for the RR algorithm where the order of the users within a round is arbitrary, is expressed as \( \gamma \). Inserting this expression together with the expressions for the mean and variance of \( b_k \) found above into (2) yields the TGVP approximation for ORR. As for the conventional RR algorithm \( p_k(0) \) is zero for \( K \geq N \).

V. Numerical Results

In this section we plot the expressions obtained in the previous section and we compare these plots with plots obtained from Monte Carlo (MC) simulations.

Figures 1, 2, and 3 show plots of the approximated TGVP for RR, MCS, and ORR, respectively. These plots have been
obtained from the expressions found in the previous section. The plots are shown for 10 users with i.i.d. Rayleigh fading channels with $\gamma = 15$ dB. The figures show how the approximated TGVP varies with the spectral efficiency guarantee per user $B/(WT_{TS})$ and the time-window the throughput guarantee is calculated for. As expected the TGVP decreases with increasing time-window and a decreasing throughput guarantee. For the RR and ORR algorithms it can be observed that the number of bits per second per Hertz that can be “hard” guaranteed (black area) is almost linearly increasing with length of the time-window $K$.

Figures 4, 5, and 6 show the corresponding TGVP found from MC simulations. For each point in the figures there were conducted 5000 MC trials. By comparing these figures with the plots of the approximate TGVP we see that our approximate results are very similar to what we would expect from exact expressions.

From our numerical results we see that the ORR algorithm are better suited than the RR and the MCS algorithms to provide throughput guarantees in wireless networks. In a real network it is possible to design hybrid schemes that allocate parts of the bandwidth to each of the different scheduling algorithms. For example can a network with a combination of real-time and non-real time traffic gain from such hybrid scheduling schemes. Because ORR gives stricter throughput guarantees over a short time horizon, it is better suited for real-time traffic. On the other hand MCS is better suited for applications that need high average bit-rates, but where the delay of the transmission is not too critical.

VI. Conclusion

In this paper we have developed a general approximation for the TGVP that can be obtained in a time-slotted wireless network with any scheduling policy with (i) a given set of system parameters, (ii) known first two moments of the bits allocated to the selected user, and, (iii) a given distribution of the number of time-slots allocated to a user within a time window. We investigated the corresponding TGVP approximations for Round Robin Scheduling, Max CNR Scheduling, and Opportunistic Round Robin Scheduling for a channel model that has uncorrelated fading from time-slot to time-slot. Our results showed that the latter scheduling algorithm is best suited when the network service provider wants to strictly guarantee moderate throughputs within a relatively short time window. It should also be noted that our simplified channel model gives optimistic TGVP values. Consequently, these violation probabilities will not be reached when the users have channels with high correlation from time-slot to time-slot. However, we expect the relative ranking of the three algorithms’ properties with respect to TGVP to be maintained.

Fig. 1. Approximated Throughput Guarantee Violation Probability for 10 users with i.i.d. Rayleigh fading channels with $\gamma = 15$ dB when Round Robin Scheduling is applied.

Fig. 2. Approximated Throughput Guarantee Violation Probability for 10 users with i.i.d. Rayleigh fading channels with $\gamma = 15$ dB when Max CNR Scheduling is applied.

Fig. 3. Approximated Throughput Guarantee Violation Probability for 10 users with i.i.d. Rayleigh fading channels with $\gamma = 15$ dB when Opportunistic Round Robin Scheduling is applied.
Fig. 4. Monte Carlo simulated Throughput Guarantee Violation Probability for 10 users with i.i.d. Rayleigh fading channels with $\gamma = 15$ dB when Round Robin Scheduling is applied.

Fig. 5. Monte Carlo simulated Throughput Guarantee Violation Probability for 10 users with i.i.d. Rayleigh fading channels with $\gamma = 15$ dB when Max CNR Scheduling is applied.

Fig. 6. Monte Carlo simulated Throughput Guarantee Violation Probability for 10 users with i.i.d. Rayleigh fading channels with $\gamma = 15$ dB when Opportunistic Round Robin Scheduling is applied.

also in such a scenario.

REFERENCES


