Cooperative Spatial Multiplexing with Hybrid Channel Knowledge

David Gesbert
Mobile Communications Department
Eurecom Institute
2229 Route des Crêtes, BP 193
F-06904 Sophia Antipolis Cédex, France
Email: gesbert@eurecom.fr

Aïre Hjørungnes
UniK - University Graduate Center
University of Oslo
Instituttveien 25, P. O. Box 70
N-2027 Kjeller, Norway
Email: arehj@unik.no

Hilde Skjeveling
Department of Informatics
University of Oslo
P. O. Box 1080, Blindern
N-0316 Oslo, Norway
Email: hildesk@ifi.uio.no

Abstract—We explore the concept of cooperative spatial multiplexing for use in MIMO multicell networks. One key application of this is the transmission of possibly correlated symbol streams jointly by several multiple-antenna access points toward multiple single antenna user terminals located in neighboring cells. To augment the realism of this setting, we consider different levels of channel state information at the transmitter (CSIT). In one case, we further introduce a constraint on hybrid channel state information (HCSI) in which any given transmitter knows its own CSI perfectly while it only has statistical information about other transmitters’ channels. This yield a game situation in which each cooperating transmitter makes a guess about the behavior of the other transmitter. We show different transmission strategies under this setting and compare them with fully cooperative (full CSI) and non cooperative schemes. Our results show a substantial cooperation gain despite the lack of instantaneous information.

I. INTRODUCTION

Recently, there has been increasing interest in so-called cooperative schemes, in which two or more transmitters collaborate to improve the quality of transmission toward a common destination. A prominent scenario for this has so far been cooperative diversity where the devices collaborate to combat the detrimental effects of fading at any one particular device. Typically, the devices are single-antenna user terminals relaying data between a source terminal and the target destination [1], [2], [3]. A specific signaling scheme is distributed space-time block coding (STBC), where the spatial elements of the codewords are distributed over the antennas of the collaborating transmitters [3], [4], [5]. Since this is space time coding, the transmitters can operate with little or no CSI, although it was recently shown that statistical channel information can be very useful there too [6].

Another form of cooperation is that of distributed spatial multiplexing in which multiple independent flows of data are jointly transmitted by distributed antennas and captured by one multiple-antenna receiver or several distributed (possibly single antenna) receivers. This is typically relevant in the downlink of a multicell scenario where multiple base stations (access points for a WLAN network) want to transmit data to multiple user terminals at once. This problem bears strong connections with multiuser MIMO. Work in this area include [7], [8], [9], [10] to cite a few. Importantly, we note that, unlike cooperative diversity, cooperative spatial multiplexing or downlink multi-user MIMO in general requires full CSI at the transmitter(s) when the user terminals have a single antenna. This case is very realistic since we expect a majority of pocket terminals to be single antenna based in the foreseeable future. We consider different levels of CSIT used for cooperation.

In one, we assume for all transmitters at the base sites to share the full joint multi-user CSIT. For cases where this would demands unbearable cell-to-cell signaling we also consider a hybrid CSI scenario where one transmitting base has full knowledge of its own CSI linking it to the terminals belonging to its coverage region but has only statistical knowledge about the other transmitters’ CSI. Here, we limit ourselves to a two cell scenario while more general cases are considered in a companion paper [11]. This scenario is equivalent to a cooperative game where each base station is optimizing a linear spatial filter based on an guess of what the collaborating base might be doing simultaneously. We investigate several possible guess strategies and obtain the corresponding optimal linear transmission schemes. We show that cooperation gains are substantial even when statistical CSI is used. However we show that one benefits a lot from well educated guesses.

II. SYSTEM DESCRIPTION

We consider a two-cell downlink situation shown in Figure 1 where two base stations BS0 and BS1, equipped with Mt antennas each, communicate with two single-antenna mobile stations MS0 and MS1. In our cooperative scenario, MSi is associated with its base station BSi but also receives part of its data multiplexed from BSj, j ̸= i, in the neighboring cell. Specifically, two symbols \( \{s_0, s_1\} \) are sent.

coverage regions of different sites are typically overlapping.
from the following symbol vector
\[ s = \begin{bmatrix} s_0 \\ s_1 \end{bmatrix}, \] (1)
such that \( s \in \mathbb{C}^{2 \times 1} \), with \( s_i \in \mathcal{A} \), where \( \mathcal{A} \) is the signal constellation alphabet.

Symbol \( s_i \) is intended for mobile station \( \text{MS}_i \) only, \( i \in \{0, 1\} \).

However, for cooperation purposes we assume that each base is given a copy of the global symbol vector \( s \) (through relay or fibers etc.).

The autocorrelation of the vector \( s \) is given by:
\[ \Phi_s = \mathbb{E} [ss^H], \] (2)
where \( \Phi_s \in \mathbb{C}^{2 \times 2} \). In the case of independent, unit-variance data, we thus get \( \Phi_s = I_2 \).

A. Complete and Hybrid Channel State Information

Assuming flat-fading, the channel from base station number \( i \) to the two mobile stations is given by matrix \( \mathbf{H}_i \in \mathbb{C}^{2 \times M_t} \). In this paper we consider a fully non line of sight situation (channels have zero mean), however, the Ricean case is also interesting [11].

Our goal is to test the impact of cooperation given various levels of knowledge shared by the transmitting bases about each other’s transmit channels. For this purpose we consider two cases i) complete channel state information, in which both bases know all CSIT from each base to each terminal, and ii) Hybrid channel state information, in the latter, CSIT is hybrid in the sense that, for each \( i=0,1 \):
- its own transmit channel \( \mathbf{H}_i \),
- but only has knowledge of the statistics of
  - the neighboring base station’s CSIT \( \mathbf{H}_j, i \neq j \)

In the hybrid CSIT case (illustrated in Fig.2), the statistics are assumed to be exchanged by the base stations via a low rate signaling channel. This scenario can be considered realistic because statistical channel information varies much more slowly than Rayleigh fading and is thus easy to broadcast to the various cells.

III.Linear Transmit Filtering

Each base \( \text{BS}_i \) transmits the common data vector \( s \) by performing linear filtering with the coefficient matrix \( \mathbf{A}_i \in \mathbb{C}^{M_t \times 2} \) such that the transmitted vector \( \mathbf{x}_i \in \mathbb{C}^{M_t \times 1} \) from \( \text{BS}_i \) is given by:
\[ \mathbf{x}_i = \mathbf{A}_i s. \] (3)

The signal received by \( \text{MS}_i \) is denoted \( y_i \), where \( i \in \{0, 1\} \). We set
\[ y = \begin{bmatrix} y_0 \\ y_1 \end{bmatrix}. \] (4)
which is the received signal vector \( y \in \mathbb{C}^{2 \times 1} \) given by
\[ y = \mathbf{H}_0 \mathbf{A}_0 s + \mathbf{H}_1 \mathbf{A}_1 s + \mathbf{v}, \] (5)
where \( \mathbf{v} \in \mathbb{C}^{2 \times 1} \) is the additive noise vector with auto correlation matrix
\[ \Phi_v = \mathbb{E} [\mathbf{v} \mathbf{v}^H], \] (6)
such that \( \Phi_v \in \mathbb{C}^{2 \times 2} \).

A. Power Constraints

We wish to optimize the transmit filters \( \mathbf{A}_i \), \( i = 0, 1 \) and we consider per-base power constraints, thus, we set for \( i \in \{0, 1\} \):
\[ \text{Tr} \{ \mathbf{A}_i \Phi_s \mathbf{A}_i^H \} = P_i. \] (7)

IV. Optimal Spatial Filtering

We consider now several strategies for linear filtering based on the transmit MMSE criterion, starting with the case where full CSI is shared by all base stations.

A. Full CSI Optimal Filtering

In this case, full CSIT is assumed however the optimization differs from that of a regular multi-user MIMO problem due to the presence of the per-base power constraints (instead of a single transmit power constraint in regular downlink MIMO) which restricts the scope of optimality. Nevertheless, we expect this case to serve as an upper bound on the cooperation gain.

Here, both bases optimize \( \mathbf{A}_0 \) and \( \mathbf{A}_1 \) jointly based on the combined knowledge of \( \mathbf{H}_0 \) and \( \mathbf{H}_1 \). The received signal is given by (5). The optimal filters are optimal in the joint MMSE sense where the mean square error (MSE) is given by
\[ \text{MSE} = \mathbb{E}_s [\| y - s \|_2^2] = \mathbb{E}_s \left[ \text{Tr} \left( (\mathbf{H}_0 \mathbf{A}_0 s + \mathbf{H}_1 \mathbf{A}_1 s + \mathbf{v} - s) (\mathbf{H}_0^H \mathbf{A}_0^H \mathbf{H}_0 + s^H s^H s + s^H s + s^H s) \right) \right] \]
\[ = \text{Tr} \left( (\mathbf{H}_0 \mathbf{A}_0 \Phi_v \mathbf{A}_0^H \mathbf{H}_0^H + \mathbf{H}_1 \mathbf{A}_1 \Phi_v \mathbf{A}_1^H \mathbf{H}_1^H + \mathbf{H}_0 \mathbf{A}_0 \Phi_v \mathbf{A}_0^H \mathbf{H}_1^H + \mathbf{H}_1 \mathbf{A}_1 \Phi_v \mathbf{A}_1^H \mathbf{H}_0^H - \mathbf{H}_0 \mathbf{A}_0 \Phi_v \mathbf{H}_0^H + \mathbf{H}_1 \mathbf{A}_1 \Phi_v \mathbf{H}_1^H + \mathbf{H}_0 \mathbf{A}_0 \Phi_v \mathbf{H}_0^H - \mathbf{H}_0 \mathbf{A}_0 \Phi_v \mathbf{H}_0^H - \mathbf{H}_1 \mathbf{A}_1 \Phi_v \mathbf{H}_1^H + \mathbf{H}_0 \mathbf{A}_0 \Phi_v \mathbf{H}_1^H + \mathbf{H}_1 \mathbf{A}_1 \Phi_v \mathbf{H}_0^H) \right) \] (8)

To optimize the filters under the distributed power constraints, we use the Lagrangian method with the objective function given by
\[ \text{MSE} + \mu_0 \text{Tr} \{ \mathbf{A}_0 \Phi_v \mathbf{A}_0^H \} + \mu_1 \text{Tr} \{ \mathbf{A}_1 \Phi_v \mathbf{A}_1^H \} \] (9)

By differentiation with respect to \( \mathbf{A}_0 \) and \( \mathbf{A}_1 \), the equations for the optimal precoders are given by
\[ \begin{bmatrix} \mathbf{H}_0^H \mathbf{H}_0 + \mu_0 \mathbf{I}_{M_t} \\ \mathbf{H}_1^H \mathbf{H}_0 \\ \mathbf{H}_0^H \mathbf{H}_1 + \mu_1 \mathbf{I}_{M_t} \end{bmatrix} \begin{bmatrix} \mathbf{A}_0 \\ \mathbf{A}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_0^H \\ \mathbf{H}_1^H \end{bmatrix}, \] (10)

which is independent of symbol correlation and where \( \mu_0 \) and \( \mu_1 \) must be chosen such that the power constraints are satisfied.

B. Hybrid CSI Optimal Filtering

We now turn to the case of hybrid CSI. In this situation, the optimal joint MMSE beamformer cannot be obtained. Instead, each base is optimizing its linear filter in the MMSE sense, based on limited knowledge. We propose to consider a game-like approach in which a base makes a guess as to what criterion the other base is optimizing its linear filter. We assume first full CSI, then we show how statistical CSI can be incorporated in this setting for two different kinds of guesses, one simplistic namely matched filter.
(MRC) guess and and one more computationally advanced, channel inversion guess. More strategies (e.g. single base MMSE guess) are discussed in [11].

C. Transmit MRC Guess

Here, BS\textsubscript{0} is operating under the assumption that BS\textsubscript{1} uses a scaled matched filter for transmission (and vice versa) given by:

$$A_1 = \sqrt{P_s} \frac{H_H^H}{\|H_H^H\|_F}.$$  \hspace{1cm} (11)

In the view of BS\textsubscript{0} the MSE expression, assuming (11), becomes

$$\text{MSE} = E_{s,M} \left[ \|y - s\|^2 \right] = E_{s,M} \left\{ \text{Tr} \left[ \left( H_0 A_0 s + \frac{\sqrt{P_s}}{\|H_H^H\|_F} H_H^H s + v - s \right) \left( s^H A_0^H H_0^H + \frac{\sqrt{P_s}}{\|H_H^H\|_F} s^H H_H^H s + v^H - s^H \right) \right] \right\}.$$  \hspace{1cm} (12)

Now, since $H_1$ is actually unknown to base station 0, we average over its realizations to obtain

$$E_{H_1} \text{[MSE]} = \text{Tr} \left\{ H_0 A_0 \Phi_s, H_1^H \right\} - H_0 A_0 \Phi_s + \frac{\sqrt{P_s}}{\|H_H^H\|_F} \left\{ H_1^H H_0^H \Phi_s + P_{r,E_{H_1}} \left[ H_1^H H_1^H \Phi_s, H_1^H H_1^H \right] \right\} - \frac{\sqrt{P_s}}{\|H_H^H\|_F} \left\{ H_1^H H_1^H \Phi_s, H_1^H H_1^H \right\} + \Phi_s \right\}.$$  \hspace{1cm} (14)

Two Lagrangian multipliers are introduced for the power constraints and then the optimization with respect to the precoder in BS\textsubscript{0} can be done. Here, the necessary conditions for optimality of $A_0$ under the considered cooperation strategy is given by:

$$H_0^H H_0 A_0 \Phi_s + \frac{\sqrt{P_s}}{\|H_H^H\|_F} E_{H_1} \left[ H_1^H H_1^H \Phi_s, H_1^H H_1^H \right] + \mu_0 A_0 \Phi_s = 0_{M_t \times 2}.$$  \hspace{1cm} (15)

Assuming that the matrix $\Phi_s$ is invertible, the necessary conditions for optimality can be rewritten as:

$$A_0 = \left[ H_0^H H_0 + \mu_0 I_{M_t} \right]^{-1} H_0^H \left( I_2 - \frac{\sqrt{P_s}}{\|H_H^H\|_F} E_{H_1} \left[ H_1^H H_1^H \right] \right).$$  \hspace{1cm} (16)

By symmetry we argue that BS\textsubscript{1} will operate with the following linear filter:

$$A_1 = \left[ H_1^H H_1 + \mu_1 I_{M_t} \right]^{-1} H_1^H \left( I_2 - \frac{\sqrt{P_0}}{\|H_H^H\|_F} E_{H_0} \left[ H_H^H H_H^H \right] \right).$$  \hspace{1cm} (17)

Note that these equation can be interpreted as modified MMSE transmit filters, where the modification makes the use of the cooperating base’s channel statistics.

Note also that in the equations derived, the following statistic $S_i \triangleq E_{H_i} \left[ H_i^H H_i^H \right]^{-1}$ is needed. An estimate of this matrix $\hat{S}_i$ can simply be found by the following estimator:

$$\hat{S}_i = \frac{1}{Q} \sum_{q=0}^{Q-1} H_i^{(q)} (H_i^{(q)})^H \left[ \|H_i^{(q)}\|_F^2 \right]^{-1}.$$  \hspace{1cm} (18)

where $H_i^{(q)}$ is a realization of the channel found from the following relation:

$$\text{vec}(H_i) = R_i^{1/2} \text{vec}(H_{w_i}),$$  \hspace{1cm} (19)

where $\text{vec}(H_{w_i}) \sim \mathcal{CN}(0_{M_t \times 1}, I_{M_t M_t})$ is used to indicate the distribution of the vector $\text{vec}(H_{w_i})$, and where $R_i \triangleq E_{H_i} \left[ \text{vec}(H_i) \text{vec}^H(H_i) \right]$ is the covariance matrix.

D. Transmit Zero-Forcing Guess

Here, we look at the case where BS\textsubscript{1} assumes that BS\textsubscript{1} uses a scaled-down zero-forcing (ZF), on the form of

$$A_1 = \sqrt{P_s} \frac{H_H^H \left( H_H^H H_H^H \right)^{-1}}{\|H_H^H \left( H_H^H H_H^H \right)^{-1}\|_F}.$$  \hspace{1cm} (20)

The expected MSE under this assumption, averaged over all realizations of the unknown $H_1$, becomes:

$$E_{H_1} \text{[MSE]} = E_{H_1} \left[ \|H_0 A_0 s + H_1 A_1 s + v - s\|^2 \right] = E_{H_1} \left[ \text{Tr} \left( H_0 A_0 s + \frac{\sqrt{P_s}}{\|H_H^H\|_F} H_H^H s + v - s \right) \left( s^H A_0^H H_0^H + \frac{\sqrt{P_s}}{\|H_H^H\|_F} s^H H_H^H s + v^H - s^H \right) \right]$$

$$= \text{Tr} \left( H_0 A_0 s + \frac{\sqrt{P_s}}{\|H_H^H\|_F} H_H^H s + v - s \right) \left[ H_1^H H_1^H \right]^{-1} \frac{\sqrt{P_s}}{\|H_H^H\|_F} \left( s^H H_H^H s + v^H - s^H \right).$$

Using again Lagrangian multipliers, we obtain the following necessary conditions for optimality of $A_0$:

$$H_0^H H_0 A_0 + \frac{\sqrt{P_s}}{\|H_H^H\|_F} E_{H_1} \left[ H_1^H H_1^H \right]^{-1} H_H^H H_0^H \Phi_s + \mu_0 A_0 \Phi_s = 0_{M_t \times 2}.$$  \hspace{1cm} (21)

This can be rewritten as:

$$A_0 = \left[ H_0^H H_0 + \mu_0 I_{M_t} \right]^{-1} H_0^H \left[ 1 - E_{H_1} \left[ H_1^H H_1^H \right]^{-1} \right].$$

By symmetry, we obtain for the transmit filter for BS\textsubscript{1}:

$$A_1 = \left[ H_1^H H_1 + \mu_1 I_{M_t} \right]^{-1} H_1^H \left[ 1 - E_{H_0} \left[ H_H^H H_H^H \right]^{-1} \right].$$

Let the operator $(\cdot)^d$ denote the Moore-Penrose inverse. In an analogous way as before, finding the matrix $U_i \triangleq E_{H_i} \left[ H_i^H H_i^H \right]^{-1} = E_{H_i} \left[ H_i^H H_i^H \right]^{-1} \|H_i^H H_i^H\|_F$ can be done using

$$\hat{U}_i = \frac{1}{Q} \sum_{q=0}^{Q-1} \frac{1}{\|H_i^{(q)}\|_F^2}.$$  \hspace{1cm} (22)

These results again correspond to modified MMSE filters, where the modification takes into account the statistics of the unknown channel and the guess in the filtering strategy of the cooperating base.
V. SIMULATION RESULTS

We have simulated a system consisting of two transmitting base stations (BS) and two receiving MSs. Each of the BSs are equipped with $M_t = 2$ antennas, while there are only $M_r = 1$ antenna on each MS.

We obtain the mean square error (MSE) versus CSNR (channel signal to noise ratio), for 4 different approaches at optimizing the precoders $A_0$ and $A_1$, based on different cases of channel state information at the BS side. The CSNR is defined as $\text{CSNR} = P / \sigma_n^2$, where $P = P_0 = P_1 = 0.5$ is the power available at both BS. Thus the noise level is what is varied to change the CSNR. Note that no receiver optimization is used, although this problem is tackled in [11]. The MSE at each CSNR-point is averaged over 1000 channel realizations.

With the fully cooperative (full CSI) case as an example, we observe that the optimal precoders $A_0$ and $A_1$ are found by jointly optimizing with respect to $A_i$ and $\mu_i$, $i \in \{0, 1\}$. The precoders are given by (10), using the per-base power constraint in (7) to determine the Lagrangian multipliers $\mu_i$, $i \in \{0, 1\}$.

We define the intercell loss ratio $r_{\text{ipl}}$ as the ratio between own-cell average channel gain and inter-cell channel gain. In Figure 3, $r_{\text{ipl}} = 0$ dB, i.e., signals from both BS experience the same average large-scale path loss on their way to both MS. Figure 4 shows the case of $r_{\text{ipl}} = 3$ dB.

In Figure 3, the best MSE-results are obtained by the full CSI cooperative approach. As expected the proposed hybrid CSI schemes yield performance in between the optimal and the non cooperative scenarios. Clearly playing the cooperative game yields better result using the ZF assumption than assuming transmit MRC, as the optimal MMSE filter is much closer to the ZF in moderate to high SNR levels.

In Figure 4, understandably we observe that $r_{\text{ipl}} = 3$ dB (more intercell loss) hurts the full CSI cooperative scheme, while it benefits the suboptimal but more practical approaches using hybrid CSI, as well as the non-cooperative approach. This means that there is clearly an interesting trade-off between the use of aggressive reuse factors of the frequency resource which increases interference but in turn amplifies cooperation gains. The trade-off seems clearly in favor of cooperation.

VI. CONCLUSIONS

We investigate spatial multiplexing signaling between two cooperating base stations/access points communicating each with users in two neighboring cells. We propose practical transmission strategies that exploit either complete or hybrid (mixed instantaneous and statistical) channel state information at the transmitters. We show that cooperation is possible thanks to a game scenario where each base station is making certain assumptions about the behavior of the cooperating base in terms of the spatial filter being used.

REFERENCES