11

Multiple Antenna Techniques

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11.1 Fundamentals of Multiple Antenna Theory

11.1.1 Overview

The value of multiple antenna systems as a means to improve communications was recognized in the very early ages of wireless transmission. However, most of the scientific progress in understanding their fundamental capabilities has occurred only in the last 20 years, driven by efforts in signal and information theory, with a key milestone being achieved with the invention of so-called Multiple-Input Multiple-Output (MIMO) systems in the mid-1990s.

Although early applications of beamforming concepts can be traced back as far as 60 years in military applications, serious attention has been paid to the utilization of multiple antenna techniques in mass-market commercial wireless networks only since around 2000. The first such attempts used only the simplest forms of space-time processing algorithms. Today, the key role which MIMO technology plays in the latest wireless communication standards for Personal, Wide and Metropolitan Area Networks (PANs, WANs and MANs) testifies to its anticipated importance. Aided by rapid progress in the areas of computation and circuit integration, this trend culminated in the adoption of MIMO for the first time in a cellular mobile network standard in the Release 7 version of HSDPA (High Speed Downlink Packet Access); soon after, the development of LTE broke new ground in being the first global mobile cellular system to be designed with MIMO as a key component from the start.

In this chapter, we first provide the reader with the theoretical background necessary for a good understanding of the role and advantages promised by multiple antenna techniques in wireless communications in general. We focus on the intuition behind the main technical

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results and show how key progress in information theory yields practical lessons in algorithm and system design for cellular communications. As can be expected, there is still a gap between the theoretical predictions and the performance achieved by schemes that must meet the complexity constraints imposed by commercial considerations.

We distinguish between single-user MIMO and multi-user MIMO theory and techniques (see below for a definition), although a common set of concepts captures the essential MIMO benefits in both cases. Single-user MIMO techniques dominate the algorithms selected for LTE, with multi-user MIMO not being used to the maximum extent in the first version of LTE, despite its potential.

Following an introduction of the key elements of MIMO theory, in both the single-user and the multi-user cases, we proceed to describe the actual methods adopted for LTE, paying particular attention to the factors leading to these choices. However, the main goal of this section is not to provide exhaustive tutorial information on MIMO systems (for which the reader may refer, for example, to [1–3]) but rather to explain the combination of underlying theoretical principles and system design constraints which influenced specific choices for LTE.

While traditional wireless communications (Single-Input Single-Output (SISO)) exploit time- or frequency-domain pre-processing and decoding of the transmitted and received data respectively, the use of additional antenna elements at either the base station (eNodeB) or User Equipment (UE) side (on the downlink and/or uplink) opens an extra spatial dimension to signal precoding and detection. So-called space-time processing methods exploit this dimension with the aim of improving the link's performance in terms of one or more possible metrics, such as the error rate, communication data rate, coverage area and spectral efficiency (bps/Hz/cell).

Depending on the availability of multiple antennas at the transmitter and/or the receiver, such techniques are classified as Single-Input Multiple-Output (SIMO), Multiple-Input Single-Output (MISO) or MIMO. Thus in the scenario of a multi-antenna enabled base station communicating with a single antenna UE, the uplink and downlink are referred to as SIMO and MISO respectively. When a (high-end) multi-antenna terminal is involved, a full MIMO link may be obtained, although the term MIMO is sometimes also used in its widest sense, thus including SIMO and MISO as special cases. While a point-to-point multiple-antenna link between a base station and one UE is referred to as Single-User MIMO (SU-MIMO), Multi-User MIMO (MU-MIMO) features several UEs communicating simultaneously with a common base station using the same frequency- and time-domain resources.[1] By extension, considering a multicell context, neighbouring base stations sharing their antennas in virtual MIMO fashion to communicate with the same set of UEs in different cells will be termed multicell multi-user MIMO (although this latter scenario is not supported in the first version of LTE, and is therefore addressed only in outline in the context of future versions in Chapter 24). The overall evolution of MIMO concepts, from the simplest diversity setup to the futuristic multicell multi-user MIMO, is illustrated in Figure 11.1.

Despite their variety and sometimes perceived complexity, single-user and multi-user MIMO techniques tend to revolve around just a few fundamental principles, which aim at

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[1] Note that in LTE a single eNodeB may in practice control multiple cells; in such a case, we consider each cell as an independent base station for the purpose of explaining the MIMO techniques; the simultaneous transmissions in the different cells address different UEs and are typically achieved using different fixed directional physical antennas; they are therefore not classified as multi-user MIMO.

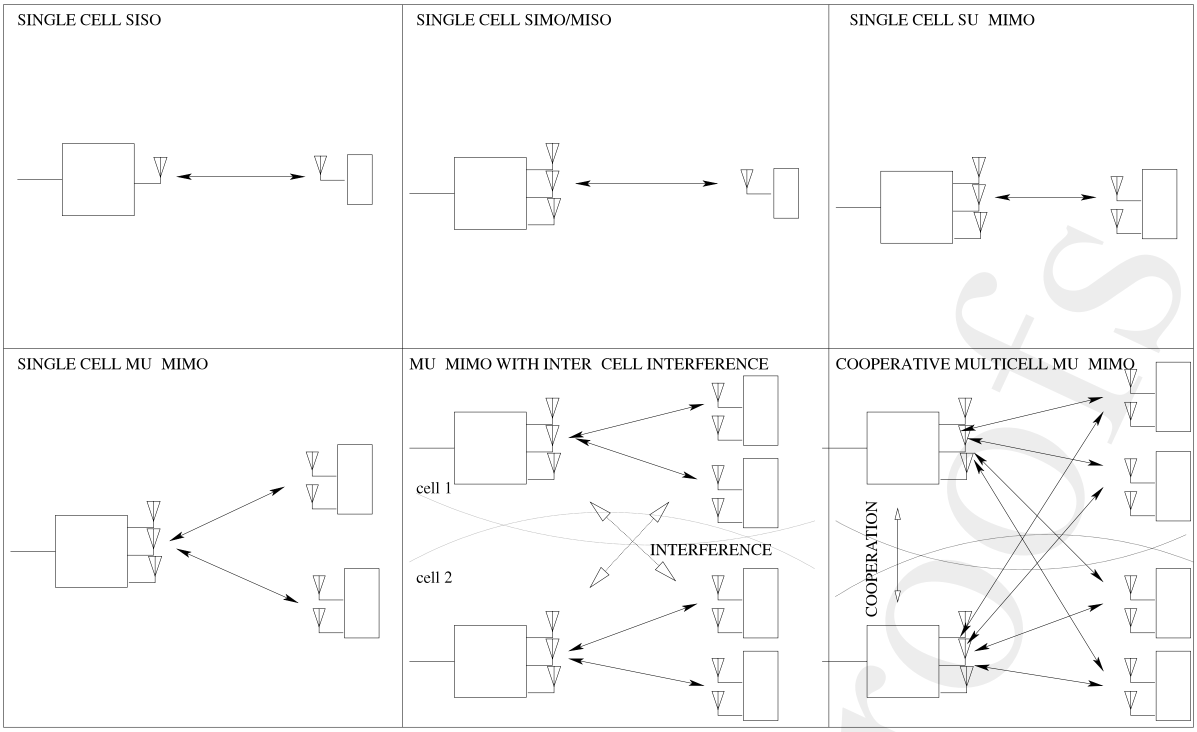

Figure 11.1 The evolution of MIMO technology, from traditional single antenna communication, to multi-user MIMO scenarios, to the possible multicell MIMO networks of the future.

leveraging some key properties of multi-antenna radio propagation channels. As introduced in Section 1.3, there are basically three advantages associated with such channels (over their SISO counterparts):

- Diversity gain.
- Array gain.
- Spatial multiplexing gain.

Diversity gain corresponds to the mitigation of the effect of multipath fading, by means of transmitting or receiving over multiple antennas at which the fading is sufficiently decorrelated. It is typically expressed in terms of an *order*, referring either to the number of effective independent diversity branches or to the slope of the bit error rate curve as a function of the Signal-to-Noise Ratio (SNR) (or possibly in terms of an SNR gain in the system's link budget).

While diversity gain is fundamentally related to improvement of the *statistics* of instantaneous SNR in a fading channel, array gain and multiplexing gain are of a different nature, rather being related to geometry and the theory of vector spaces. Array gain corresponds to a spatial version of the well-known matched-filter gain in time-domain receivers, while multiplexing gain refers to the ability to send multiple data streams in parallel and to separate them on the basis of their spatial signature. The latter is much akin to the multiplexing of users separated by orthogonal spreading codes, timeslots or frequency assignments, with the great advantage that, unlike Code, Time or Frequency Division Multiple Access (CDMA, TDMA or FDMA), MIMO multiplexing does not come at the cost of bandwidth expansion; it does, however, suffer the expense of added antennas and signal processing complexity.

These aspects are analysed further by introducing a common signal model and notation for the main families of MIMO techniques. The model is valid for single-user MIMO, yet it is sufficiently general to capture all the key principles mentioned above, as well as being easily extensible to the multi-user MIMO case (see Section 11.2.3). The model is first presented in a general way, covering theoretically optimal transmission schemes, and then particularized to popular MIMO approaches. We consider models for both uplink and downlink, or when possible a generic formulation which includes both possibilities. LTE-related schemes, specifically for the downlink, are addressed subsequently. We focus on the Frequency Division Duplex (FDD) case. Discussion of some aspects of MIMO which are specific to Time Division Duplex (TDD) operation can be found in Section 23.5.

11.1.2 MIMO Signal Model

Let $\mathbf{Y}$ be a matrix of size $N \times T$ denoting the set of (possibly precoded) signals being transmitted from N distinct antennas over T symbol durations (or, in the case of some frequency-domain systems, T subcarriers), where T is a parameter of the MIMO algorithm (defined below). Thus the n^{th} row of $\mathbf{Y}$ corresponds to the signal emitted from the n^{th} transmit antenna. Let $\mathbf{H}$ denote the $M \times N$ channel matrix modelling the propagation effects from each of the N transmit antennas to any one of M receive antennas, over an arbitrary subcarrier whose index is omitted here for simplicity. We assume $\mathbf{H}$ to be invariant over T symbol durations. The matrix channel is represented by way of example in Figure 11.2. Then the $M \times T$ signal $\mathbf{R}$ received over T symbol durations over this subcarrier can be conveniently written as

$$\mathbf{R} = \mathbf{H}\mathbf{Y} + \mathbf{N} \tag{11.1}$$

where $\mathbf{N}$ is the additive noise matrix of dimension $M \times T$ over all M receiving antennas. We will use $\mathbf{h}_i$ to denote the i^{th} column of $\mathbf{H}$, which will be referred to as the *receive spatial signature* of (i.e. corresponding to) the i^{th} transmitting antenna. Likewise, the j^{th} row of $\mathbf{H}$ can be termed the *transmit spatial signature* of the j^{th} receiving antenna.

Mapping the symbols to the transmitted signal

Let $\mathbf{X} = (x_1, x_2, \ldots, x_P)$ be a group of P QAM symbols to be sent to the receiver over the T symbol durations. Thus these symbols must be *mapped* to the transmitted signal $\mathbf{Y}$ before launching into the air. The choice of this mapping function $\mathbf{X} \rightarrow \mathbf{Y}(\mathbf{X})$ determines which one out of several possible MIMO transmission methods results, each yielding a different combination of the diversity, array and multiplexing gains. Meanwhile, the so-called *spatial* rate of the chosen MIMO transmission method is given by the ratio P/T.

Note that, in the most general case, the considered transmit (or receive) antennas may be attached to a single transmitting (or receiving) device (base station or UE), or distributed over different devices. The symbols in $(x_1, x_2, \ldots, x_P)$ may also correspond to the data of one or possibly multiple users, giving rise to the so-called single-user MIMO or multi-user MIMO models.

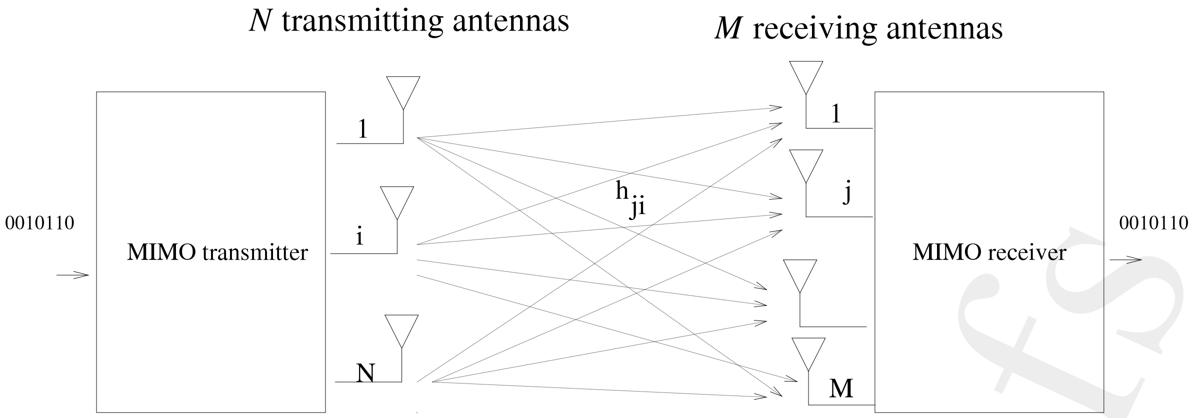

Figure 11.2 Simplified transmission model for a MIMO system with N transmit antennas, M receive antennas, giving rise to a $M \times N$ channel matrix, with MN links.

In the following sections, we explain classical MIMO techniques to illustrate the basic principles of this technology. We first assume a base station to single-user communication. The techniques are then generalized to multi-user MIMO situations.

11.1.3 Single-User MIMO Techniques

Several classes of SU-MIMO transmission methods are discussed below, both optimal and suboptimal.

11.1.3.1 Optimal Transmission over MIMO Systems

The optimal way of communicating over the MIMO channel involves a channel-dependent precoder, which fulfils the roles of both transmit beamforming and power allocation across the transmitted streams, and a matching receive beamforming structure. Full channel knowledge is therefore required at the transmit side for this method to be applicable. Consider a set of $P = NT$ symbols to be sent over the channel. The symbols are separated into N streams (or layers) of T symbols each. Stream i consists of symbols $[x_{i,1}, x_{i,2}, \ldots, x_{i,T}]$. Note that in an ideal setting, each stream may adopt a distinct code rate and modulation. This is clarified below. The transmitted signal can now be written as

$$\mathbf{Y}(\mathbf{X}) = \mathbf{V}\mathbf{P}\bar{\mathbf{X}} \tag{11.2}$$

where

$$\bar{\mathbf{X}} = \begin{pmatrix} x_{1,1} & x_{1,2} & \ldots & x_{1,T} \\ \vdots & \vdots & & \vdots \\ x_{N,1} & x_{N,2} & \ldots & x_{N,T} \end{pmatrix} \tag{11.3}$$

and where $\mathbf{V}$ is an $N \times N$ transmit beamforming matrix, and $\mathbf{P}$ is a $N \times N$ diagonal power-allocation matrix with $\sqrt{p_i}$ as its i^{th} diagonal element, where p_i is the power allocated to the i^{th} stream. Of course, the power levels must be chosen so as not to exceed the available transmit power, which can often be conveniently expressed as a constraint on the total normalized transmit power P_t.[2] Under this model, the information-theoretic capacity of the MIMO channel in bps/Hz can be obtained as [3]

$$C_{\text{MIMO}} = \log_2 \det(I + \rho \mathbf{H}\mathbf{V}\mathbf{P}^2\mathbf{V}^{\text{H}}\mathbf{H}^{\text{H}}) \tag{11.4}$$

where $\{\cdot\}^{\text{H}}$ denotes the Hermitian operator for a matrix or vector and ρ is the so-called transmit SNR, given by the ratio of the transmit power over the noise power.

The optimal (capacity-maximizing) precoder (**VP**) in Equation (11.4) is obtained by the concatenation of *singular vector beamforming* and the so-called *waterfilling power allocation*.

Singular vector beamforming means that **V** should be a unitary matrix (i.e. $\mathbf{V}^{\text{H}}\mathbf{V}$ is the identity matrix of size N) chosen such that $\mathbf{H} = \mathbf{U}\Sigma\mathbf{V}^{\text{H}}$ is the Singular-Value Decomposition[3] (SVD) of the channel matrix **H**. Thus the i^{th} right singular vector of **H**, given by the i^{th} column of **V**, is used as a transmit beamforming vector for the i^{th} stream. At the receiver side, the optimal beamformer for the i^{th} stream is the i^{th} left singular vector of **H**, obtained as the i^{th} row of $\mathbf{U}^{\text{H}}$:

$$\mathbf{u}_i^{\text{H}}\mathbf{R} = \lambda_i \sqrt{p_i}[x_{i,1}, x_{i,2}, \ldots, x_{i,T}] + \mathbf{u}_i^{\text{H}}\mathbf{N} \tag{11.5}$$

where λ_i is the i^{th} singular value of **H**.

Waterfilling power allocation is the optimal power allocation and is given by

$$p_i = [\mu - 1/(\rho\lambda_i^2)]_+ \tag{11.6}$$

where $[x]_+$ is equal to x if x is positive and zero otherwise. μ is the so-called 'water level', a positive real variable which is set such that the total power constraint is satisfied.

Thus the optimal SU-MIMO multiplexing scheme uses SVD-based transmit and receive beamforming to decompose the MIMO channel into a number of parallel non-interfering subchannels, dubbed 'eigen-channels', each one with an SNR being a function of the corresponding singular value λ_i and chosen power level p_i.

Contrary to what would perhaps be expected, the philosophy of optimal power allocation across the eigen-channels is *not* to equalize the SNRs, but rather to render them more unequal, by 'pouring' more power into the better eigen-channels, while allocating little power (or even none at all) to the weaker ones because they are seen as not contributing enough to the total capacity. This waterfilling principle is illustrated in Figure 11.3.

The underlying information-theoretic assumption here is that the information rate on each stream can be adjusted finely to match the eigen-channel's SNR. In practice this is done by selecting a suitable Modulation and Coding Scheme (MCS) for each stream.

11.1.3.2 Beamforming with Single Antenna Transmitter or Receiver

In the case where either the receiver or the transmitter is equipped with only a single antenna, the MIMO channel exhibits only one active eigen-channel, and hence multiplexing of more than one data stream is not possible.

In *receive* beamforming, $N = 1$ and $M > 1$ (assuming a single-stream). In this case, one symbol is transmitted at a time, such that the symbol-to-transmit-signal mapping function is

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[2] In practice there may be a limit on the maximum transmission power from each antenna.

[3] The reader is referred to [4] for an explanation of generic matrix operations and terminology.

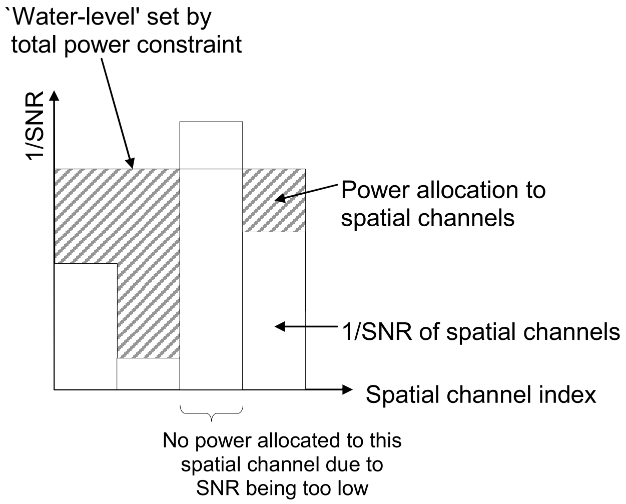

Figure 11.3 The waterfilling principle for optimal power allocation.

characterized by $P = T = 1$, and $\mathbf{Y}(\mathbf{X}) = \mathbf{X} = x$, where x is the one QAM symbol to be sent. The received signal vector is given by

$$\mathbf{R} = \mathbf{H}x + \mathbf{N} \tag{11.7}$$

The receiver combines the signals from its M antennas through the use of weights $\mathbf{w} = [w_1, \ldots, w_M]$. Thus the received signal after antenna combining can be expressed as

$$z = \mathbf{wR} = \mathbf{wH}s + \mathbf{wN} \tag{11.8}$$

After the receiver has acquired a channel estimate (as discussed in Chapter 8), it can set the beamforming vector $\mathbf{w}$ to its optimal value to maximize the received SNR. This is done by aligning the beamforming vector with the UE's channel, via the so-called Maximum Ratio Combining (MRC) $\mathbf{w} = \mathbf{H}^{\mathrm{H}}$, which can be viewed as a spatial version of the well-known matched filter. Note that cancellation of an interfering signal can also be achieved, by selecting the beamforming vector to be orthogonal to the channel from the interference source. These simple concepts are illustrated vectorially in Figure 11.4.

The maximum ratio combiner provides a factor of M improvement in the received SNR compared to the $M = N = 1$ case – i.e. an array gain of $10\log_{10}(M)$ dB in the link budget.

In *transmit* beamforming, $M = 1$ and $N > 1$. The symbol-to-transmit-signal mapping function is characterized by $P = T = 1$, and $\mathbf{Y}(\mathbf{X}) = \mathbf{w}x$, where x is the one QAM symbol to be sent. $\mathbf{w}$ is the transmit beamforming vector of size $N \times 1$, computed based on channel knowledge, which is itself often obtained via a receiver-to-transmitter feedback link.[4] Assuming perfect channel knowledge at the transmitter side, the SNR-maximizing solution is given by the transmit MRC, which can be seen as a matched prefilter:

$$\mathbf{w} = \frac{\mathbf{H}^{\mathrm{H}}}{\|\mathbf{H}\|} \tag{11.9}$$

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[4] In some situations other techniques such as receive/transmit channel reciprocity may be used, as discussed in Section 23.5.2.

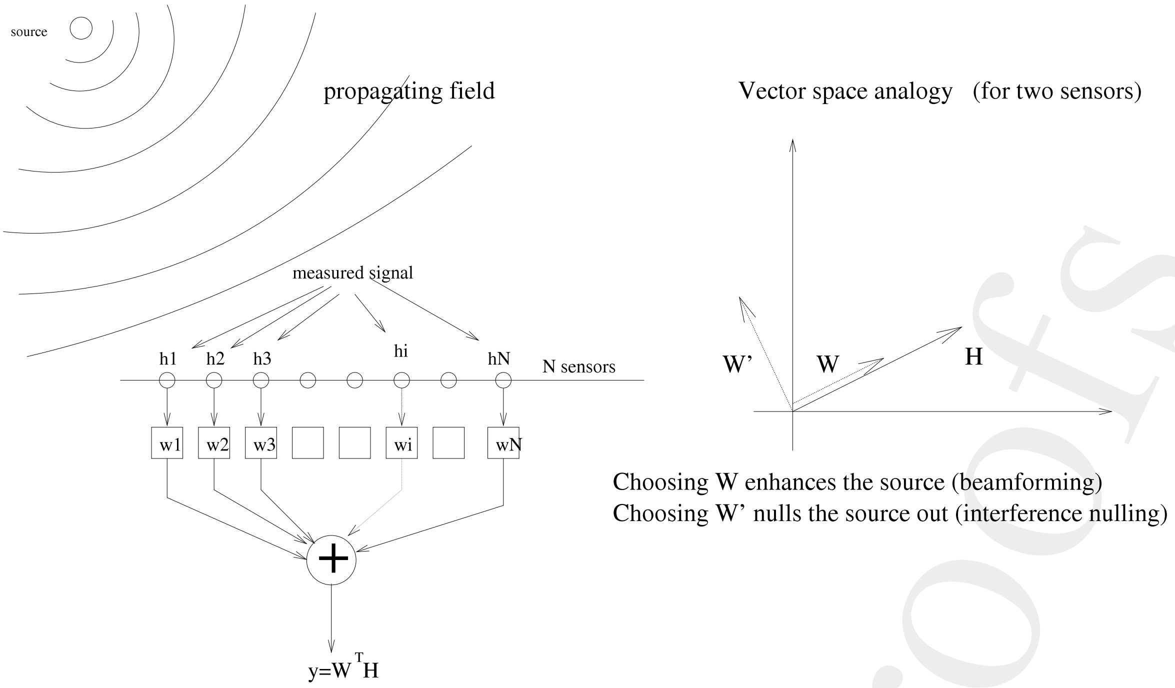

Figure 11.4 The beamforming and interference cancelling concepts.

where the normalization by $\|\mathbf{H}\|$ enforces a total power constraint across the transmit antennas. The transmit MRC pre-filter provides a similar gain as its receive counterpart, namely $10\log_{10}(N)$ dB in average SNR improvement.

11.1.3.3 Spatial Multiplexing without Channel Knowledge at the Transmitter

When $N > 1$ and $M > 1$, multiplexing of up to $\min(M, N)$ streams is theoretically possible even without transmit channel knowledge. Assume for instance that $M \geq N$. In this case one considers N streams, each transmitted using one different transmitted antenna. As the transmitter does not have knowledge of the matrix $\mathbf{H}$, the design of the spatial multiplexing scheme cannot be improved by the use of a channel-dependent precoder. Thus the precoder is simply the identity matrix. In this case, the symbol-to-transmit-signal mapping function is characterized by $P = NT$ and by

$$\mathbf{Y}(\mathbf{X}) = \bar{\mathbf{X}} \tag{11.10}$$

At the receiver, a variety of linear and non-linear detection techniques may be implemented to recover the symbol matrix $\bar{\mathbf{X}}$. A low-complexity solution is offered by the linear case, whereby the receiver superposes N beamformers $\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_N$.

The detection of stream $[x_{i,1}, x_{i,2}, \ldots, x_{i,T}]$ is achieved by applying $\mathbf{w}_i$ as follows:

$$\mathbf{w}_i\mathbf{R} = \mathbf{w}_i\mathbf{H}\bar{\mathbf{X}} + \mathbf{w}_i\mathbf{N} \tag{11.11}$$

The design criterion for the beamformer $\mathbf{w}_i$ can be interpreted as a compromise between single-stream beamforming and cancelling of interference (created by the other $N-1$ streams). Inter-stream interference is fully cancelled by selecting the Zero-Forcing (ZF) receiver given by

$$\mathbf{W} = \begin{pmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \vdots \\ \mathbf{w}_N \end{pmatrix} = (\mathbf{H}^{\mathrm{H}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{H}} \tag{11.12}$$

However, for optimal performance, $\mathbf{w}_i$ should strike a balance between alignment with respect to $\mathbf{h}_i$ and orthogonality with respect to all other signatures $\mathbf{h}_k$, $k \neq i$. Such a balance is achieved by, for example, a Minimum Mean-Squared Error (MMSE) receiver.

Beyond classical linear detection structures such as the ZF or MMSE receivers, more advanced but nonlinear detectors can be exploited which provide a better error rate performance at the chosen SNR operating point, at the cost of extra complexity. Examples of such detectors include the Successive Interference Cancelling (SIC) detector and the Maximum Likelihood Detector (MLD). The principle of the SIC detector is to treat individual streams, which are channel-encoded, like layers which are peeled off one by one by a processing sequence consisting of linear detection, decoding, remodulating, re-encoding and subtraction from the total received signal $\mathbf{R}$. On the other hand, the MLD attempts to select the most likely set of all streams, simultaneously, from $\mathbf{R}$, by an exhaustive search procedure or a lower-complexity equivalent such as the sphere-decoding technique [3].

Multiplexing gain

The multiplexing gain corresponds to the multiplicative factor by which the spectral efficiency is increased by a given scheme. Perhaps the single most important requirement for MIMO multiplexing gain to be achieved is for the various transmit and receive antennas to experience a sufficiently different channel response. This translates into the condition that the spatial signatures of the various transmitters (the $\mathbf{h}_i$'s) (or receivers) be sufficiently decorrelated and linearly independent to allow for the channel matrix $\mathbf{H}$ to be invertible (or more generally, well-conditioned). An immediate consequence of this condition is the limitation to $\min(M, N)$ of the number of independent streams which may be multiplexed into the MIMO channel, or more generally to $\mathrm{rank}(\mathbf{H})$ streams. As an example, single-user MIMO communication between a four-antenna base station and a dual antenna UE can, at best, support multiplexing of two data streams, and thus a doubling of the UE's data rate compared with a single stream.

11.1.3.4 Diversity

Unlike the basic multiplexing scenario in Equation (11.10), where the design of the transmitted signal matrix $\mathbf{Y}$ exhibits no redundancy between its entries, a diversity-oriented design will feature some level of repetition between the entries of $\mathbf{Y}$. For 'full diversity', each transmitted symbol $x_1, x_2, \ldots, x_P$ must be assigned to each of the transmit antennas at least once in the course of the T symbol durations. The resulting symbol-to-transmit-signal mapping function is called a Space-Time Block Code (STBC). Although many designs of STBC exist, additional properties such as the orthogonality of matrix $\mathbf{Y}$ allow improved performance and easy decoding at the receiver. Such properties are realized by the so-called Alamouti space-time code [5], explained later in this chapter. The total diversity order which can be realized in the N to M MIMO channel is MN when entries of the MIMO channel matrix are statistically uncorrelated. The intuition behind this is that $MN - 1$ represents the number of SISO links simultaneously in a state of severe fading which the system can sustain while still being able to convey the information to the receiver. The diversity order is equal to this number plus one. As in the previous simple multiplexing scheme, an advantage of diversity-oriented transmission is that the transmitter does not need knowledge of the channel $\mathbf{H}$, and therefore no feedback of this parameter is necessary.

Diversity versus multiplexing trade-off

A fundamental aspect of the benefits of MIMO lies in the fact that any given multiple antenna configuration has a limited number of degrees of freedom. Thus there exists a compromise between reaching full beamforming gain in the detection of a desired stream of data and the perfect cancelling of undesired, interfering streams. Similarly, there exists a trade-off between the number of streams that may be multiplexed across the MIMO channel and the amount of diversity that each one of them will enjoy. Such a trade-off can be formulated from an information-theoretic point of view [6]. In the particular case of spatial multiplexing of N streams over a N to M antenna channel, with $M \geq N$, and using a linear detector, it can be shown that each stream will enjoy a diversity order of $M - N + 1$.

To some extent, increasing the spatial load of MIMO systems (i.e. the number of spatially-multiplexed streams) is akin to increasing the user load in CDMA systems. This correspondence extends to the fact that an optimal load level exists for a given target error rate in both systems.

11.1.4 Multi-User Techniques

11.1.4.1 Comparing Single-User and Multi-User MIMO

The set of MIMO techniques featuring data streams being communicated to (or from) antennas located on distinct UEs in the model is referred to as Multi-User MIMO (MU-MIMO). Although this situation is just as well described by our model in Equation (11.1), the MU-MIMO scenario differs in a number of crucial ways from its single-user counterpart. We first explain these differences qualitatively, and then present a brief survey of the most important MU-MIMO transmission techniques.

In MU-MIMO, K UEs are selected for simultaneous communication over the same time-frequency resource, from a set of U active UEs in the cell. Typically, K is much smaller than U. Each UE is assumed to be equipped with J antennas, so the selected UEs together form a set of $M = KJ$ UE-side antennas. Since the number of streams that may be communicated over an N to M MIMO channel is limited to $\min(M, N)$ (if complete interference suppression is intended using linear combining of the antennas), the upper bound on the number of streams in MU-MIMO is typically dictated by the number of base station antennas N. The number of streams which may be allocated to each UE is limited by the number of antennas J at that UE. For instance, with single-antenna UEs, up to N streams can be multiplexed, with a distinct stream being allocated to each UE. This is in contrast to SU-MIMO, where the transmission of N streams necessitates that the UE be equipped with at least N antennas. Therefore a great advantage of MU-MIMO over SU-MIMO is that the MIMO multiplexing benefits are preserved even in the case of low-cost UEs with a small number of antennas. As a result, it is generally assumed that in MU-MIMO it is the base station which bears the burden of spatially separating the UEs, be it on the uplink or the downlink. Thus the base station performs receive beamforming from several UEs on the uplink and transmit beamforming towards several UEs on the downlink.

Another fundamental contrast between SU-MIMO and MU-MIMO comes from the difference in the underlying channel model. While in SU-MIMO the decorrelation between the spatial signatures of the antennas requires rich multipath propagation or the use of orthogonal polarizations, in MU-MIMO the decorrelation between the signatures of the different UEs occurs naturally due to fact that the separation between such UEs is typically large relative to the wavelength.

11.1.4.2 Techniques for Single-Antenna UEs

In considering the case of MU-MIMO for single-antenna UEs, it is worth noting that the number of antennas available to a UE for transmission is typically less than the number available for reception. We therefore examine first the uplink scenario, followed by the downlink.

With a single antenna at each UE, the MU-MIMO uplink scenario is very similar to the one described by Equation (11.10): because the UEs in mobile communication systems such as LTE typically cannot cooperate and do not have knowledge of the uplink channel coefficients, no precoding can be applied and each UE simply transmits an independent message. Thus, if K UEs are selected for transmission in the same time-frequency resource, each UE k transmitting symbol s_k, the received signal at the base station, over a single $T = 1$ symbol period, is written

$$\mathbf{R} = \mathbf{H}\bar{\mathbf{X}} + \mathbf{N} \tag{11.13}$$

where

$$\bar{\mathbf{X}} = \begin{pmatrix} x_1 \\ \vdots \\ x_K \end{pmatrix} \tag{11.14}$$

In this case, the columns of $\mathbf{H}$ correspond to the receive spatial signatures of the different UEs. The base station can recover the transmitted symbol information by applying beamforming filters, for example using MMSE or ZF solutions (as in Equation (11.12)). Note that no more than N UEs can be served (i.e. $K \leq N$) if inter-user interference is to be suppressed fully.

MU-MIMO in the uplink is sometimes referred to as ‘Virtual MIMO’, as from the point of view of a given UE there is no knowledge of the simultaneous transmissions of the other UEs. This transmission mode and its implications for LTE are discussed in Section 17.5.2.

On the downlink, which is illustrated in Figure 11.5, the base station must resort to transmit beamforming in order to separate the data streams intended for the various UEs. Over a single $T = 1$ symbol period, the signal received by UEs 1 to K can be written compactly as

$$\mathbf{R} = \begin{pmatrix} r_1 \\ \vdots \\ r_K \end{pmatrix} = \mathbf{HVP}\bar{\mathbf{X}} + \mathbf{N} \tag{11.15}$$

This time, the *rows* of $\mathbf{H}$ correspond to the transmit spatial signatures of the various UEs. $\mathbf{V}$ is the transmit beamforming matrix and $\mathbf{P}$ is the (diagonal) power allocation matrix selected

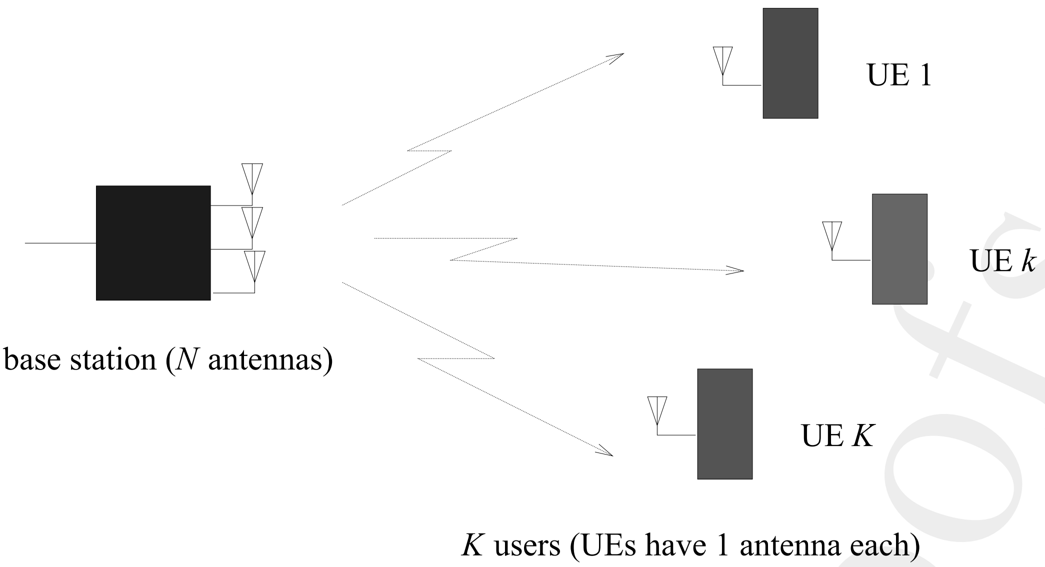

Figure 11.5 A MU-MIMO scenario in the downlink with single-antenna UEs: the base station transmits to K selected UEs simultaneously. Their contributions are separated by multiple-antenna precoding at the base station side, based on channel knowledge.

such that it fulfils the total normalized transmit power constraint P_t. To cancel out fully the inter-user interference when $K \leq N$, a transmit ZF beamforming solution may be employed (although this is not optimal due to the fact that it may require a high transmit power if the channel is ill-conditioned). Such a solution would be given by

$$\mathbf{V} = \mathbf{H}^{\mathrm{H}}(\mathbf{H}\mathbf{H}^{\mathrm{H}})^{-1} \tag{11.16}$$

Note that regardless of the channel realization, the power allocation must be chosen to satisfy any power constraints at the base station, for example such that trace$(\mathbf{V}\mathbf{P}\mathbf{P}^{\mathrm{H}}\mathbf{V}^{\mathrm{H}}) = P_t$.

11.1.4.3 Techniques for Multiple-Antenna UEs

The ideas presented above for single-antenna UEs can be generalized to the case of multiple-antenna UEs. There could, in theory, be essentially two ways of exploiting the additional antennas at the UE side. In the first approach, the multiple antennas are simply treated as multiple *virtual* UEs, allowing high-capability terminals to receive or transmit more than one stream, while at the same time spatially sharing the channel with other UEs. For instance, a four-antenna base station could theoretically communicate in a MU-MIMO fashion with two UEs equipped with two antennas each, allowing two streams per UE, resulting in a total multiplexing gain of four. Another example would be that of two single-antenna UEs, receiving one stream each, and sharing access with another two-antenna UE, the latter receiving two streams. Again, the overall multiplexing factor remains limited to the number of base-station antennas.

The second approach for making use of additional UE antennas is to treat them as extra degrees of freedom for the purpose of strengthening the link between the UE and the base station. Multiple antennas at the UE may then be combined in MRC fashion in the case of the downlink, or in the case of the uplink space-time coding could be used. Antenna selection is another way of extracting more diversity out of the channel, as discussed in Section 17.5.

11.1.4.4 Comparing Single-User and Multi-User capacity

To illustrate the gains of multi-user multiplexing over single-user transmission, we compare the sum-rate achieved by both types of system from an information theoretic standpoint, for single antenna UEs. We compare the Shannon capacity in single-user and multi-user scenarios both for an idealized synthetic channel and for a channel obtained from real measurement data.

The idealized channel model assumes that the entries of the channel matrix $\mathbf{H}$ in Equation (11.13) are independently and identically distributed (i.i.d.) Rayleigh fading. For **Q10** the measured channel case, a channel sounder was used[5] to perform real-time wideband channel measurements synchronously for two UEs moving at vehicular speed in an outdoor semi-urban hilly environment with Line-Of-Sight (LOS) propagation predominantly present. The most important parameters of the platform are summarized in Table 11.1.

Table 11.1 Parameters of the measured channel for SU-MIMO/MU-MIMO comparison. More details can be found in references [7, 8].

Parameter	Value
Centre frequency	1917.6 MHz
Bandwidth	4.8 MHz
Base station transmit power	30 dBm
Number of antennas at base station	4 (2 cross polarized)
Number of UEs	2
Number of antennas at UE	1
Number of subcarriers	160

The sum-rate capacity of a two-UE MU-MIMO system (calculated assuming a zero-forcing precoder as described in Section 11.1.4) is compared with the capacity of an equivalent MISO system serving a single UE at a time (i.e. in TDMA), employing beamforming (see Section 11.1.3.2). The base station has four antennas and the UE has a single antenna. Full Channel State Information at the Transmitter (CSIT) is assumed in both cases.

Figure 11.6 shows the ergodic (mean) sum-rate of both schemes in both channels. The mean is taken over all frames and all subcarriers and subsequently normalized to bps/Hz. It can be seen that in both the ideal and the measured channels, MU-MIMO yields a higher sum-rate than SU-MISO in general. In fact, at high SNR, the multiplexing gain of the MU-MIMO system is two while it is limited to one for the SU-MISO case.

However, for low SNR, the SU-MISO TDMA and MU-MIMO schemes perform very similarly. This is because a sufficiently high SNR is required to excite more than one MIMO transmission mode. Interestingly, the performance of both SU-MISO TDMA and MU-MIMO

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[5] The Eurecom MIMO OpenAir Sounder (EMOS) [7].

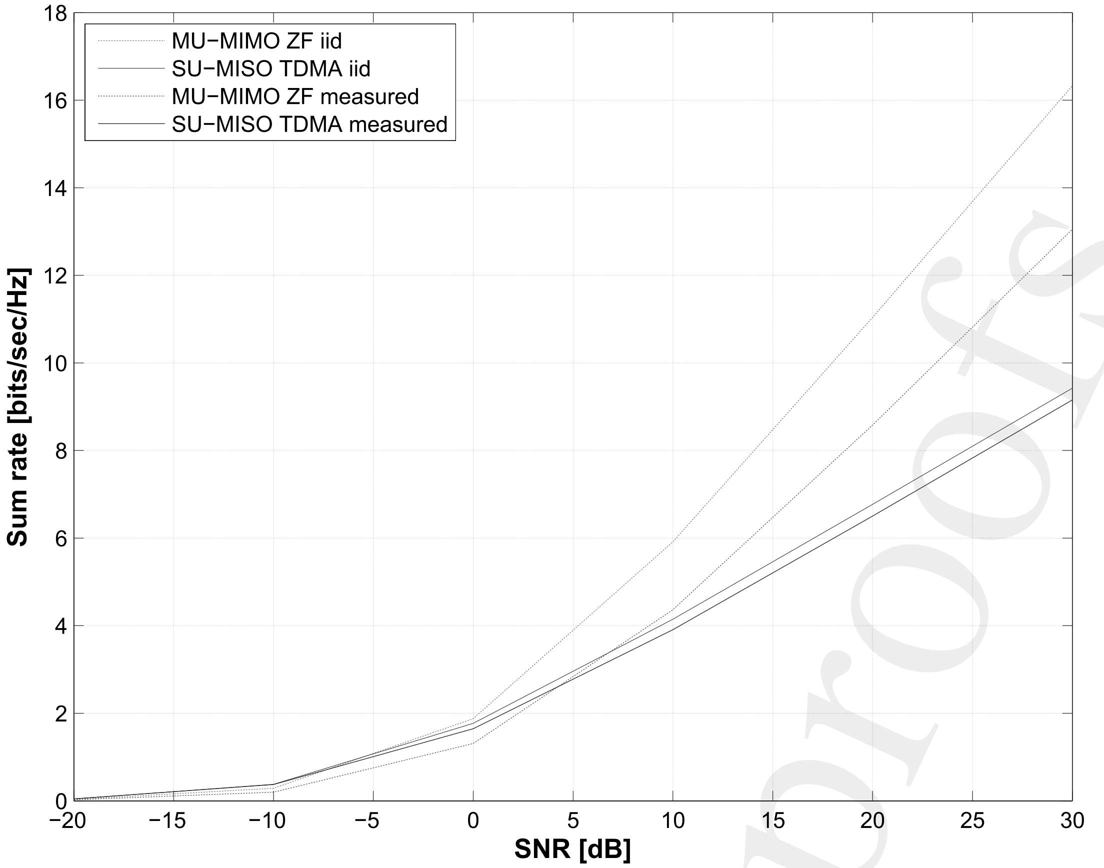

Figure 11.6 Ergodic sum-rate capacity of SU-MISO TDMA and MU-MIMO with two UEs, for an i.i.d. Rayleigh fading channel and for a measured channel.

is slightly worse in the measured channels than in the idealized i.i.d. channels. This can be attributed to the correlation of the measured channel in time (due to the relatively slow movement of the users), in frequency (due to the LOS propagation), and in space (due to the transmit antenna correlation). In the MU-MIMO case the difference between the i.i.d. and the measured channel is much higher than in the single-user TDMA case, since these correlation effects result in a rank-deficient channel matrix.

11.2 MIMO Schemes in LTE

Building on the theoretical background of the previous section, the MIMO schemes adopted for LTE are reviewed and explained. These schemes relate to the downlink unless otherwise mentioned.

11.2.1 Practical Considerations

First, a few important practical constraints are briefly reviewed which affect the real-life performance of the theoretical MIMO systems considered above, and which often are