Coordination on the MISO Interference Channel Using the Virtual SINR Framework

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Outline

Motivation
  Cooperation in multi-cell/link systems
  MISO IC

System Model and Performance Measures

Virtual SINR Framework
  Definition
  Two link case

Proposed Algorithm

Numerical Results
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Why Cooperate?

- In cellular systems, *reuse 1* considered for increased spectral efficiency.
- But cells are not isolated.

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- Hence, interest in cooperative schemes:
  1. Network MIMO
  2. Interference Avoidance
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Scenario considered

Cooperation Issues:

▶ Data sharing
▶ Channel state information (CSI) sharing

We consider:

▶ MISO interference channel (IC)
▶ local channel information
  ➤ Transmitter $k$ knows:

\[ h_{ki}, i = 1, \ldots, K \]
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System Model
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\[ h_{11}, \ldots, h_{1K} \]
System Model
Linear precoding at each TX:

\[ x_k = \sqrt{p_k} w_k s_k, \]
\[ \text{s.t. } \|w_k\| = 1, \ p_k \leq P \]

Single user decoding at each RX:

\[ \gamma_k = \frac{p_k |h_{kk} w_k|^2}{\sigma^2 + \sum_{j \neq k} p_j |h_{jk} w_j|^2} \]
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How to distributely design the \( \mathbf{w}_k \)?
Performance Measures

The rate region $\mathcal{R}$ is defined as the set of rates that may be achieved simultaneously at the different base stations, given the power constraints at each base station. I.e.:

$$\mathcal{R} = \{(R_1, \ldots, R_K) \in \mathbb{R}_+^K \mid R_k = \log_2(1 + \gamma_k), p_k \leq P \forall k \in \{1, \ldots, K\}\}$$ (1)

Its boundary is the set of Pareto optimal rate-tuples: one cannot increase any $R_k$ without decreasing at least one of the other rates.
The higher the sum rate, the better. Being close to the Pareto boundary is also desirable.
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Virtual SINR

Definition

- General form:

\[
\gamma^\text{virtual}_k = \frac{p_k |h_{kk}w_k|^2}{\sigma^2 + \sum_{j\neq k} \alpha_{kj} p_k |h_{kj}w_k|^2},
\]

where \( \alpha_{kj} \in \mathbb{R}_+ \), \( j, k = 1, \ldots, K \) are a given set of weights.

- Ratio of useful power generated to sum of noise plus weighted sum of interference caused.

- Specialization to full-power use:

\[
\gamma^\text{virtual}_k = \frac{|h_{kk}w_k|^2}{1/p + \sum_{j\neq k} \alpha_{kj} |h_{kj}w_k|^2},
\]

where \( \rho = \frac{P}{\sigma^2} \).
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- General form:

\[ \gamma_k^{\text{virtual}} = \frac{p_k |h_{kk} w_k|^2}{\sigma^2 + \sum_{j \neq k} \alpha_{kj} p_k |h_{kj} w_k|^2}, \] (2)

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- Ratio of useful power generated to sum of noise plus weighted sum of interference caused.

- Specialization to full-power use:

\[ \gamma_k^{\text{virtual}} = \frac{|h_{kk} w_k|^2}{\frac{1}{\rho} + \sum_{j \neq k} \alpha_{kj} |h_{kj} w_k|^2}, \] (3)

where \( \rho = \frac{P}{\sigma^2} \).
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Theorems

Theorem
Any point on the Pareto boundary may be attained by solving the virtual SINR optimization problem with full power use, for an appropriate choice of $\alpha_{12}, \alpha_{21} \in \mathbb{R}^+$.  

Theorem
The rate pair obtained by using virtual SINR maximizing beamformers with $\alpha_{12} = \alpha_{21} = 1$ lies on the Pareto boundary of the two-link rate region.
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Results from the Two-link case

Illustration
Proposed Algorithm

- Always use full power (this is also optimal for the multi-link case provided $N_t \geq K$): $p_k = P, \forall k = 1, \ldots, K$.

- Design beamforming vectors as the solutions to the following virtual SINR maximization problem:

$$w_k = \arg \max_{\|w\|^2=1} \frac{|h_{kk}w|^2}{\frac{1}{\rho} + \sum_{j \neq k} |h_{kj}w|^2}.$$  \hfill (4)
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Numerical Results

Sum rate

For 7 cells and a channel model including path loss, lognormal slow fading and Rayleigh fast fading:

![Graph showing sum rate vs SNR edge for different algorithms and number of transmit antennas.](image-url)
Numerical Results For 7 cells and a channel model including path loss, lognormal slow fading and Rayleigh fast fading:

![Graph showing extra power used vs. SNR edge for different numbers of tiers, Nt = 2, 4, 5.]
Conclusion

- A distributed algorithm for beamforming on the MISO IC was proposed.
- Its optimality was illustrated for the two-link case, in terms of achieving rates on the Pareto boundary.
- Simulation results illustrate that gains are achieved in the more general case as well.

Outlook

- Different models of local CSI could be considered.
- A more general model of cooperation could be looked at under different CSI conditions.
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Thank You! Questions?