Distributed Admission Control and Resource Allocation in Ad-hoc Networks

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Ad Hoc Network
Introduction

An example of the pure physical layer approach

⇒ alternate between possible power allocations per node and per fading state → produce basic rate matrix (transmission scheme) → optimizing a utility function (e.i. linear combination of transmission schemes in TDMA mode).

An example of cross-layer approach

Cross-layer approach ⇒ define the transmission rate as a function of topology state and link control action and add buffers → Stability and network capacity → Networking inside and outside the network capacity region → admission control
Motivations

We are searching for a model that:

- Makes the connection between channel capacity, and the real transmission rates (number of packets to be transmitted) ⇒ joint power and rate allocation
- Works both inside and outside the stability region ⇒ finite length buffers and admission control mechanism
- Supports the self-forming and self-haling properties of ad hoc network ⇒ decentralized algorithms
System Model(1)

$N_T$ transmitters, $N_R$ receivers; $N_R \leq N_T$, $N_T$ one-hop transmissions

Information theoretic models of such a network are:

- Full cooperation of the receivers in the sense that they even share the data $\Rightarrow$ MAC
- Full cooperation of the transmitters in the sense that they even share the data $\Rightarrow$ MIMO-BC
- No cooperation in the sense that the data is not shared but still some side information (e.i. state information) can be exchanged $\Rightarrow$ Interference Channel
  - no/partial/perfect side information

Our problem: interference channel with partial side information

Note: One node has an exact knowledge of its own state and a stochastical knowledge of states of other communication pairs. The Random variables are choosen from discrete sets.
System Model(2): Channel

- The time is uniformly slotted.
- The channel in time slot $t \in \mathbb{N}$: $N_T \times N_R$ matrix $Y(t)$ where $y^j_i(t)$ is the power attenuation of the channel between transmitter $i$ and receiver $j$.
- Each CS $y^j_i(t)$ is modelled as an ergodic Markov chain taking values in the discrete set $E$ of cardinality $L$.
- we define a bijection between the set $E$ and the set of the natural numbers $\{0, 1, ..., L-1\}$, $\varphi : E \rightarrow \{0, 1, ..., L-1\}$.
- $T(i, j)$: Transition matrix of the Markov chain; $\sum_{\ell=1}^{L} T_{k\ell}(i, j) = 1$. 
System Model(3): Buffer State and Arrival Process

Each transmitter is endowed with a buffer of finite length

- $B_i$: maximum length of the buffer at node $i$
- $q_i(t)$, queue state (QS): number of queued packets at the beginning of slot $t$.

At each node, packets arrive from the upper layer according to

- iid arrival process $\gamma_i(t)$, $t \in \mathbb{N}$ with arrival rate $\lambda_i$.
- Packets have constant length.
System Model(4): Definition of State

The following different state definitions will be later used in the formulations:

- Transmitter state (TS): $x_i(t) = (y_i(t), q_i(t)) \Rightarrow$ definition of the policy
- Receiver state (RS): $x^i(t) = (y^i(t), q_i(t))$ \Rightarrow calculation of the SINR at $d_i$
- Network state (NS): $x_{-i} = (y_{-i}, q_{-i}) \Rightarrow$ calculation of the probability of outage
System Model(5): Definition of Action

In each time slot, on the basis of the available information at time $t$ transmitter $i$ decides:

- **Transmission power level** $p_i \in P_i$, while $P_i$ is a finite set of nonnegative reals including zero
- **Transmission rate** (Number of packets to transmit) $\mu_i \in M_i$, while $M_i = \{0, 1, \ldots, M_i\}$ and $M_i \leq B_i$;
- **Accept** ($c_i = 1$) or **Reject** ($c_i = 0$) new packets arriving from upper layers.

Therefore, the action of the node $i$ at time slot $t$ is described by the triplet $a_i(t) = (p_i(t), \mu_i(t), c_i(t))$.

We define $\mathcal{K}_i = \mathcal{X}_i \times \mathcal{A}_i = \{(x_i, a_i) : x_i = (y_i, q_i) \in \mathcal{X}_i, a_i = (p_i, \mu_i, c_i) \in a_i(x_i)\}$
System Model(6): Definition of Rate for SU/SIC decoder

\[ r_i(x^i(t), p) = \log_2(1 + \text{SINR}_i(x^i(t), p)) \]

\[ \text{SINR}^\text{SU}_i(x^i(t), p) = \begin{cases} \frac{y^d_i(t)p_i(t)}{\sigma^2 + \sum_{j \neq i} y^d_j(t)p_j(t)}, & q_i(t) > 0 \\ 0, & \text{otherwise.} \end{cases} \]

\[ \text{SINR}^\text{SIC}_i(x^i(t), p) = \begin{cases} \frac{y^d_i(t)p_i(t)}{\sigma^2 + \sum_{j \neq i} y^d_j(t)p_j(t)}, & q_i(t) > 0 \\ 0, & \text{otherwise.} \end{cases} \]
Problem Statement(1): Optimization problem

Let $R$ be the rate required to transmit a packet in a time slot. The probability that $\mu_i(t)$ packets can be transmitted successfully in a time slot $t$ by source $i$ is

$$\Pr\{r_i(x^i(t), p) \geq \mu_i(t)R\}$$

We want to maximize the average throughput for source $i$ with the following definition

$$\limsup_{T \to +\infty} \frac{1}{T} \sum_{t=0}^{T-1} E(\Pr\{r_i(x^i(t), p) \geq \mu_i(t)R \mid x_i(0) = \beta_i\}\mu_i(t)R)$$

For physical and QoS reasons the transmitters are subjected to constraints on the average transmitted powers, average queue length, and maximum outage probability:

$$\limsup_{T \to +\infty} \frac{1}{T} \sum_{t=0}^{T-1} E\{p_i(x_i(t), a_i(t)) \mid x_i(0) = \beta_i\} \leq \bar{p}_i$$

$$\limsup_{T \to +\infty} \frac{1}{T} \sum_{t=0}^{T-1} E\{q_i(t) \mid x_i(0) = \beta_i\} \leq \bar{q}_i$$

$$\lim_{T \to +\infty} \Pr\{r_i(x_i(t), p) < \mu_i(t)R_i(t) \mid x_i(0) = \beta_i\} \leq \bar{F}_i^{out}$$
Problem Statement(2): BIG TROUBLE!!!

We will end up facing this: a non-convex non-linear constrained optimization problem!!!
⇒ we will use the approach in reference [1]
Definition: A policy of transmitter $i$ is a deterministic or probabilistic application from the space of TS $\mathcal{X}_i$ to the action space $\mathcal{A}_i$.

- Since Markov policies are dominating, we assume that the policy of a transmitter at time $t$ is only a function of its current state and we omit the time indices.
- A mixed policy of transmitter $i$ is $u_i(a_i|x_i)$, i.e. the probability that mobile $i$ chooses the action $a_i$ when the state is $x_i$. 
**Problem Statement(4): N-player Game**

We formulate the previous problem as a stochastic $N_T$-player game. We denote by $g_i$ the cardinality of the product set $\mathcal{K}_i = \mathcal{X}_i \times \mathcal{A}_i = \{(x_i, a_i) : x_i = (y_i, q_i) \in x_i, a_i = (p_i, \mu_i, c_i) \in \mathcal{A}_i(x_i)\}$ and by $<x_i, a_i>_{n_i}$ the $n_i$-th element of $\mathcal{K}_i$. The payoff matrix $C^{(i)}$ of transmitter $i$ is a $g_1 \times g_2 \times \cdots g_n$ matrix having $N_T$-dimensions and its element $c^{(i)}_{n_1,n_2...n_{N_T}}$ is the payoff of transmitter $i$ when correspondingly to TS $x_i$ it performs action $a_i$ while the remaining users adopt the strategies $<x_k, a_k>_{n_k}$ with $k \neq i$.

Payoff in the selfish game

$$c^{(i)}_{n_1,n_2...n_{N_T}} = R\mu_i1_{(r_i(x_i,p_i) \geq \mu_i R)};$$

Payoff in the cooperative game

$$c^{(i)}_{n_1,n_2...n_{N_T}} = R \sum_{j=1}^{N_T} \mu_j1_{(r_j(x_j,p_j) \geq \mu_j R)}. $$
Problem Statement(5): Multilinear presentation

Let $z_i = z_i(x_i, a_i)$ be the joint probability that transmitter $i$ performs action $a_i$ while being in state $x_i$. It can be expressed by the row vector $z_i = (z_i^1, z_i^2, ..., z_i^{g_i})$.

\[ \rho_i = \sum_{n_1=1}^{g_1} \sum_{n_2=1}^{g_2} \cdots \sum_{n_{NT}=1}^{g_{NT}} c^{i}_{n_1n_2\ldots n_{NT}} z_{1}^{n_1} z_{2}^{n_2} \cdots z_{NT}^{n_{NT}} = z_i f^i \quad (1) \]

where

\[ f^i = \sum_{n_{1}=1}^{g_1} \cdots \sum_{n_{i-1}=1}^{g_{i-1}} \sum_{n_{i+1}=1}^{g_{i+1}} \cdots \sum_{n_{NT}=1}^{g_{NT}} c^{(i)}_{n_1\ldots n_{i-1}kn_{i+1}\ldots n_{NT}} z_{1}^{n_1} \cdots z_{i-1}^{n_{i-1}} z_{i+1}^{n_{i+1}} \cdots z_{NT}^{n_{NT}} . \]
Problem Statement(6): Linear Programming

\[
\max_{x_i, a_i} \sum_{x_i \in x_i} \sum_{a_i \in a_i} z_i(x_i, a_i) Pr\{r_i(x^i, p) \geq \mu_i R\} \mu_i R
\]

Subject to:

\[
\sum_{x_i \in x_i} \sum_{a_i \in a_i} z_i(x_i, a_i) [\delta_{r_i}(x_i) - P_{x_i a_i r_i}] = 0 \quad \forall r_i \in x_i
\]

\[
\sum_{x_i \in x_i} \sum_{a_i \in a_i} q_i z_i(x_i, a_i) \leq \bar{q}_i
\]

\[
\sum_{x_i \in x_i} \sum_{a_i \in a_i} p(x_i, a_i) z_i(x_i, a_i) \leq \bar{p}_i
\]

\[
\sum_{x_i \in x_i} \sum_{a_i \in a_i} Pr\{r_i(x^i, p) < \mu_i R\} z_i(x_i, a_i) \leq \bar{P}_i^{out}
\]

\[
z_i(x_i, a_i) = z_i((y_i, q_i), (\mu_i, p_i, c_i) | q_i \leq \mu_i) = 0
\]

\[
z_i(x_i, a_i) \geq 0; \quad \forall (x_i, a_i) \in K_i; \quad \sum_{(x_i, a_i) \in K_i} z_i(x_i, a_i) = 1
\]
Queue state transitions

Diagram of state transitions between queues q_0, q_1, q_2, and q_3 with probabilities P_{q_i q_j}.
Nash Equilibrium

COP:

$$\min_{z_i} -z_i f^{(i)}$$

(b) $A_i z_i^T + a_i = 0$  (c) $B_i z_i^T + b_i \leq 0$  (d) $z_i^T \geq 0$

The existence of the Nash equilibrium for a general class of constrained stochastic games, where players have independent state processes, is proven in [3].

KKt conditions:

$$\mathcal{L}_i(z_i, u_i, v_i) = -\rho_i + u_i (A_i z_i^T + a_i) + v_i (B_i z_i^T + b_i)$$

$$\begin{align*}
\theta_{1i} &= \nabla_{z_i} \mathcal{L}_i = -f_i + u_i A_i + v_i B_i \\
\theta_{2i} &= \nabla_{u_i} \mathcal{L}_i = A_i z_i^T \\
\theta_{3i} &= \nabla_{v_i} \mathcal{L}_i = -B_i z_i^T \\
z_i \geq 0 & \quad u_i \geq 0 & \quad v_i \geq 0 \\
z_i \theta_{1i} = 0 & \quad u_i \theta_{2i} = 0 & \quad v_i \theta_{3i} = 0.
\end{align*}$$

Let $w_i = (z_i, u_i, v_i)$, $\Theta_i(w_i) = (\theta_{1i}^T, \theta_{2i}^T, \theta_{3i}^T)^T$, and by concatenating different transmitters’ vectors $w = (w_1, w_2, \ldots, w_{N_T})$ and $\Theta(w) = (\Theta_1^T, \Theta_2^T, \ldots, \Theta_{N_T}^T)^T$, we obtain the nonlinear complementarity problem

$$\begin{align*}
\Theta(w) \geq 0 & \quad w^T \geq 0 & \quad w \Theta(w) = 0
\end{align*}$$
Symmetric Network

Definition:

- All the transmitters have the same statistics for the channel and arrival process
- The constraint parameters are identical for different users.
- The same objective and constraint functions for different users.

→ NLCP in $g_i + N_i^{eq} + N_i^{ineq}$ unknowns

→ Standard solution: extragradient method for variational inequality problem → quadratic programming
Best Response Algorithm

Based on the fact that, the game reduces to a LP when the strategies of \( N_T - 1 \) players is known, our best response algorithm is:

step 0: Choose a transmitter \( i \) and the policies \( U_{-i} \) arbitrarily.

step 1: solve the corresponding LP. Go back to step 0.

Note: If the algorithm converges, its fixed points are Nash equilibria of the \( N_T \)-player game. The convergence of the best response algorithm to Nash equilibrium is guaranteed only in a symmetric case, while in the general case the algorithm could converge to a point which is not a Nash equilibrium.
Simulation parameters

The CS varies according to a Markov chain with the following transition probabilities: $T_0^0(i, j) = \frac{1}{2}, T_0^1(i, j) = \frac{1}{2}, T_{L-1}^L(i, j) = \frac{1}{2}, (2 \leq k \leq N - 2) T_k^k(i, j) = \frac{1}{3}, T_k^{k+1}(i, j) = \frac{1}{3}$.

Network Parameters

|          | $N_T$ | $N_R$ | $B_i$ | $L$ | $M_i$ | $|P_i|$ | $\bar{p}_i$ | $\bar{q}_i$ |
|----------|-------|-------|-------|-----|-------|--------|------------|------------|
| Scn1     | 2     | 2     | 5     | 3   | 4     | 4      | 2          | 4          |
| Scn2     | 3     | 3     | 4     | 3   | 4     | 3      | 2          | 4          |

Labeling of states

<table>
<thead>
<tr>
<th></th>
<th>state index</th>
<th>queue state</th>
<th>channel state</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 1 2 3 4 5 6 ... 17</td>
<td>0 0 1 1 1 2 ... 6</td>
<td>0 1 2 0 1 2 0 ... 2</td>
</tr>
</tbody>
</table>

Labeling of actions

<table>
<thead>
<tr>
<th></th>
<th>action index</th>
<th>Num of packets</th>
<th>power level</th>
<th>accept/reject</th>
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<tbody>
<tr>
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<td>0 1 2 3 4 5 6 7 8 9 ... 47</td>
<td>0 0 0 0 0 0 0 0 1 1 1 ... 6</td>
<td>0 0 1 1 2 2 3 3 0 0 1 ... 3</td>
<td>0 1 0 1 0 1 0 1 0 1 0 ... 1</td>
</tr>
</tbody>
</table>
Optimal policies of Scenario 1

Two communication-flow network

Transmitter State Index

Action Index

SU
SIC1
SIC2

0 tx pkt
1 tx pkt
2 tx pkt
3 tx pkt
4 tx pkt
5 tx pkt

0 q pkt
1 q pkt
2 q pkt
3 q pkt
4 q pkt
5 q pkt
Performance

Performance metrics are:

- Throughput (TP), i.e. the number of packets per time slot correctly decoded by the receiver
- Outage rate, i.e. the fraction of transmitted packets which can not be decoded correctly,
- Drop rate, i.e. the fraction of arriving packets from upper layer which are rejected due to the admission control.

<table>
<thead>
<tr>
<th></th>
<th>TP</th>
<th>Outage Rate</th>
<th>Drop rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A - self - SU$</td>
<td>0.49</td>
<td>0.42</td>
<td>0.15</td>
</tr>
<tr>
<td>$A - self - SIC1$</td>
<td>0.64</td>
<td>0.24</td>
<td>0.16</td>
</tr>
<tr>
<td>$A - self - SIC2$</td>
<td>0.69</td>
<td>0.19</td>
<td>0.15</td>
</tr>
<tr>
<td>$A - coop - SU$</td>
<td>0.5</td>
<td>0.4</td>
<td>0.17</td>
</tr>
<tr>
<td>Policy in [1]</td>
<td>0.41</td>
<td>0.35</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Policy in [1]: Under the assumption of fixed transmission rate for all users and the assumption that reliable communication are always possible in the decentralized context. The utility function is the average maximum achievable rate.
Optimal policies of Scenario 2
THANK YOU! Questions?
References


