Efficient Multi-Carrier Communication over Mobile Wideband Channels

Phil Schniter

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(Joint work with Mr. Sungjun Hwang and Dr. Sib Das)
Multipath, Mobility, and Bandwidth:

Trends:

- As the signal bandwidth increases, there are more gain variations across the signal bandwidth, and hence more channel parameters. 

  \[ \text{Increased BW } \Rightarrow \text{ longer channel impulse response.} \]

- As the mobilities of the transmitter, receiver, and reflectors increase, there are more channel gain variations per unit time.

  \[ \text{Increased mobility } \Rightarrow \text{ faster impulse response variation.} \]
Multipath Fading (cont.):

- Implication:
  
  *As bandwidth & mobility increase, time-domain receivers need to implement longer filters and adapt them at faster rates.*

- Example — North American Digital TV:
  Typical receivers use decision feedback equalization (DFE) with 1000 taps in the feedforward path and 500 in the feedback path.

Even with a fixed transmitter and receiver, it is very difficult to adapt such long filters quickly enough!
Orthogonal Frequency Division Multiplexing (OFDM):

Main Ideas:

1. Transmit data in parallel over non-interfering narrowband subchannels.
   - Flat frequency response across each subchannel ⇒ 1 \( \frac{\text{channel coefficient}}{\text{subchannel}} \).
   - Equalization = one complex gain adjustment per subchannel!

2. Implement subchannelization using the Fast Fourier Transform (FFT).
   - No calibration or drift issues like with analog modulators.
   - Minimal frequency spacing (⇒ good spectral efficiency).
   - Fast implementation: \( N \log_2 N \) multiplications per \( N \)-FFT.
   - Requires a guard interval of length-\( L-1 \).
Equalization Complexity:

Single-carrier:

- Linear: \[ 3L \] decisions \[ 3L \] mults per QAM symbol
- DFE: \[ 2L \] decisions \[ 3L \] mults per QAM symbol
- MLSD: \[ |S|^L \] mults per QAM symbol

*Complexity scales linearly or exponentially in the channel length \( L \).*

Multi carrier:

\[ N + N \log_2 N \] mults per block, where typically \( N \approx 4L \).

\[ 1 + \log_2 N = 3 + \log_2 L \] mults per QAM symbol.

*Complexity scales logarithmically in the channel length \( L \).*

Example: When \( L = 512 \), we have \( 3L = 1536 \) and \( 3 + \log_2 L = 12 \).
CP-OFDM:

Principal advantage:

- Low-complexity demod with delay-spreading (i.e., freq selective) chans.

Some disadvantages:

- Sensitive to Doppler-spreading (i.e., time selective) channels.

- Loss of spectral efficiency due to the insertion of guards.

    *What if we increased $N$ relative to $L$ (i.e., $P \triangleq \frac{N}{L} \gg 4$)?*

    - Complexity increases to $1 + \log_2 P + \log_2 L$ \(\frac{\text{mults}}{\text{QAM symbol}}\) ...not bad.
    - Reduced subcarrier spacing \(\Rightarrow\) more sensitive to Doppler spread!

- Slow spectral roll-off causes interference to adjacent-band systems.

    Improves with raised-cosine pulse, but at further loss in efficiency:

    - High peak-to-average power ratio (PAPR).
The Big Question:

Can we fix CP-OFDM’s

- *sensitivity to Doppler spread*
- *loss in spectral efficiency*, and
- *slow spectral roll-off*,

without spoiling its $O\left(\log_2 L\right) \frac{\text{mults}}{\text{QAM symbol}}$ complexity scaling?
The Big Question:

Can we fix CP-OFDM’s

- sensitivity to Doppler spread
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- slow spectral roll-off,

without spoiling its $O(\log_2 L) \frac{\text{mults}}{\text{QAM symbol}}$ complexity scaling?

Yes!

Re-think the role of “pulse shaping” in multi-carrier modulation...
Rectangular Pulses:

A standard CP-OFDM block can be recognized as a sum of $N$ infinite-length complex exponentials windowed by a rectangular pulse of width $N + L - 1$.

$\Rightarrow$ Dirichlet sinc in DTFT domain, whose slow side-lobe decay causes:

- strong interference to adjacent-band communication systems, and
- high sensitivity to Doppler spreading:
Smooth Overlapping Pulses:

What if we applied a smooth window instead?

The main-lobe is wider but the sidelobes decay more quickly, implying:

- reduced adjacent-band interference, and
- strong interference from adjacent subcarriers, but very little interference from all other subcarriers, even under large Doppler spreads:
Smooth Overlapping Pulses:

**Challenge:** The use of smooth overlapping pulses potentially causes *inter-carrier interference (ICI)* and *inter-block interference (IBI)*:

\[
x(i) = \sum_{q=-\infty}^{\infty} H(i, q) s(i - q) + z(i).
\]

*Difficult to equalize!*

**Solution:** Design the pulse shapes with the goal of . . .

1. Completely suppressing IBI: \( H(i, q) \big|_{q\neq0} = 0 \).

2. Allowing ICI only within a radius of \( D \ll N \) subcarriers. (Often \( D = 1 \).)

\[
\begin{array}{c|c|c|c}
\hline
x(i) & H(i, 0) & s(i) & z(i) \\
\hline
\end{array}
\]

*Not difficult to equalize.*
Receiver Pulse-Shaping:

Though so far we’ve considered a non-rectangular *transmission pulse* \( \{a_n\} \),

\[
t_n = \sum_{i=-\infty}^{\infty} a_{n-iN_s} \sum_{k=0}^{N-1} s_k(i) e^{j\frac{2\pi}{N} kn}, \quad n = -\infty \ldots \infty,
\]

we can use, in addition, a non-rectangular *reception pulse* \( \{b_n\} \):

\[
x_k(i) = \sum_{n=-\infty}^{\infty} r_{n-iN_s} b_n e^{-j\frac{2\pi}{N} kn}, \quad k = 0 \ldots N - 1.
\]

\( N_s \) specifies the interval between the start of one OFDM block and the next.

- Modulation efficiency \( \eta \equiv \frac{N}{N_s} \) symbols/sec Hz
- For OFDM, \( N_s = N + L - 1 \), but now there is *no constraint* on \( N_s \)!

*We focus on* \( N_s = N \iff \text{no guard interval} \iff \eta = 1. \)
Max-SINR Pulse Design:

Writing the received signal energy components due to

\[ \mathcal{E}_s = \sum_{(q,k,l) \in \mathcal{E}} \mathbb{E}\{|H_{k,l}(\cdot, q)|^2\} \]  
“signal” and

\[ \mathcal{E}_i = \sum_{(q,k,l) \in \mathcal{E}} \mathbb{E}\{|H_{k,l}(\cdot, q)|^2\} \]  
“interference” (IBI+ICI)

where \( \{H(\cdot, q)\} = [\text{interference} \cdots \text{interference} \text{don’t care} \text{signal}] \)

we can write

\[ \text{SINR} = \frac{\mathcal{E}_s}{\mathcal{E}_i + \mathcal{E}_n} = \frac{a^H P_1(b)a}{a^H P_2(b)a} = \frac{b^H P_3(a)b}{b^H P_4(a)b} \]

where

\[ a = \text{transmission pulse coefficients} \]

\[ b = \text{reception pulse coefficients} \]

\[ P_1(\cdot), P_2(\cdot), P_3(\cdot), P_4(\cdot) = \text{matrix fxns dependant on Doppler & SNR.} \]

\[ \Rightarrow \text{SINR-maximizing pulses are generalized eigenvectors.} \]
Max-SINR Pulse Examples:

ZP-OFDM ($\eta = 0.803$)

Optimized receiver ($\eta = 1$)

Optimized transmitter ($\eta = 1$)

Jointly optimized ($\eta = 1$)
IBI/ICI Energy Profiles (same for each subcarrier):

\[
D = 1, \quad \text{SNR} = 15\text{dB}, \quad L = 64, \quad f_D T_c = 7.6 \times 10^{-4}, \quad \text{Jakes Doppler spectrum.}
\]

(For example, \(f_c = 20\text{GHz}, \quad \text{BW}=3\text{MHz}, \quad T_h = 5.4\mu\text{s}, \quad v = 120\text{km/hr}.\))
Non-Orthogonal FDM:

To summarize, Orthogonal FDM is possible only when

1. the channel is time-invariant, and
2. an adequate-length guard is included.

With a properly-designed *Non-Orthogonal FDM*, we can

- eliminate the guard, and
- tolerate large delay and Doppler spreads,

at the cost of

- a short span of intercarrier-interference,

which can be properly handled via low-complexity equalization.

Thus, we advocate *shaping ISI/ICI rather than suppressing ISI/ICI*. 
Outage Capacity vs $N f_D T_s$ for various ICI-radii $D$:

- The outage-capacity optimal $D$ obeys $D \approx \lceil N f_D T_s \rceil$!
- ICI shaping is better than ICI suppression when $2 f_D T_s \geq \frac{1}{N}$. 
## ICI Equalization:

<table>
<thead>
<tr>
<th>Coherent approaches (i.e., known channel):</th>
<th>Non-coherent approaches (i.e., unknown channel):</th>
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<tr>
<td>2. Iterative Soft [Das/Schniter Asilomar 04]</td>
<td>2. Soft Tree Search [Hwang/Schniter Asilomar 07]</td>
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<td>3. Linear MMSE [Rugini/Banelli/Leus SPL 05]</td>
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<td>4. MMSE DFE [Rugini/Banelli/Leus SPAWC 05]</td>
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<tr>
<td>5. Tree Search [Hwang/Schniter SPAWC 06]</td>
<td></td>
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</tbody>
</table>

### Complexity:

- Viterbi: \( \mathcal{O}(|S|^PD) \)
- Iterative Soft: \( \mathcal{O}(D^2) \)
- Linear MMSE: \( \mathcal{O}(D^2) \)
- MMSE DFE: \( \mathcal{O}(D^2) \)
- Tree Search: \( \mathcal{O}(D^2) \)
- Hard Tree Search: \( \mathcal{O}(D^2L^2) \)
- Soft Tree Search: \( \mathcal{O}(D^2L^2) \)
1) **Coherent Tree Search:**

Two-step procedure:

1. **MMSE-GDFE pre-processing**  
   \[ \text{[Damen/ElGamal/Caire TIT 03]}: \]
   
   \[
   \begin{array}{cc}
   D + 1 & D \\
   D & 2D + 1 \\
   N & 2D \\
   \end{array}
   \]

   \[ \rightarrow \]

   \[ \mathcal{O}(D^2 N) \text{ algorithm} \quad \text{[Rugini/Banelli/Leus SPAWC 05]}. \]

2. **Near-optimal yet efficient tree search.** Options include:
   
   - Depth-first search (e.g., Schnor-Euchner sphere decoder),
   - Best-first search (e.g., Fano alg, stack alg),
   - Breadth-first search (e.g., M-alg, T-alg, Pohst sphere decoder).
Suboptimal tree search is almost indistinguishable from ML!
Average Complexity (MACs/frame):

When $f_DT_c = 0.001$, breadth-first & DFE stay cheap, while depth-first & Fano explode!
Error Masking due to V-shaped Channel Matrix:

After MMSE-GDFE pre-processing, we get the following system:

\[ 0 \leq 2D + 1 \quad N - 4D - 2 \]

\[ = \quad N - 2D - 1 \quad N - 2D - 1 \]

Key point: The blue symbol does not affect any of the red observations.

Error-masking explains the complexity explosion of the depth-first and Fano searches!
2) Iterative Soft ICI Cancellation:

\[ x_k = H_k s_k + z_k \]

- Soft interference cancellation using mean of \( s_k \).
- Assuming Gaussian residual interference and using the covariance of \( s_k \), compute LLRs(\( s_k \)).
- Using LLRs(\( s_k \)), update mean/covariance of \( s_k \).
- \( k \rightarrow (k + 1)_N \).
Iterative Soft ICI Cancellation (BPSK example):

\[ \text{LLR}^{(i)}_k, v^{(i)}_k, g^{(i)}_k \]

\[ \hat{s}^{(i)}_k \triangleq \mathbb{E}\{s_k | \hat{s}_k\} = \tanh(\text{LLR}^{(i)}_k / 2) \]

\[ v^{(i)}_k \triangleq \text{var}(s_k | \hat{s}_k) = 1 - (\hat{s}^{(i)}_k)^2 \]

\[ y^{(i)}_k = x_k - H_k \hat{s}^{(i)}_k \]

\[ g^{(i)}_k = y^{(i)}_kH \left( R_z + H_k D(v^{(i)}_k)H^H_k \right)^{-1} h_k \]

\[ \text{LLR}^{(i+1)}_k = \text{LLR}^{(i)}_k + 2 \text{Re}(g^{(i)}_k) \]

**Complexity:** \( M \times \mathcal{O}(D^2) \) per BPSK symbol.
Turbo Equalization:

Soft bit information (i.e., LLRs) can be exchanged with a soft-input soft-output (SISO) decoder:
Turbo Equalization Performance:

rate-$\frac{1}{2}$ conv code
QPSK
$N = 64$
$D = 2$
$L = 16$
$f_D T_c = 0.003$

for example:
$f_c = 20\text{GHz}$
$\text{BW} = 3\text{MHz}$
$T_h = 5.4\mu\text{s}$
$v = 3 \times 160\text{km/hr}$
3) Non-Coherent Equalization:

- So far we have assumed that the equalizer is provided with channel estimates.
  
  But with long and quickly varying channels, accurate channel estimation requires a high proportion of pilots!

- Instead, one might consider joint estimation of channel and symbols.
  
  Actually, we are not interested in explicitly estimating the channel, but rather doing non-coherent equalization: inferring transmitted bits using knowledge of channel statistics but not channel state.

For this it helps to re-parameterize the system model...
**Basis-Expansion Model:**

From the pulse-shaped FDM model:

\[
x(n) = H(n)s(n) + z(n) \quad H(n) \text{ is banded with band radius } D
\]

\[
= S(n)h(n) + z(n) \quad h(n) \in \mathbb{C}^{(2D+1)N} \text{ ICI coefs}
\]

we can use a basis-expansion model (BEM) for the ICI coefs:

\[
h(n) = B\theta(n) \quad \theta(n) \in \mathbb{C}^{(2D+1)L} \text{ delay/Doppler coefs}
\]

\[
B = \begin{pmatrix} F_L & \cdots & F_L \end{pmatrix} \quad F_L \in \mathbb{C}^{N \times L} \text{ truncated DFT matrix}
\]

to rewrite the observation as

\[
x(n) = S(n)B\theta(n) + z(n).
\]

Note: if we knew the locations of \( K \) active sparse taps, we would only include those columns of the DFT matrix, giving \( \theta(n) \in \mathbb{C}^{(2D+1)K} \).
**Non-coherent MLSD:**

Treating the delay/Doppler coefs $\theta$ as nuisance parameters,

$$
\hat{s}_{\text{ML}} = \arg \max_s p(x|s)
$$

$$
= \arg \max_s \int p(x|s, \theta)p(\theta)d\theta
$$

Assuming $\theta \sim \mathcal{CN}(0, R_\theta)$,

$$
\hat{s}_{\text{ML}} = \arg \max_s \left\{ x^H S B \Sigma^{-1}_s B^H S^H x - \sigma^2 \log |\Sigma_s| \right\}
$$

$$
\Sigma_s \triangleq B^H S^H S B + \sigma^2 R_\theta^{-1}
$$

Since $\hat{\theta}_{\text{MMSE}|s} = \Sigma^{-1}_s B^H S^H x$, we can rewrite

$$
\hat{s}_{\text{ML}} = \arg \max_s \left\{ \|x - S B \hat{\theta}_{\text{MMSE}|s}\|^2 - \sigma^2 \log |\Sigma_s| \right\}
$$

But how do we avoid an exhaustive search over $s$?
**Fast Tree Search:**

By turning off the first and last $D$ subcarriers, $H$ becomes upper-triangular, facilitating the use of tree-search.

\[
\begin{align*}
    s_0 &= [s_0]^T \\
    s_1 &= [s_0, s_1]^T \\
    s_2 &= [s_0, s_1, s_2]^T \\
    s_3 &= [s_0, s_1, s_2, s_3]^T
\end{align*}
\]

(for BPSK)

The important thing here is that the partial ML metric

\[
\mu_{ML}(s_k) = \|x_k - S_k B_k \hat{\theta}_{MMSE}\|_2^2 - \sigma^2 \log |\Sigma_k|
\]

can be computed recursively. Thus, total search complexity via the $M$-algorithm is only

\[
2M|S|(2D + 1)^2L^2 \text{ mults per QAM-symbol!}
\]
Complexity Reduction via Pilots:

- With non-coherent decoding, we require only a single pilot subcarrier.
- But, with more pilots, the initial channel estimate $\hat{\theta}_{\text{MMSE}}$ improves, allowing more aggressive branch pruning in the tree search.

Example: M-algorithm (BPSK, 25% pilots, $M=8$) compared to coherent MLSD with genie-aided $\hat{\theta}_{\text{MMSE}}$: 
Non-coherent Turbo Equalization:

\[ L_e(b_k|\mathbf{x}) = \ln \frac{\sum_{s: b_k=1} \exp \mu_{\text{MAP}}(s)}{\sum_{s: b_k=0} \exp \mu_{\text{MAP}}(s)} - L_a(b_k) \quad \text{"extrinsic LLR"} \]

\[ \mu_{\text{MAP}}(s) = \ln p(\mathbf{x}|s) + \sum_{k: b_k=1} L_a(b_k) \quad \text{"MAP metric"} \]

Need \( \mathcal{O}(2^{QN}) \) evaluations of \( \mu(s) \) \( \rightsquigarrow \) Computationally infeasible!
**Simplified LLR Evaluation:**

The “max-log” approximation:

\[
L_e(b_k|x) \approx \max_{s \in \mathcal{L} \cap \{s:b_k=1\}} \mu_{\text{MAP}}(s) - \max_{s \in \mathcal{L} \cap \{s:b_k=0\}} \mu_{\text{MAP}}(s) - L_a(b_k)
\]

\[
\mathcal{L} : \text{ set containing the } M \text{ most probable } s,
\]

requires only a few evaluations of \(\mu_{\text{MAP}}(s)\).

To find \(\mathcal{L}\), the set of most probable \(s\), and the MAP metrics \(\{\mu_{\text{MAP}}(s)\}_{s \in \mathcal{L}}\), we use a *tree search*, as in the uncoded case.

Here again, there exists a fast metric update such that the total search complexity, with the M-algorithm, is only

\[
2M|\mathcal{S}|(2D+1)^2L^2 \text{ mults per QAM-symbol!}
\]
Non-coherent Turbo Performance:

rate-$\frac{1}{2}$ LDPC
QPSK
$N = 64$
$D = 2$
$L = 16$
$K = 3$
$f_D T_c = 0.003$
**Related Work:**

- Channel estimation techniques (for coherent equalization).
- Single-carrier frequency-domain equalization.
- Pilot pattern designs:
  - MMSE optimal (under CE-BEM assumption).
  - Achievable-rate optimal (under CE-BEM assumption).
- Theoretical analysis of pulse-shaped multicarrier modulation:
  - Achievable-rate characterized.
- Analysis of doubly selective channel:
  - Capacity characterized (under CE-BEM assumption).

(See http://www.ece.osu.edu/~schniter/pubs_by_topic.html)
Conclusions:

Single Carrier:

- $\mathcal{O}(L) \frac{\text{mults}}{\text{QAM symbol}}$ equalization of delay-spread channels.
- Challenging to track quickly time-varying channels.

Orthogonal FDM:

- $\mathcal{O}(\log_2 L) \frac{\text{mults}}{\text{QAM symbol}}$ equalization of delay-spread channels.
- Loss in spectral efficiency due to guard interval.
- Sensitive to Doppler spread.
- Slow spectral roll-off $\Rightarrow$ high adjacent-band interference.

Non-Orthogonal FDM:

- No need for a guard interval; high spectral efficiency.
- Large simultaneous delay & Doppler spreads $\Rightarrow$ no IBI and short ICI.
- Fast spectral roll-off $\Rightarrow$ low adjacent-band interference.
Equalization/Decoding of Short ICI Span:

- **Uncoded Coherent**: Tree search gives ML-like performance with DFE-like complexity.

- **Coded Coherent**: Iterative soft ICI cancellation in turbo configuration performs close to perfect-interference-cancellation bound.

- **Uncoded Non-Coherent**: Tree-search with fast metric update gives ML-like performance with $O(D^2L^2)$ complexity.

- **Coded Non-Coherent**: Tree-search with fast metric update performs close to genie-aided bound with $O(D^2L^2)$ complexity, or $O(D^2K^2)$ if sparseness leveraged.
Thanks for listening!