Efficient Multi-Carrier Communication over Mobile Wideband Channels

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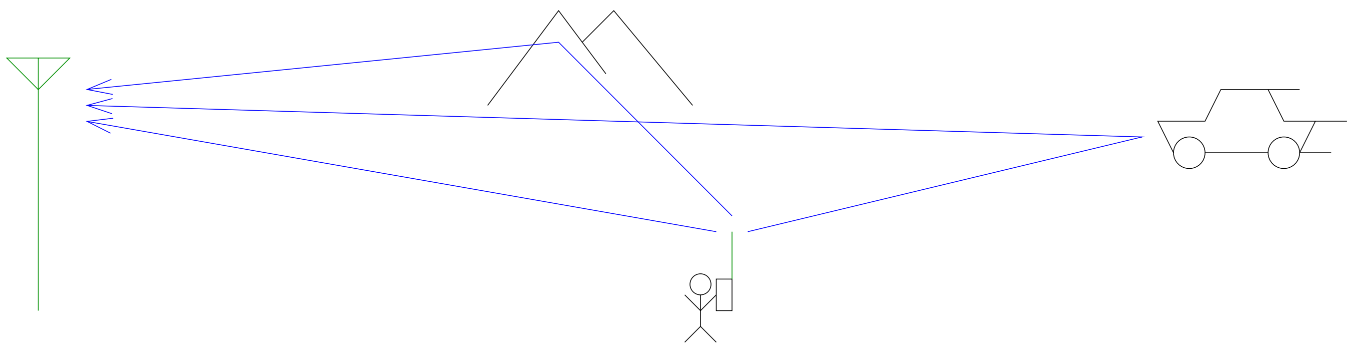
(Joint work with Mr. Sungjun Hwang and Dr. Sib Das)

Multipath, Mobility, and Bandwidth:



Trends:

- As the signal bandwidth increases, there are more gain variations across the signal bandwidth, and hence more channel parameters.

  Increased BW $\Rightarrow$ longer channel impulse response.

- As the mobilities of the transmitter, receiver, and reflectors increase, there are more channel gain variations per unit time.

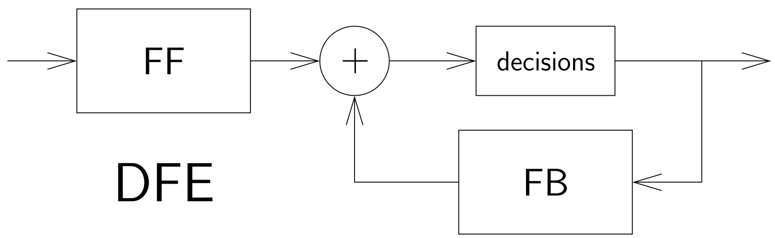
  Increased mobility $\Rightarrow$ faster impulse response variation.

Multipath Fading (cont.):

- Implication:

  As bandwidth & mobility increase, time-domain receivers need to implement longer filters and adapt them at faster rates.

- Example — North American Digital TV:
  Typical receivers use decision feedback equalization (DFE) with 1000 taps in the feedforward path and 500 in the feedback path.

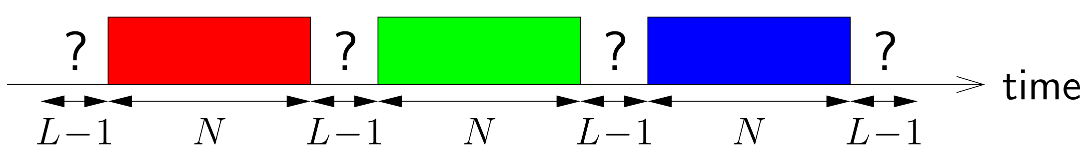


Even with a fixed transmitter and receiver, it is very difficult to adapt such long filters quickly enough!

Orthogonal Frequency Division Multiplexing (OFDM):

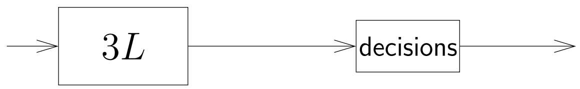
Main Ideas:

1. *Transmit data in parallel over non-interfering narrowband subchannels.*
   - Flat frequency response across each subchannel $\Rightarrow 1\ \frac{\text{channel coefficient}}{\text{subchannel}}$.
   - Equalization $=$ one complex gain adjustment per subchannel!
2. *Implement subchannelization using the Fast Fourier Transform (FFT).*
   - No calibration or drift issues like with analog modulators.
   - Minimal frequency spacing ($\Rightarrow$ good spectral efficiency).
   - Fast implementation: $N \log_2 N$ multiplications per N-FFT.
   - Requires a guard interval of length-$L-1$.

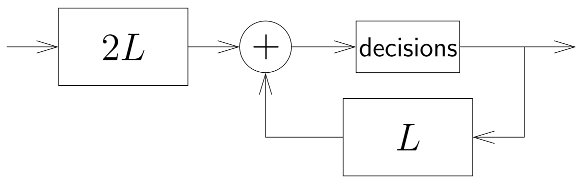


Equalization Complexity:

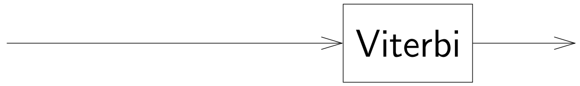
Single-carrier:

Linear: 

$3L$ mults per QAM symbol

DFE: 

$3L$ mults per QAM symbol

MLSD: 

$|\mathcal{S}|^L$ mults per QAM symbol

Complexity scales linearly or exponentially in the channel length L.

Multi carrier:

$N + N\log_2 N$ mults per block, where typically $N \approx 4L$.

$\Rightarrow 1 + \log_2 N = \boxed{3 + \log_2 L}$ mults per QAM symbol.

Complexity scales logarithmically in the channel length L.

Example: When $L = 512$, we have $3L = 1536$ and $3 + \log_2 L = 12$.

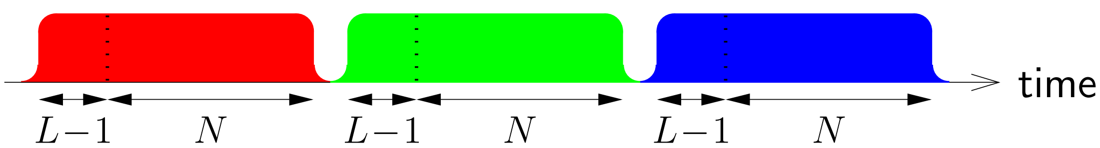
CP-OFDM:

Principal advantage:

- Low-complexity demod with delay-spreading (i.e., freq selective) chans.

Some disadvantages:

- Sensitive to Doppler-spreading (i.e., time selective) channels.
- Loss of spectral efficiency due to the insertion of guards.
  What if we increased N relative to L (i.e., $P \triangleq \frac{N}{L} \gg 4$)?
  - Complexity increases to $1 + \log_2 P + \log_2 L \ \ \frac{\text{mults}}{\text{QAM symbol}}$...not bad.
  - Reduced subcarrier spacing $\Rightarrow$ more sensitive to Doppler spread!
- Slow spectral roll-off causes interference to adjacent-band systems.
  Improves with raised-cosine pulse, but at further loss in efficiency:



- High peak-to-average power ratio (PAPR).

The Big Question:

Can we fix CP-OFDM's

- *sensitivity to Doppler spread*
- *loss in spectral efficiency*, and
- *slow spectral roll-off*,

without spoiling its $\mathcal{O}(\log_2 L)$ $\frac{\text{mults}}{\text{QAM symbol}}$ complexity scaling?

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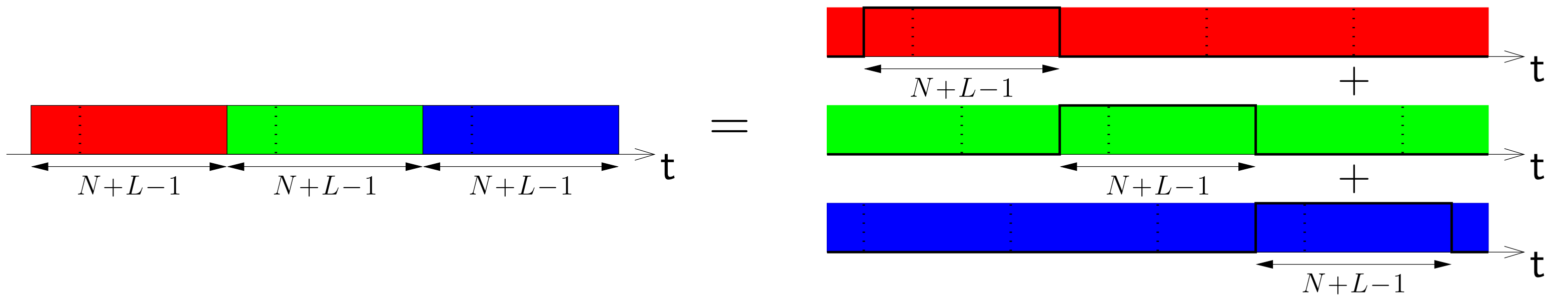
Yes!

Re-think the role of "pulse shaping" in multi-carrier modulation...

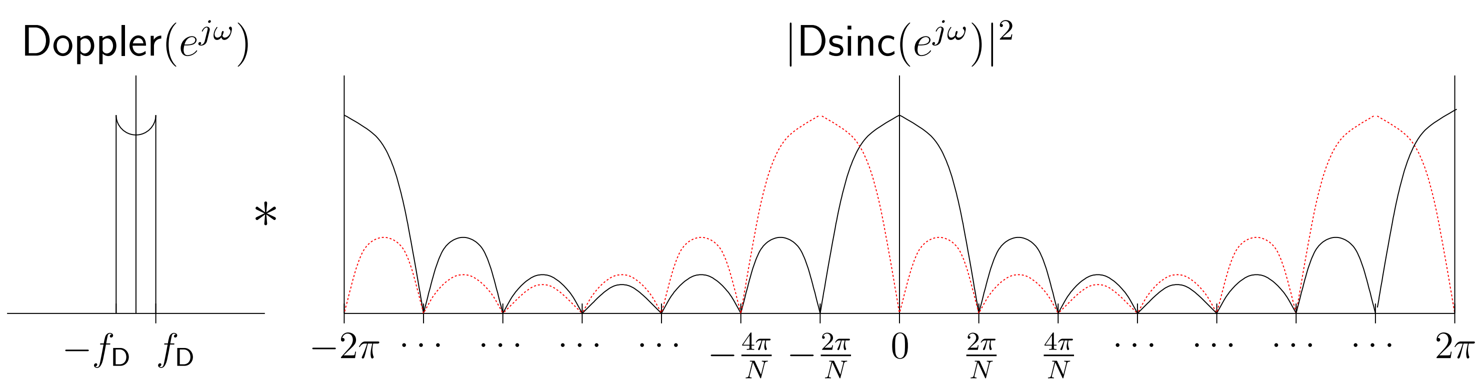
Rectangular Pulses:

A standard CP-OFDM block can be recognized as a sum of N infinite-length complex exponentials windowed by a rectangular pulse of width $N+L-1$.



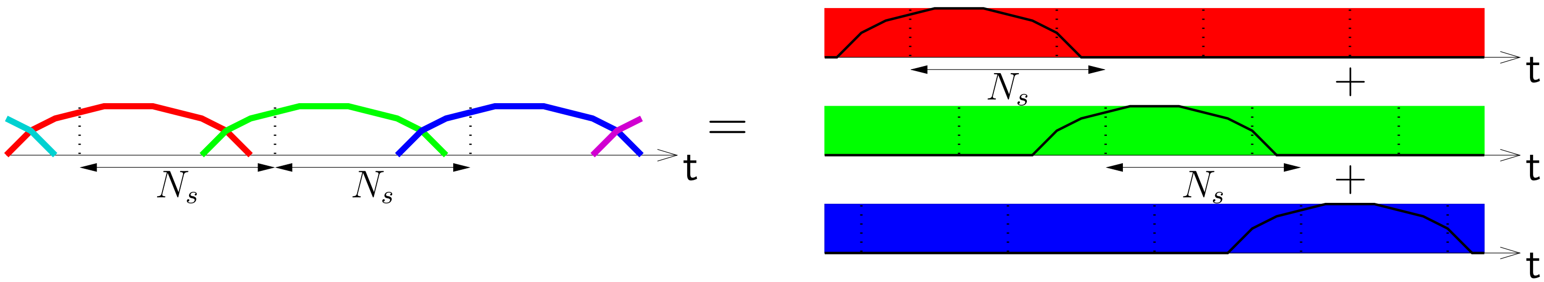
$\Rightarrow$ *Dirichlet sinc* in DTFT domain, whose slow side-lobe decay causes:

- strong interference to adjacent-band communication systems, and
- high sensitivity to Doppler spreading:



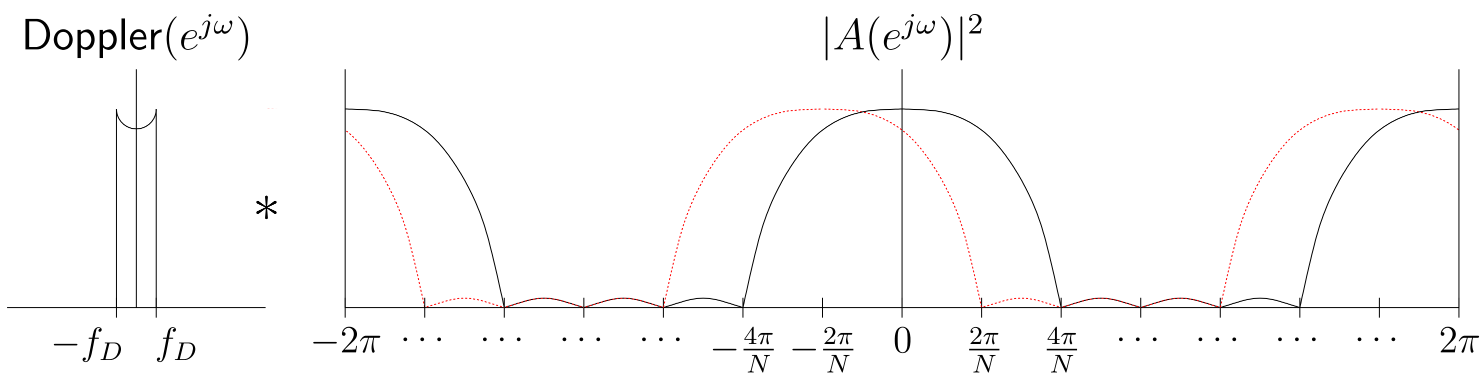
Smooth Overlapping Pulses:

What if we applied a *smooth* window instead?



The main-lobe is *wider* but the sidelobes *decay more quickly*, implying:

- reduced adjacent-band interference, and
- strong interference from adjacent subcarriers, but very little interference from all other subcarriers, even under large Doppler spreads:



Smooth Overlapping Pulses:

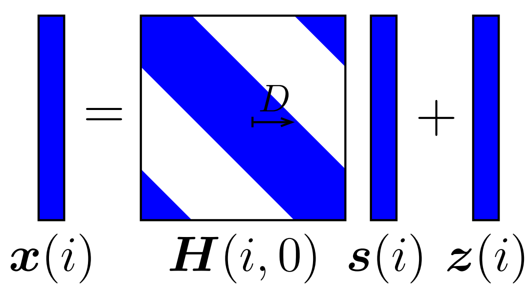
Challenge: The use of smooth overlapping pulses potentially causes *inter-carrier interference* (ICI) and *inter-block interference* (IBI):

$$\boldsymbol{x}(i) = \sum_{q=-\infty}^{\infty} \underbrace{\boldsymbol{H}(i,q)}_{\substack{\text{subcarrier}\\ \text{coupling matrix}}} \boldsymbol{s}(i-q) + \boldsymbol{z}(i).$$

Difficult to equalize!

Solution: Design the pulse shapes with the goal of. . .

1. Completely suppressing IBI: $\boldsymbol{H}(i,q)\big|_{q\neq 0} = \boldsymbol{0}$.
2. Allowing ICI only within a radius of $D \ll N$ subcarriers. (Often $D = 1$.)



Not difficult to equalize.

Receiver Pulse-Shaping:

Though so far we've considered a non-rectangular *transmission pulse* $\{a_n\}$,

$$t_n = \sum_{i=-\infty}^{\infty} a_{n-iN_s} \sum_{k=0}^{N-1} s_k(i) e^{j\frac{2\pi}{N}kn}, \qquad n = -\infty \ldots \infty,$$

we can use, in addition, a non-rectangular *reception pulse* $\{b_n\}$:

$$x_k(i) = \sum_{n=-\infty}^{\infty} r_{n-iN_s}\, b_n\, e^{-j\frac{2\pi}{N}kn}, \qquad k = 0 \ldots N-1.$$

N_s specifies the interval between the start of one OFDM block and the next.

- Modulation efficiency $\eta \triangleq \frac{N}{N_s} \frac{\text{symbols}}{\text{sec Hz}}$
- For OFDM, $N_s = N+L-1$, but now there is *no constraint* on N_s!

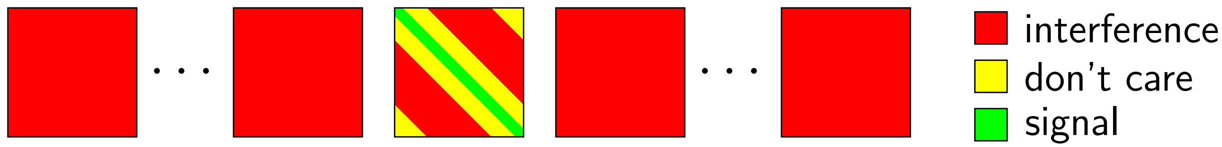
We focus on $N_s = N$ $\Leftrightarrow$ *no guard interval* $\Leftrightarrow$ $\eta = 1$.

Max-SINR Pulse Design:

Writing the received signal energy components due to

$$\mathcal{E}_s = \sum_{(q,k,l)\in\blacksquare} \mathrm{E}\{|H_{k,l}(\cdot,q)|^2\} \quad \text{“signal” and}$$

$$\mathcal{E}_i = \sum_{(q,k,l)\in\blacksquare} \mathrm{E}\{|H_{k,l}(\cdot,q)|^2\} \quad \text{“interference” (IBI+ICI)}$$

where $\{\boldsymbol{H}(\cdot,q)\} =$ 

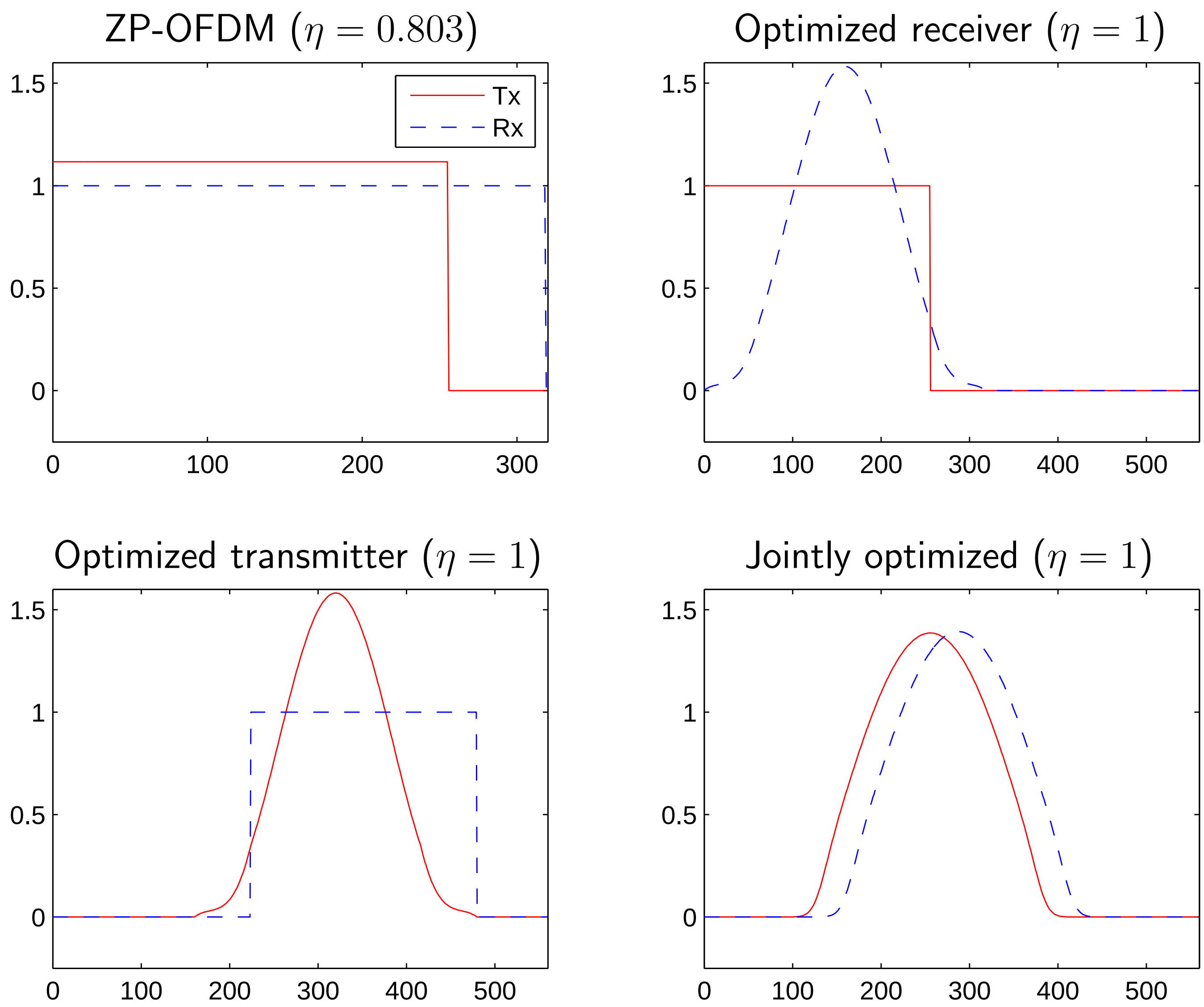
we can write $\quad \text{SINR} = \dfrac{\mathcal{E}_s}{\mathcal{E}_i + \mathcal{E}_n} = \dfrac{\boldsymbol{a}^H \boldsymbol{P}_1(\boldsymbol{b})\boldsymbol{a}}{\boldsymbol{a}^H \boldsymbol{P}_2(\boldsymbol{b})\boldsymbol{a}} = \dfrac{\boldsymbol{b}^H \boldsymbol{P}_3(\boldsymbol{a})\boldsymbol{b}}{\boldsymbol{b}^H \boldsymbol{P}_4(\boldsymbol{a})\boldsymbol{b}}$

where

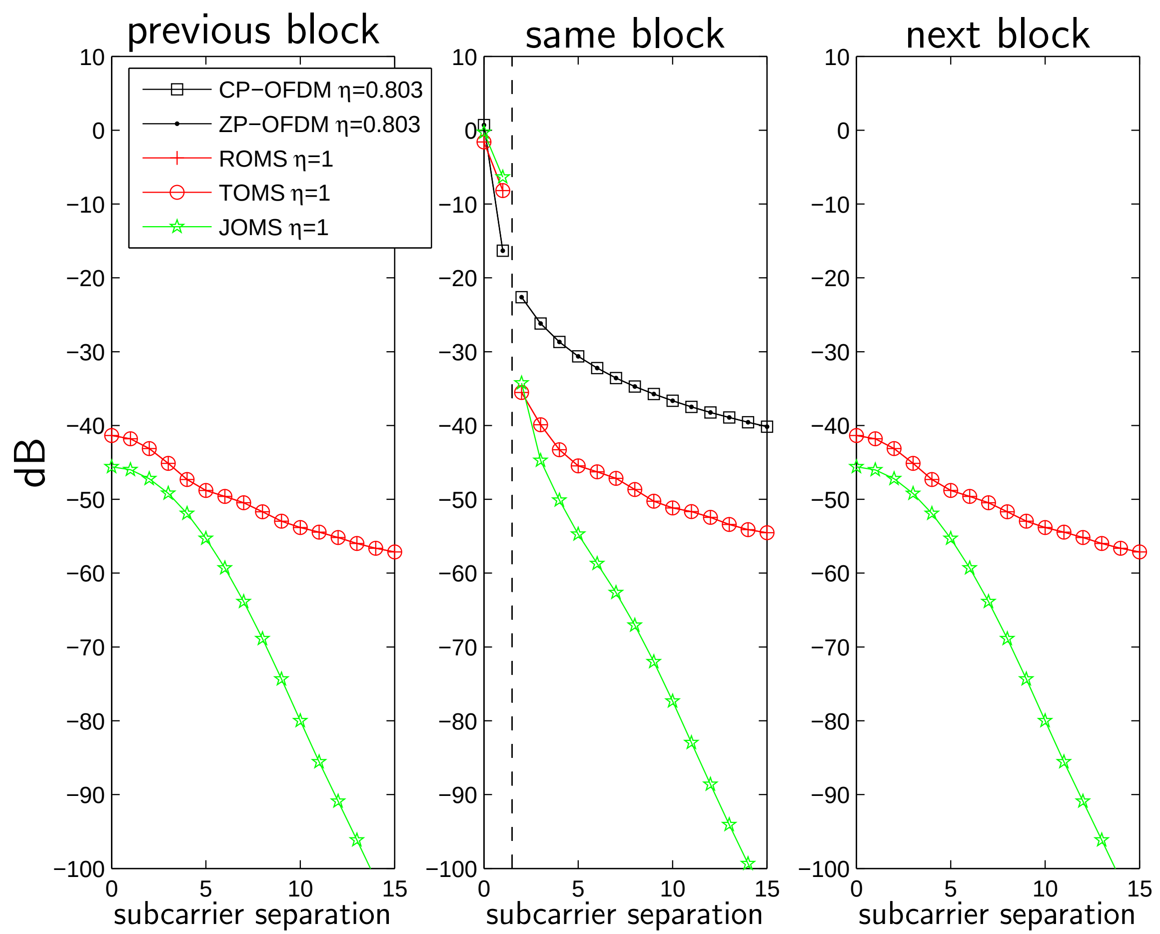
$$\begin{aligned} \boldsymbol{a} &= \text{transmission pulse coefficients} \\ \boldsymbol{b} &= \text{reception pulse coefficients} \\ \boldsymbol{P}_1(\cdot),\boldsymbol{P}_2(\cdot),\boldsymbol{P}_3(\cdot),\boldsymbol{P}_4(\cdot) &= \text{matrix fxns dependant on Doppler \& SNR.} \end{aligned}$$

$\Rightarrow$ *SINR-maximizing pulses are generalized eigenvectors.*

Max-SINR Pulse Examples:



IBI/ICI Energy Profiles (same for each subcarrier):



$D = 1$, $\mathsf{SNR} = 15\mathsf{dB}$, $L = 64$, $f_\mathsf{D}T_c = 7.6 \times 10^{-4}$, Jakes Doppler spectrum.

(For example, $f_c = 20\mathsf{GHz}$, BW=$3\mathsf{MHz}$, $T_h = 5.4\mu\mathsf{s}$, $v = 120\mathsf{km/hr}$.)

Non-Orthogonal FDM:

To summarize, Orthogonal FDM is possible only when

1. the channel is time-invariant, and
2. an adequate-length guard is included.

With a properly-designed *Non-Orthogonal FDM*, we can

- eliminate the guard, and
- tolerate large delay and Doppler spreads,

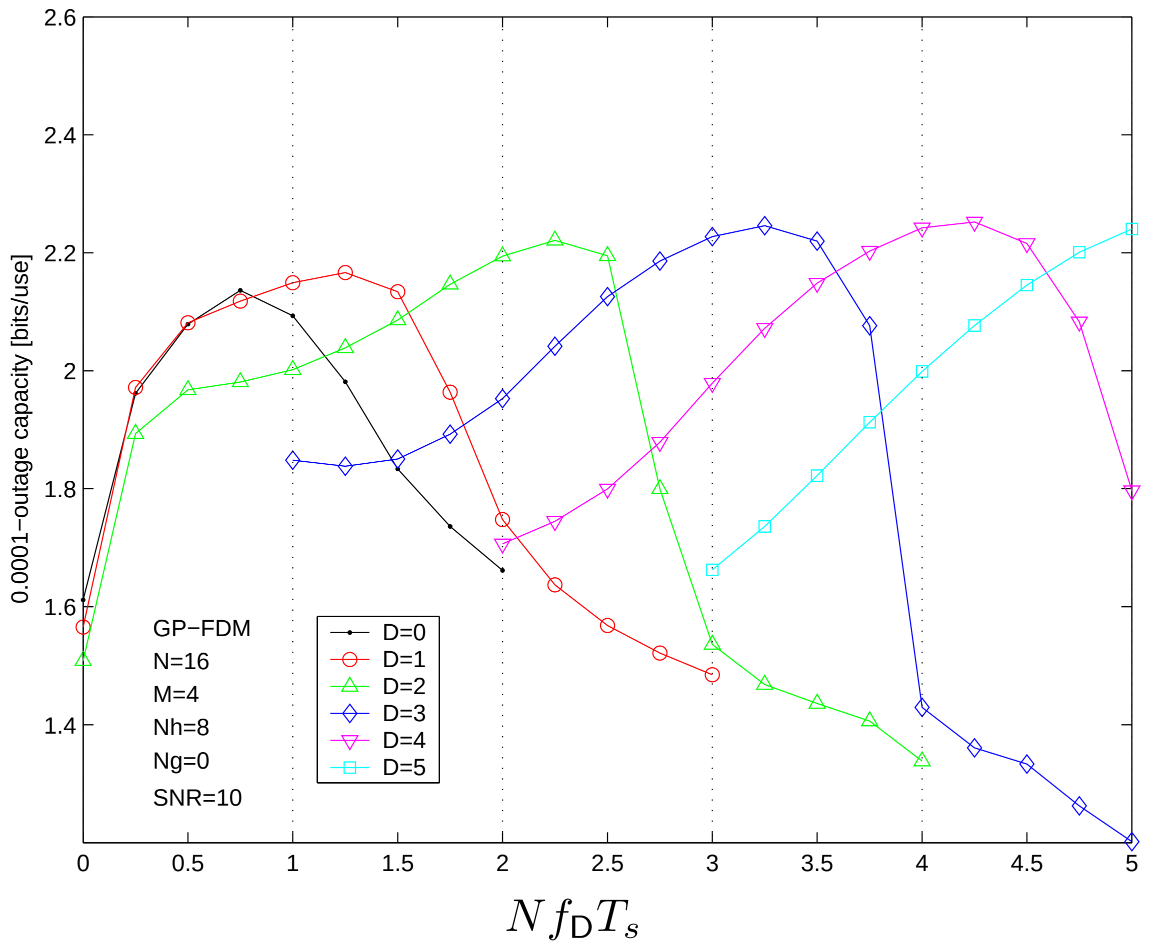
at the cost of

- a short span of intercarrier-interference,

which can be properly handled via low-complexity equalization.

Thus, we advocate *shaping ISI/ICI rather than suppressing ISI/ICI*.

Outage Capacity vs $N f_\mathsf{D} T_s$ for various ICI-radii D:



- The outage-capacity optimal D obeys $D \approx \lfloor N f_\mathsf{D} T_s \rceil$!
- ICI shaping is better than ICI suppression when $2 f_\mathsf{D} T_s \geq \frac{1}{N}$.

ICI Equalization:

Coherent approaches (i.e., known channel): $\frac{\text{mults}}{\text{QAM symbol}}$

1. Viterbi [Matheus/Kammeyer GLOBE 97] $\mathcal{O}(|\mathcal{S}|^D D)$
2. Iterative Soft [Das/Schniter Asilomar 04] $\mathcal{O}(D^2)$
3. Linear MMSE [Rugini/Banelli/Leus SPL 05] $\mathcal{O}(D^2)$
4. MMSE DFE [Rugini/Banelli/Leus SPAWC 05] $\mathcal{O}(D^2)$
5. Tree Search [Hwang/Schniter SPAWC 06] $\mathcal{O}(D^2)$

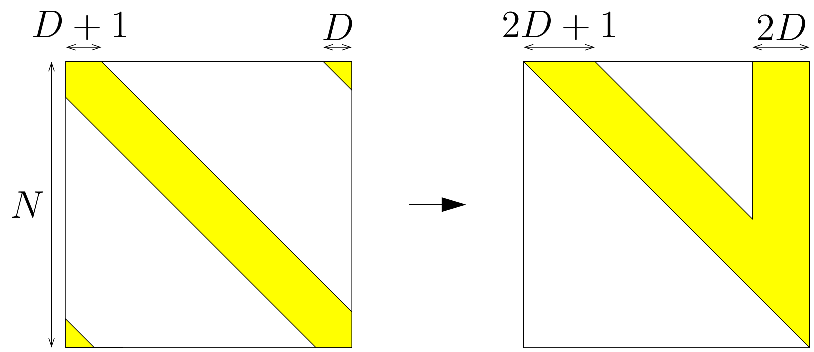
Non-coherent approaches (i.e., unknown channel):

1. Hard Tree Search [Hwang/Schniter WUWNet 07] $\mathcal{O}(D^2 L^2)$
2. Soft Tree Search [Hwang/Schniter Asilomar 07] $\mathcal{O}(D^2 L^2)$

1) Coherent Tree Search:

Two-step procedure:

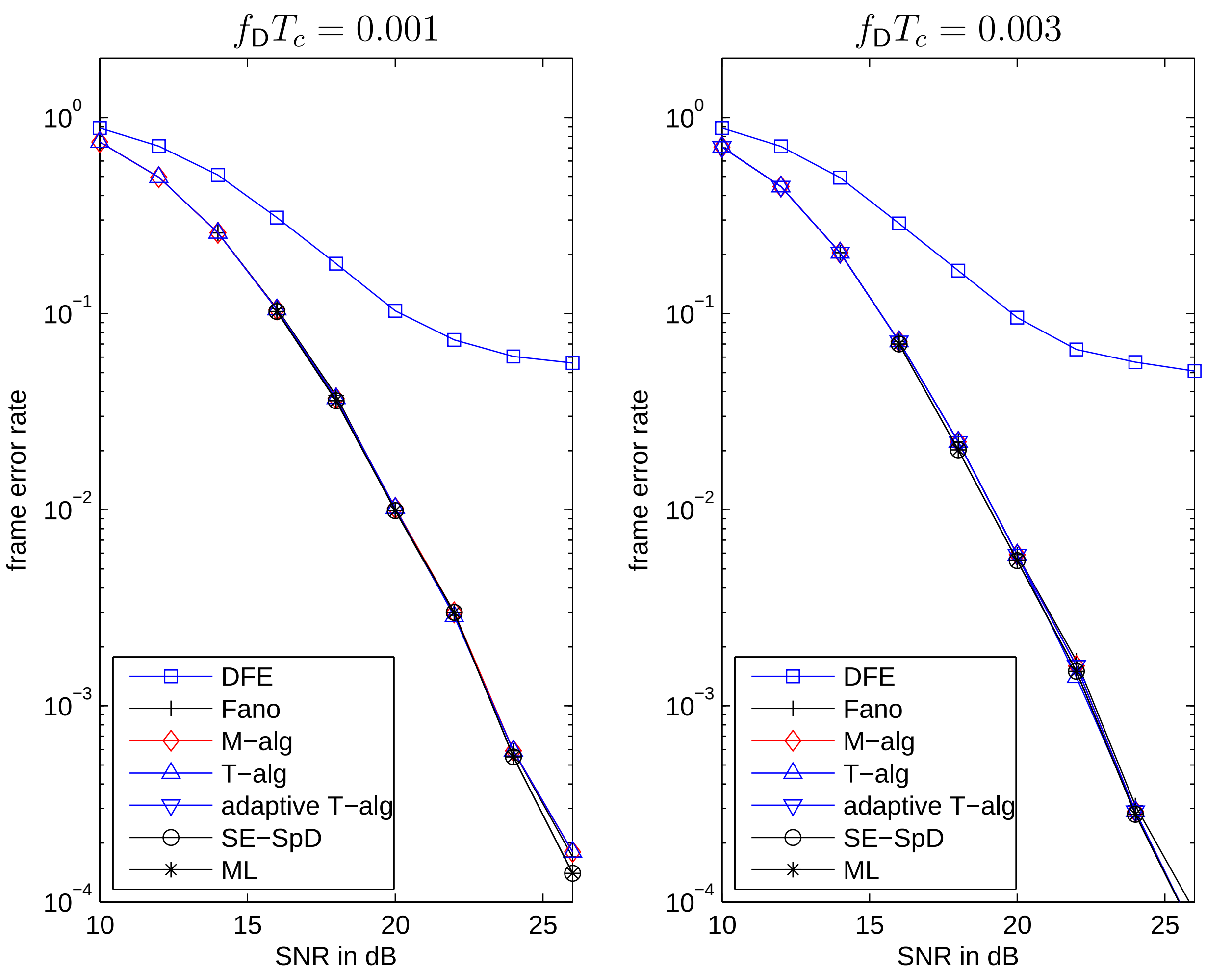
1. MMSE-GDFE pre-processing [Damen/ElGamal/Caire TIT 03]:



$\rightsquigarrow \mathcal{O}(D^2N)$ algorithm [Rugini/Banelli/Leus SPAWC 05].

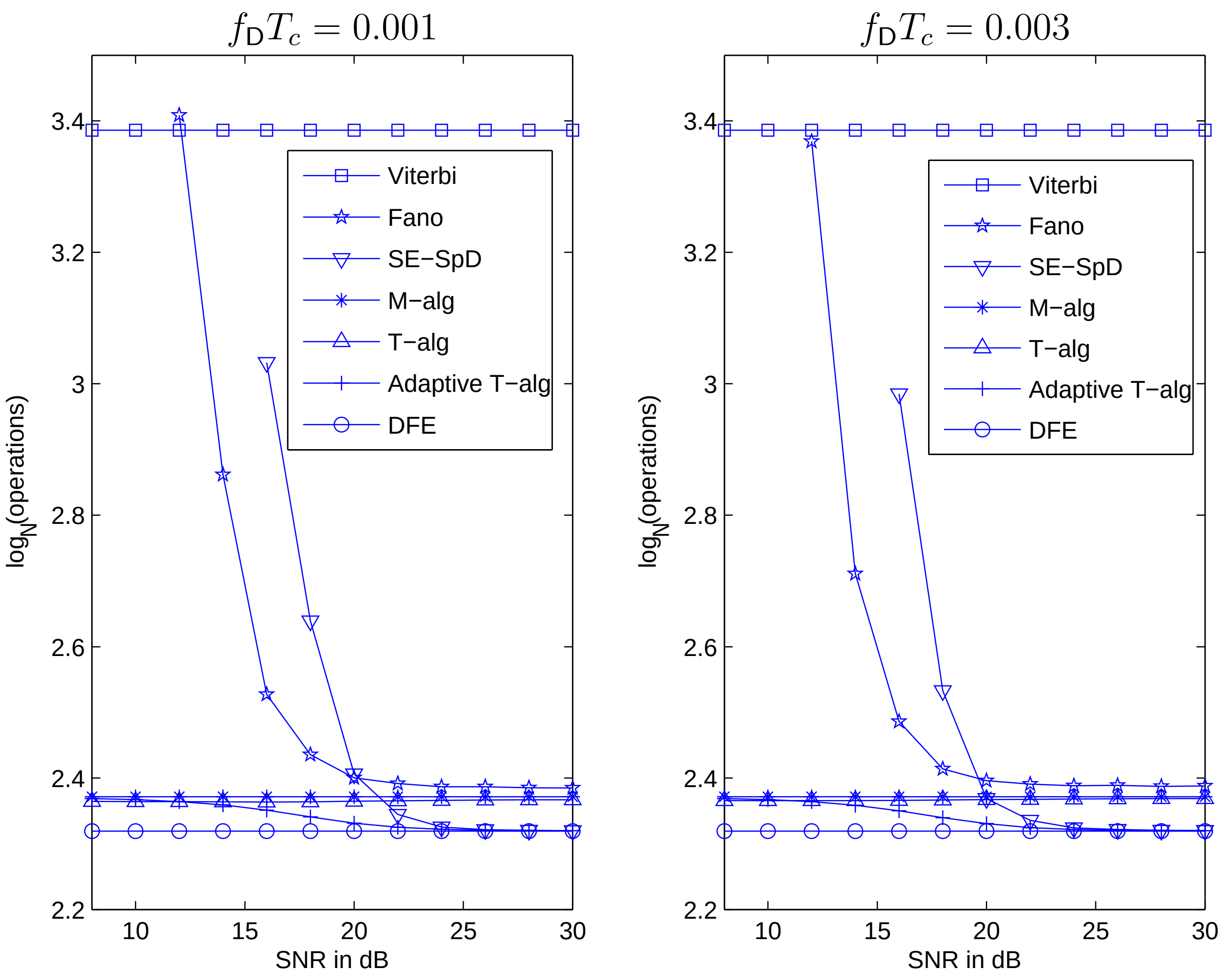
2. Near-optimal yet efficient tree search. Options include:
   - Depth-first search (e.g., Schnor-Euchner sphere decoder),
   - Best-first search (e.g., Fano alg, stack alg),
   - Breadth-first search (e.g., M-alg, T-alg, Pohst sphere decoder).

Performance:



Suboptimal tree search is almost indistinguishable from ML!

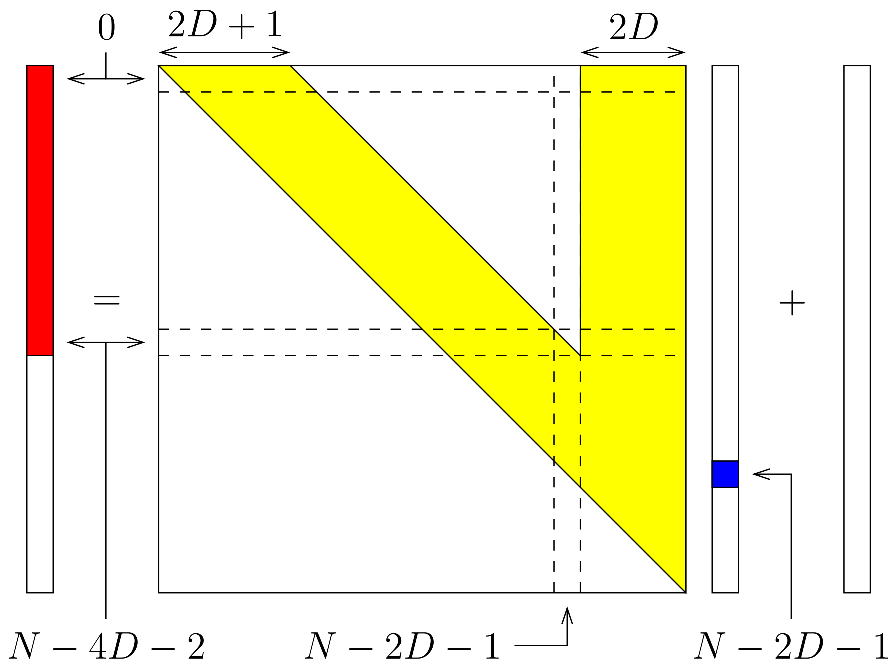
Average Complexity (MACs/frame):



Breadth-first & DFE stay cheap, while depth-first & Fano explode!

Error Masking due to V-shaped Channel Matrix:

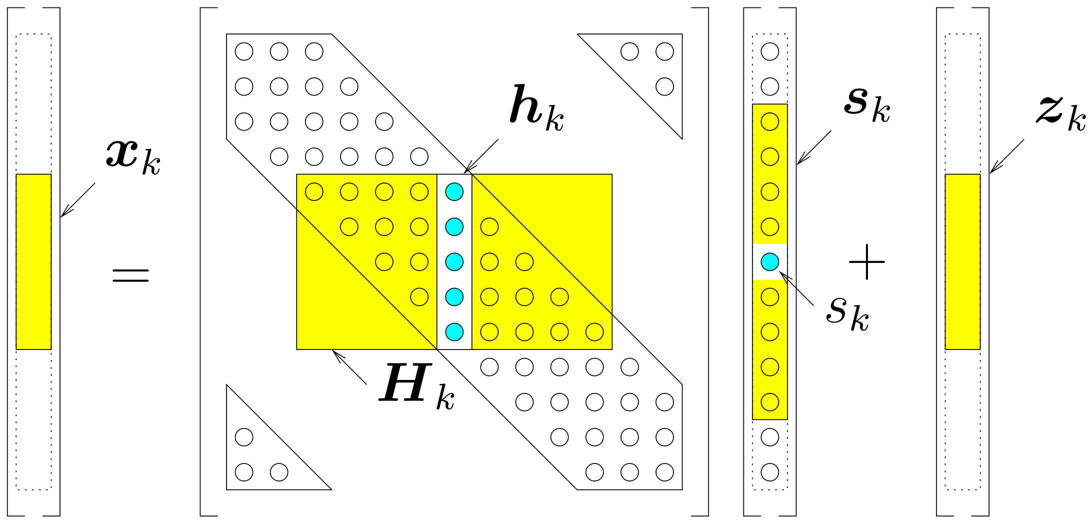
After MMSE-GDFE pre-processing, we get the following system:



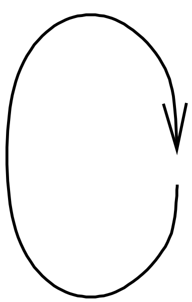
Key point: The blue symbol does not affect any of the red observations.

Error-masking explains the complexity explosion of the depth-first and Fano searches!

2) Iterative Soft ICI Cancellation:

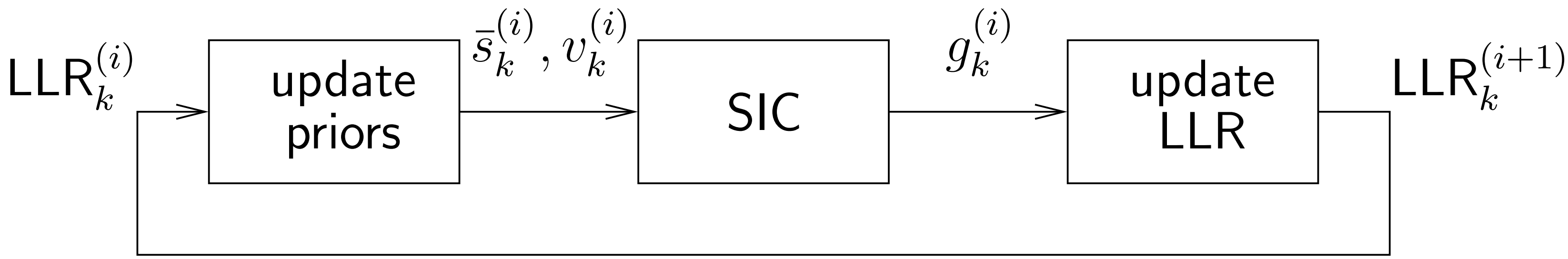


$$\boxed{\boldsymbol{x}_k = \boldsymbol{H}_k \boldsymbol{s}_k + \boldsymbol{z}_k}$$



- Soft interference cancellation using mean of $\boldsymbol{s}_k$.
- Assuming Gaussian residual interference and using the covariance of $\boldsymbol{s}_k$, compute LLRs(s_k).
- Using LLRs(s_k), update mean/covariance of s_k.
- $k \to \langle k+1 \rangle_N$.

Iterative Soft ICI Cancellation (BPSK example):



$$\bar{s}_k^{(i)} \triangleq \widehat{\mathrm{E}\{s_k|\hat{s}_k\}} = \tanh(\mathsf{LLR}_k^{(i)}/2)$$

$$v_k^{(i)} \triangleq \widehat{\mathrm{var}(s_k|\hat{s}_k)} = 1 - (\bar{s}_k^{(i)})^2$$

$$\boldsymbol{y}_k^{(i)} = \boldsymbol{x}_k - \boldsymbol{H}_k \bar{\boldsymbol{s}}_k^{(i)}$$

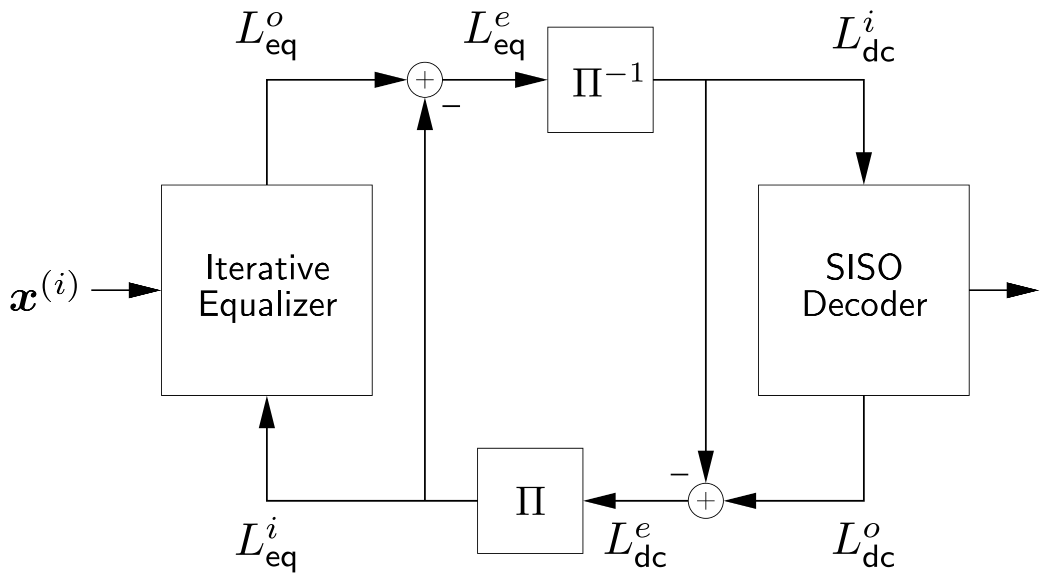
$$g_k^{(i)} = \boldsymbol{y}_k^{(i)H} \big(\boldsymbol{R}_z + \boldsymbol{H}_k \mathcal{D}(\boldsymbol{v}_k^{(i)}) \boldsymbol{H}_k^H\big)^{-1} \boldsymbol{h}_k$$

$$\mathsf{LLR}_k^{(i+1)} = \mathsf{LLR}_k^{(i)} + 2\,\mathrm{Re}(g_k^{(i)})$$

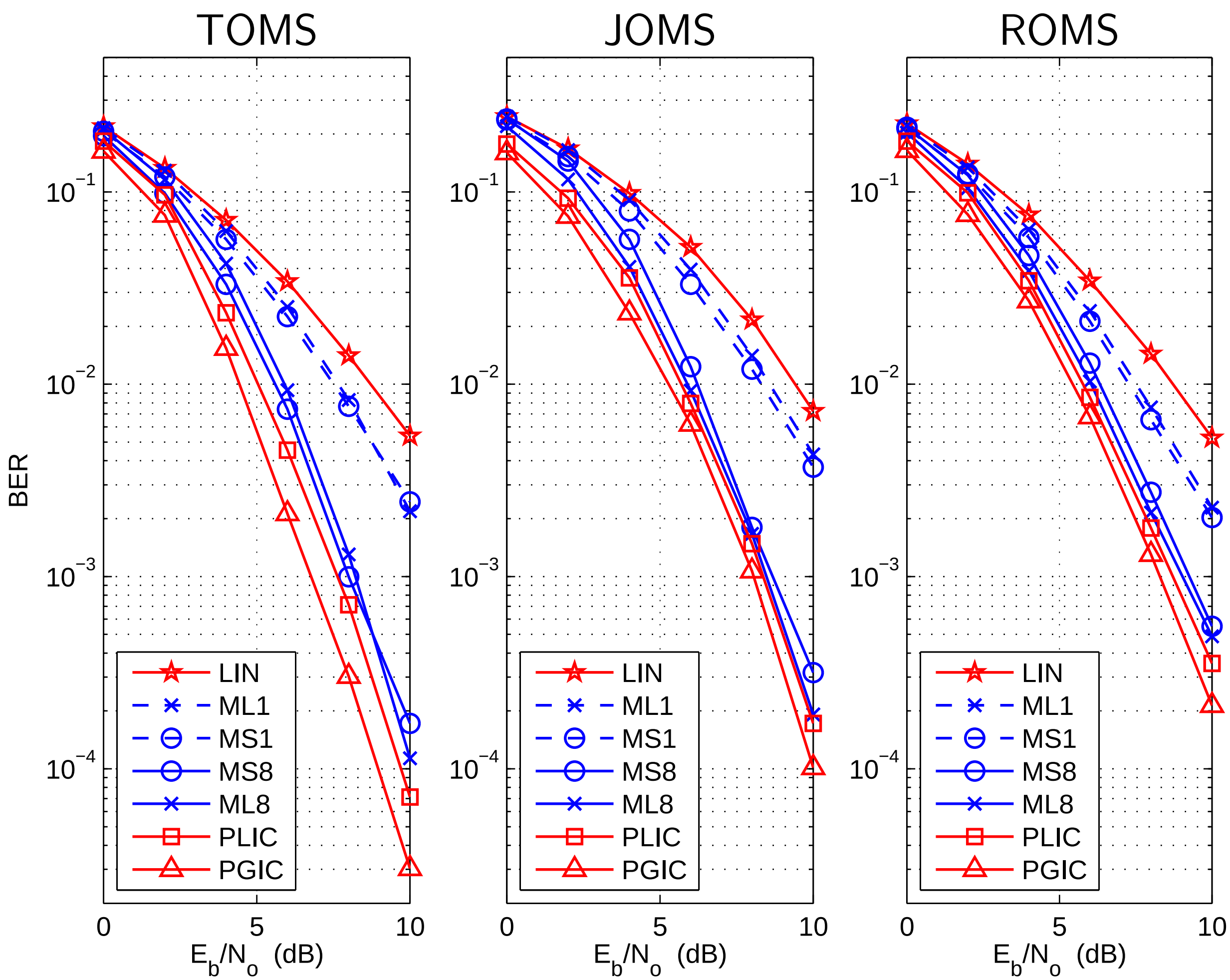
Complexity: $\underbrace{M}_{\textit{iters}} \times \underbrace{\mathcal{O}(D^2)}_{\textit{mtx inv}}$ per BPSK symbol.

Turbo Equalization:

Soft bit information (i.e., LLRs) can be exchanged with a soft-input soft-output (SISO) decoder:



Turbo Equalization Performance:



rate-$\frac{1}{2}$ conv code

QPSK

$N = 64$

$D = 2$

$L = 16$

$f_{\mathsf{D}}T_c = 0.003$

for example:

$f_c = 20$GHz

BW$= 3$MHz

$T_h = 5.4\mu$s

$v = 3\times 160$km/hr

3) Non-Coherent Equalization:

- So far we have assumed that the equalizer is provided with channel estimates.

  But with long and quickly varying channels, accurate channel estimation requires a high proportion of pilots!

- Instead, one might consider *joint* estimation of channel and symbols.

  Actually, we are not interested in explicitly estimating the channel, but rather doing *non-coherent equalization*: inferring transmitted bits using knowledge of channel statistics but not channel state.

For this it helps to re-parameterize the system model...

Basis-Expansion Model:

From the pulse-shaped FDM model:

$$\begin{aligned} \boldsymbol{x}(n) &= \boldsymbol{H}(n)\boldsymbol{s}(n) + \boldsymbol{z}(n) \qquad && \boldsymbol{H}(n) \text{ is banded with band radius } D \\ &= \boldsymbol{S}(n)\boldsymbol{h}(n) + \boldsymbol{z}(n) && \boldsymbol{h}(n) \in \mathbb{C}^{(2D+1)N} \text{ ICI coefs} \end{aligned}$$

we can use a basis-expansion model (BEM) for the ICI coefs:

$$\begin{aligned} \boldsymbol{h}(n) &= \boldsymbol{B\theta}(n) \qquad && \boldsymbol{\theta}(n) \in \mathbb{C}^{(2D+1)L} \text{ delay/Doppler coefs} \\ \boldsymbol{B} &= \begin{pmatrix} \boldsymbol{F}_L & & \\ & \ddots & \\ & & \boldsymbol{F}_L \end{pmatrix} && \boldsymbol{F}_L \in \mathbb{C}^{N \times L} \text{ truncated DFT matrix} \end{aligned}$$

to rewrite the observation as

$$\boldsymbol{x}(n) = \boldsymbol{S}(n)\boldsymbol{B\theta}(n) + \boldsymbol{z}(n).$$

Note: if we knew the locations of K active *sparse* taps, we would only include those columns of the DFT matrix, giving $\boldsymbol{\theta}(n) \in \mathbb{C}^{(2D+1)K}$.

Non-coherent MLSD:

Treating the delay/Doppler coefs $\boldsymbol{\theta}$ as nuisance parameters,

$$\begin{aligned}\hat{\boldsymbol{s}}_{\mathsf{ML}} &= \arg\max_{\boldsymbol{s}} p(\boldsymbol{x}|\boldsymbol{s}) \\ &= \arg\max_{\boldsymbol{s}} \int_{\boldsymbol{\theta}} p(\boldsymbol{x}|\boldsymbol{s},\boldsymbol{\theta})p(\boldsymbol{\theta})d\boldsymbol{\theta}\end{aligned}$$

Assuming $\boldsymbol{\theta} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{R}_\theta)$,

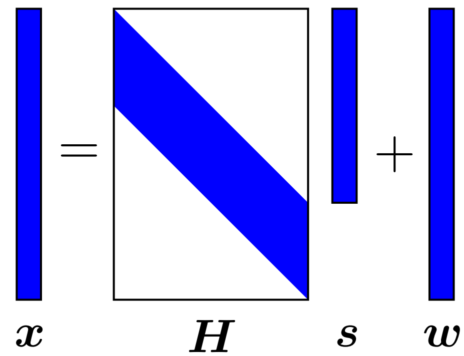
$$\begin{aligned}\hat{\boldsymbol{s}}_{\mathsf{ML}} &= \arg\max_{\boldsymbol{s}} \left\{\boldsymbol{x}^H \boldsymbol{S}\boldsymbol{B}\boldsymbol{\Sigma}_{\boldsymbol{s}}^{-1}\boldsymbol{B}^H\boldsymbol{S}^H\boldsymbol{x} - \sigma^2 \log|\boldsymbol{\Sigma}_{\boldsymbol{s}}|\right\} \\ \boldsymbol{\Sigma}_{\boldsymbol{s}} &\triangleq \boldsymbol{B}^H\boldsymbol{S}^H\boldsymbol{S}\boldsymbol{B} + \sigma^2\boldsymbol{R}_\theta^{-1}\end{aligned}$$

Since $\hat{\boldsymbol{\theta}}_{\mathsf{MMSE}|\boldsymbol{s}} = \boldsymbol{\Sigma}_{\boldsymbol{s}}^{-1}\boldsymbol{B}^H\boldsymbol{S}^H\boldsymbol{x}$, we can rewrite

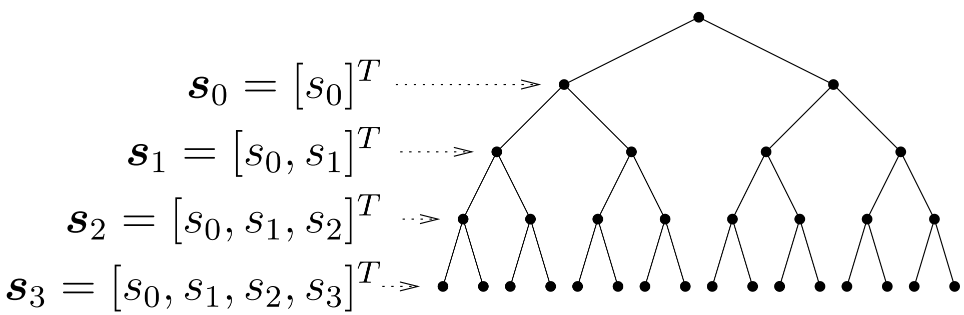
$$\hat{\boldsymbol{s}}_{\mathsf{ML}} = \arg\max_{\boldsymbol{s}} \left\{\|\boldsymbol{x} - \boldsymbol{S}\boldsymbol{B}\hat{\boldsymbol{\theta}}_{\mathsf{MMSE}|\boldsymbol{s}}\|^2 - \sigma^2 \log|\boldsymbol{\Sigma}_{\boldsymbol{s}}|\right\}$$

But how do we avoid an exhaustive search over $\boldsymbol{s}$?

Fast Tree Search:



By turning off the first and last D subcarriers, $\boldsymbol{H}$ becomes upper-triangular, facilitating the use of **tree-search**.



(for BPSK)

The important thing here is that the *partial* ML metric

$$\mu_{\mathsf{ML}}(\boldsymbol{s}_k) \;=\; \|\boldsymbol{x}_k - \boldsymbol{S}_k \boldsymbol{B}_k \hat{\boldsymbol{\theta}}_{\mathsf{MMSE}|\boldsymbol{s}_k}\|^2 - \sigma^2 \log|\boldsymbol{\Sigma}_k|$$

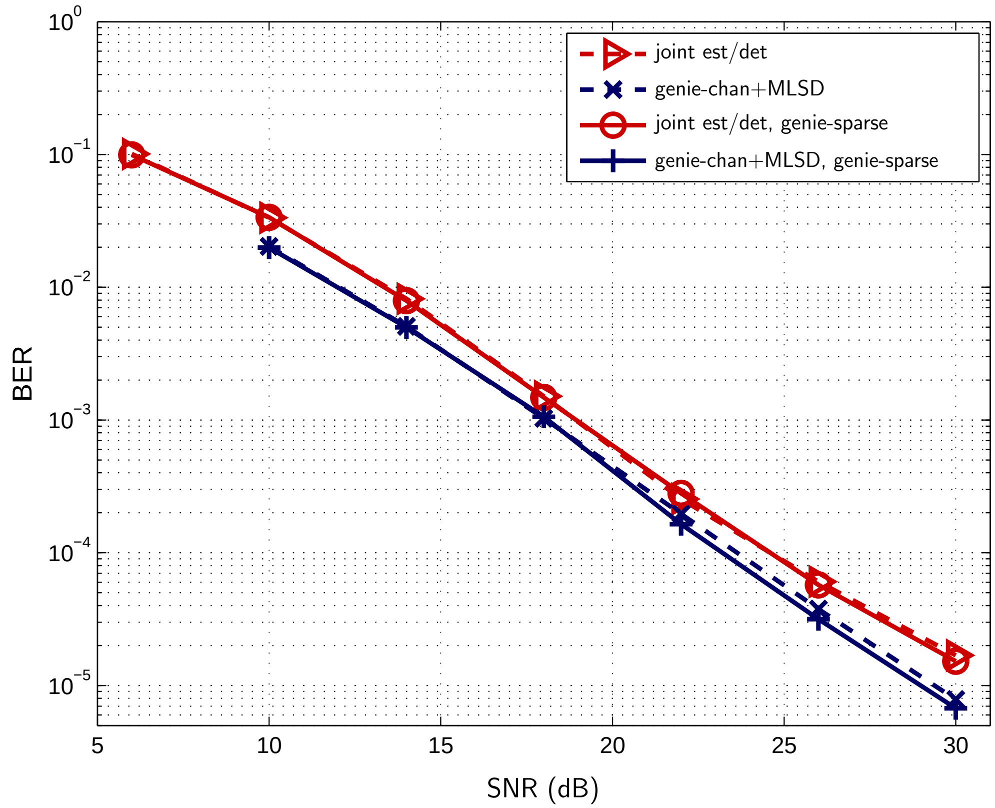
can be computed *recursively*. Thus, total search complexity via the M-algorithm is only

$$2M|\mathcal{S}|(2D+1)^2L^2 \;\; \text{mults per QAM-symbol!}$$

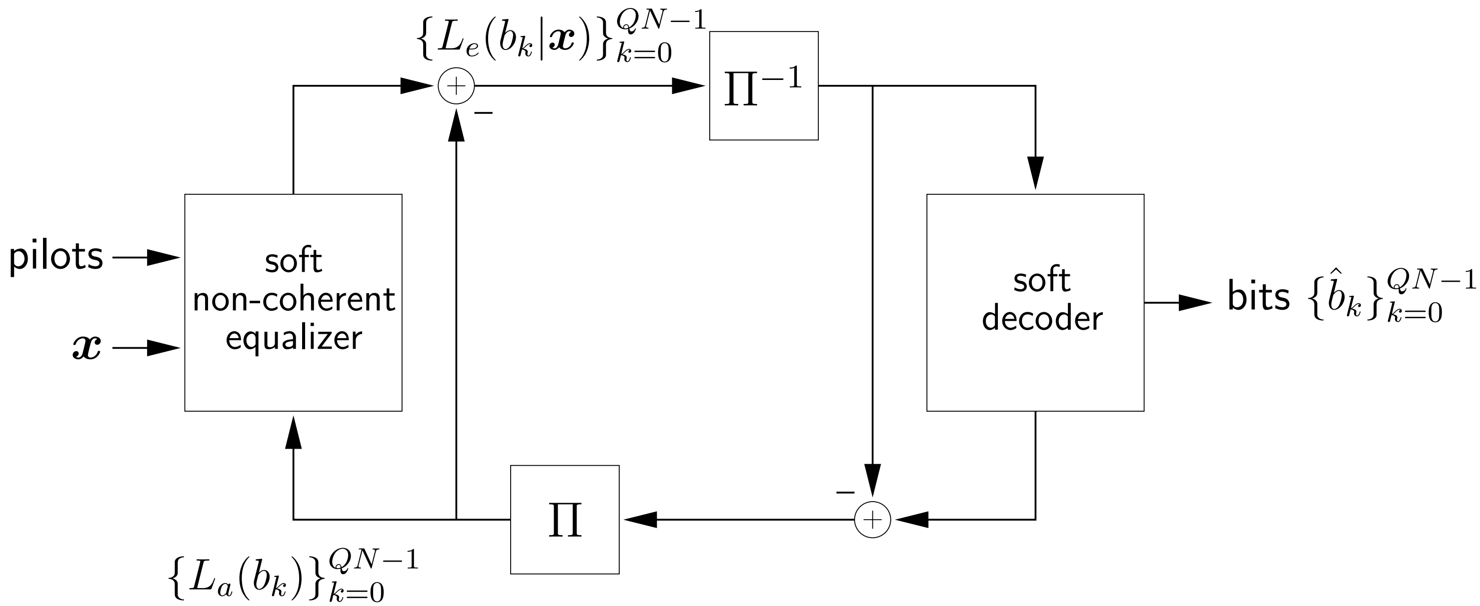
Complexity Reduction via Pilots:

- With non-coherent decoding, we require only a *single* pilot subcarrier.
- But, with more pilots, the initial channel estimate $\hat{\boldsymbol{\theta}}_{\text{MMSE}}$ improves, allowing more aggressive branch pruning in the tree search.

Example: M-algorithm (BPSK, 25% pilots, $M{=}8$) compared to coherent MLSD with genie-aided $\hat{\boldsymbol{\theta}}_{\text{MMSE}}$:



Non-coherent Turbo Equalization:



$$L_e(b_k|\boldsymbol{x}) \;=\; \ln \frac{\sum_{\boldsymbol{s}:\, b_k=1} \exp \mu_{\mathsf{MAP}}(\boldsymbol{s})}{\sum_{\boldsymbol{s}:\, b_k=0} \exp \mu_{\mathsf{MAP}}(\boldsymbol{s})} - L_a(b_k) \qquad \text{"extrinsic LLR"}$$

$$\mu_{\mathsf{MAP}}(\boldsymbol{s}) \;=\; \ln p\big(\boldsymbol{x}|\boldsymbol{s}\big) + \sum_{k:\, b_k=1} L_a(b_k) \qquad \text{"MAP metric"}$$

Need $\mathcal{O}(2^{QN})$ evaluations of $\mu(\boldsymbol{s}) \rightsquigarrow$ *Computationally infeasible!*

Simplified LLR Evaluation:

The “max-log” approximation:

$$L_e(b_k|\boldsymbol{x}) \approx \max_{\boldsymbol{s}\in\mathcal{L}\cap\{\boldsymbol{s}:b_k=1\}} \mu_{\mathsf{MAP}}(\boldsymbol{s}) - \max_{\boldsymbol{s}\in\mathcal{L}\cap\{\boldsymbol{s}:b_k=0\}} \mu_{\mathsf{MAP}}(\boldsymbol{s}) - L_a(b_k)$$

$$\mathcal{L} \; : \; \text{set containing the } M \text{ most probable } \boldsymbol{s},$$

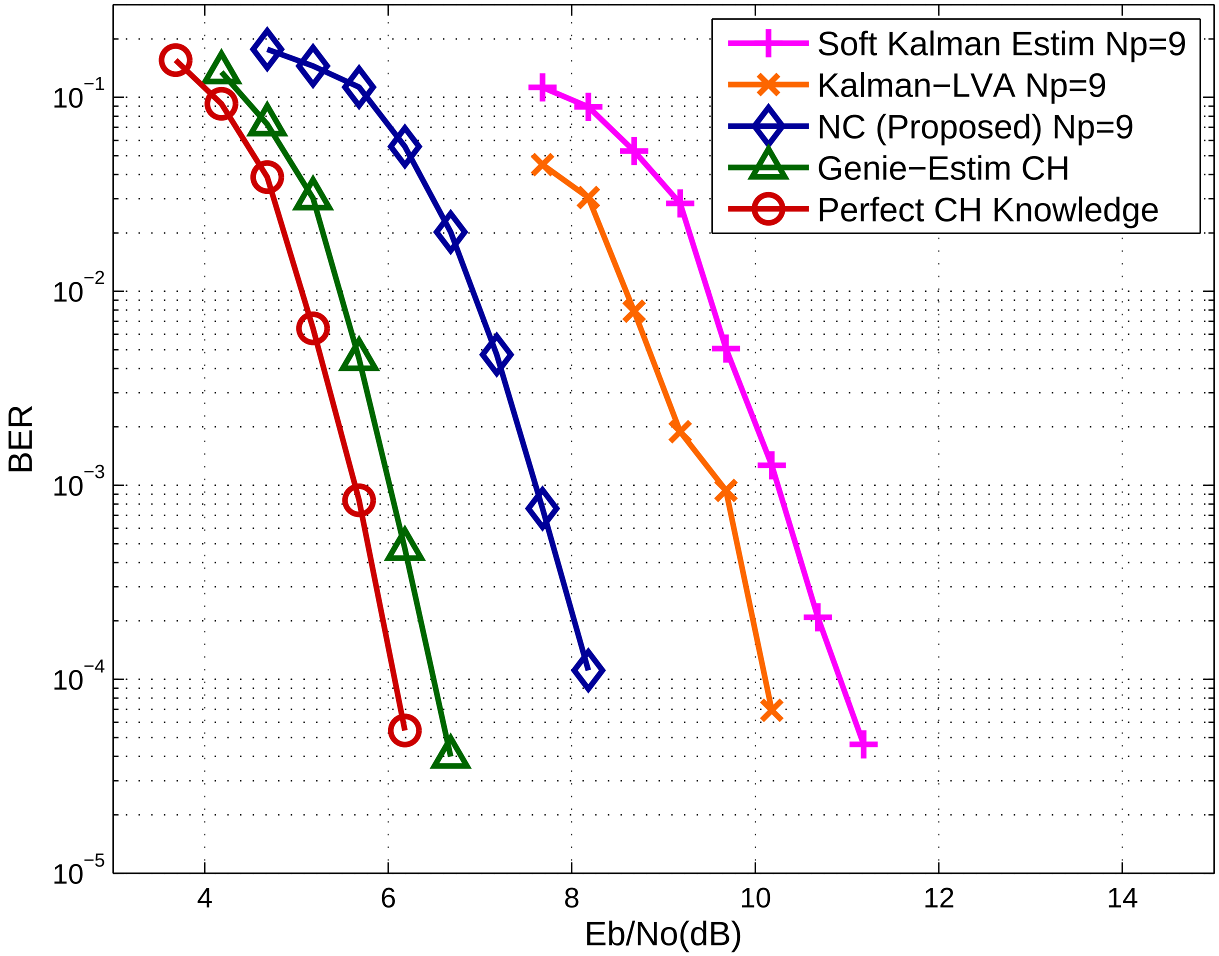
requires only a few evaluations of $\mu_{\mathsf{MAP}}(\boldsymbol{s})$.

To find $\mathcal{L}$, the set of most probable $\boldsymbol{s}$, and the MAP metrics $\{\mu_{\mathsf{MAP}}(\boldsymbol{s})\}_{\boldsymbol{s}\in\mathcal{L}}$, we use a *tree search*, as in the uncoded case.

Here again, there exists a fast metric update such that the total search complexity, with the M-algorithm, is only

$$2M|\mathcal{S}|(2D+1)^2L^2 \;\; \text{mults per QAM-symbol!}$$

Non-coherent Turbo Performance:



rate-$\frac{1}{2}$ LDPC

QPSK

$N = 64$

$D = 2$

$L = 16$

$K = 3$

$f_{\mathsf{D}} T_c = 0.003$

Related Work:

- Channel estimation techniques (for coherent equalization).
- Single-carrier frequency-domain equalization.
- Pilot pattern designs:
  - MMSE optimal (under CE-BEM assumption).
  - Achievable-rate optimal (under CE-BEM assumption).
- Theoretical analysis of pulse-shaped multicarrier modulation:
  - Achievable-rate characterized.
- Analysis of doubly selective channel:
  - Capacity characterized (under CE-BEM assumption).

(See `http://www.ece.osu.edu/~schniter/pubs_by_topic.html`)

Conclusions:

Single Carrier:

- $\mathcal{O}(L)\ \frac{\text{mults}}{\text{QAM symbol}}$ equalization of delay-spread channels.
- Challenging to track quickly time-varying channels.

Orthogonal FDM:

- $\mathcal{O}(\log_2 L)\ \frac{\text{mults}}{\text{QAM symbol}}$ equalization of delay-spread channels.
- Loss in spectral efficiency due to guard interval.
- Sensitive to Doppler spread.
- Slow spectral roll-off $\Rightarrow$ high adjacent-band interference.

Non-Orthogonal FDM:

- No need for a guard interval; high spectral efficiency.
- Large simultaneous delay & Doppler spreads $\Rightarrow$ no IBI and short ICI.
- Fast spectral roll-off $\Rightarrow$ low adjacent-band interference.

Equalization/Decoding of Short ICI Span:

- *Uncoded Coherent*:
  Tree search gives ML-like performance with DFE-like complexity.
- *Coded Coherent*:
  Iterative soft ICI cancellation in turbo configuration performs close to perfect-interference-cancellation bound.
- *Uncoded Non-Coherent*:
  Tree-search with fast metric update gives ML-like performance with $\mathcal{O}(D^2L^2)$ complexity.
- *Coded Non-Coherent*:
  Tree-search with fast metric update performs close to genie-aided bound with $\mathcal{O}(D^2L^2)$ complexity, or $\mathcal{O}(D^2K^2)$ if sparseness leveraged.

Thanks for listening!