Near-far gain in a multiuser diversity system

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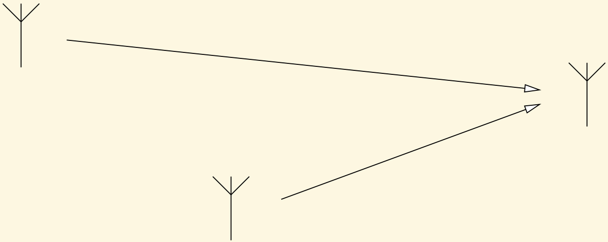
Outline

1. Introduction
2. Overview of PFS
3. Group scheduling
4. Asymptotic behavior analysis
5. Summary

Multiuser diversity and near-far



- Time varying channels
- Unbalance between channels
- Selection diversity over users

Multiuser diversity and near-far, examples

Sum-rate greedy scheduler

Schedule one user with strongest channel $\rightarrow$ time division.

Hard-fair scheduler

Schedule all users (invert channel, guarantee rate) $\rightarrow$ code division.

Motivation

Initial thought

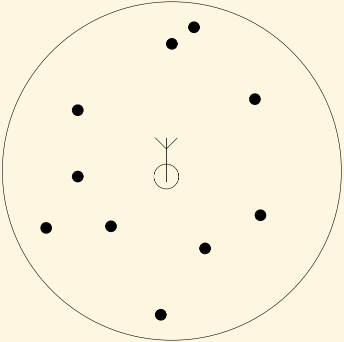
- PFS is sum-rate suboptimal
  (maximizes $\sum_{k=1}^{K} \log\left(E\left[R_k\right]\right)$).
- We should be able to improve by simultaneous scheduling

What shall we analyze

- Average spectral efficiency / sum rate.
- Focus on high and low spectral efficiency regimes
  (wideband regime)

Single cell system – uplink (multiple access channel)

- K users.
- Each user has fixed random pathloss s_k (exponential).
- Each user has i.i.d. short term fading f_k (Rayleigh).
- Channel gain of user k is $d_k = s_k f_k$.



Proportional Fairness 1

At time t PFS schedules the user k with maximum metric

$$\frac{R_k(t)}{T_k(t)}$$

where

$$R_k(t) = \log_2\left(1 + \frac{d_k(t)}{N_0}\right) \quad (1)$$

$$T_k(t+1) = \lambda T_k(t) + (1-\lambda) R_k(t) \quad (2)$$

The forgetting factor $\lambda \in [0, 1)$

if $\lambda \to 0$, PFS is more round-robin
if $\lambda \to 1$, PFS is at its greediest

Proportional Fairness 2

When $\lambda \to 1$ (infinite forgetting), PFS

- schedules the user with largest short term fading f,
- allocates rate according to channel state d,
- enforces orthogonal transmission (TDMA),
- provides no rate guarantee, and
- is (still) not sum rate optimal.

Key idea

- Choose K_A users with largest short term fading f_k
- Allocate each user the power $1/K_A$
- Allocate each user rate assuming superposition coding.

Answers we have

- System capacity behavior.
- Optimal number of simultaneous users.

Answers to find

- Can we formulate a simple recursive allocation cf. PFS?
- What did we just do to “fairness”?

Scheduling $k \in \mathscr{A}$

- Received energy $y = \frac{1}{K_A} \sum_{k \in \mathscr{A}} f_k s_k$.
- Sum rate $R(y) = \log_2(1 + y\mathsf{SNR})$.

Average system capacity and system energy per bit

$$\mathsf{C} = \int_0^\infty R(y) dF(y) \qquad (3)$$

$$\left(\frac{E_b}{N_0}\right)_{\mathsf{sys}} = \frac{\mathsf{SNR}}{\mathsf{C}} \qquad (4)$$

$F(y)$ is the distribution of the K_A largest f_k (out of K) each multiplied with a random path loss s_k.

System capacity

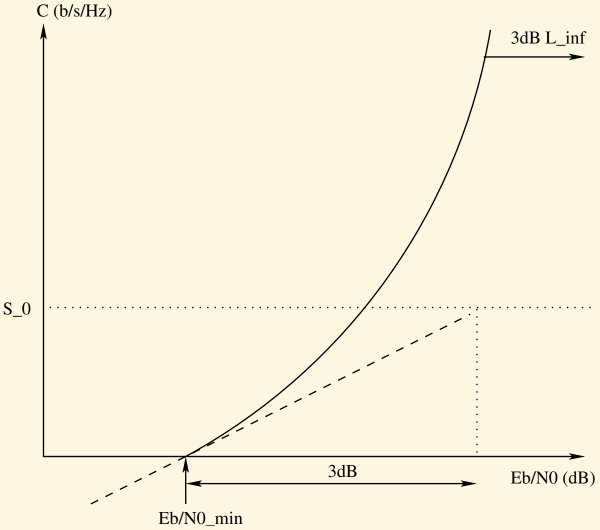
Low spectral efficiency approximation

$$\left.\left(\frac{E_b}{N_0}\right)_{\mathsf{sys}}\right|_{\mathsf{dB}} = \left.\left(\frac{E_b}{N_0}\right)_{\mathsf{min}}\right|_{\mathsf{dB}} + \frac{\mathsf{3dB}}{\mathscr{S}_0}\mathsf{C} + o(\mathsf{C}), \quad (5)$$

High spectral efficiency approximation

$$\left.\left(\frac{E_b}{N_0}\right)_{\mathsf{sys}}\right|_{\mathsf{dB}} = \frac{\mathsf{3dB}}{\mathscr{S}_\infty}\mathsf{C} - 10\log_{10}(\mathsf{C}) + \mathscr{L}_\infty 10\log_{10}(2) + o(1), \quad (6)$$

The same visually



Minimum E_b/N_0

$$\left(\frac{E_b}{N_0}\right)_{\mathsf{min}} = \frac{\log(2)}{\mathsf{C}'(0)} \tag{7}$$

$$= \frac{\log(2)}{\int_0^\infty \frac{y}{1+\mathsf{SNR}y} dF_y(y)\Big|_{\mathsf{SNR}=0}} \tag{8}$$

$$= \frac{\log(2)}{\mathsf{E}[y]} \tag{9}$$

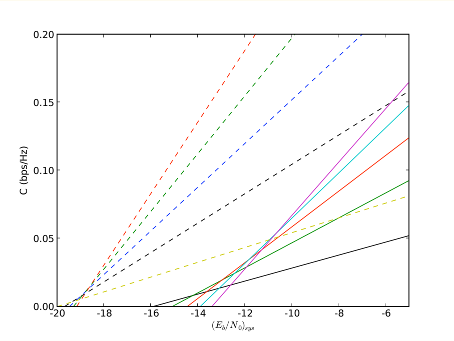
$$= \frac{\log(2)}{\mathsf{E}[s]\frac{1}{K_A}\sum_{i=K-K_A+1}^{K}\mathsf{E}[f_{i:K}]}. \tag{10}$$

Wideband slope

$$\begin{aligned}
\mathscr{S}_0 &= \frac{2\mathsf{C}'(0)^2}{-\mathsf{C}''(0)} && (11)\\
&= \frac{2\left(\int_0^\infty \frac{y}{1+\mathsf{SNR}y}\, dF_y(y)\Big|_{\mathsf{SNR}\to 0}\right)^2}{\int_0^\infty \left(\frac{y}{1+\mathsf{SNR}y}\right)^2 dF_y(y)\Big|_{\mathsf{SNR}\to 0}} && (12)\\
&= \frac{2\mathsf{E}[y]^2}{\mathsf{E}[y^2]} && (13)\\
&= \frac{2\left(\sum_{i=K-K_A+1}^{K} \mathsf{E}[f_{i:K}]\right)^2}{\sum_{i=K-K_A+1}^{K}\left(\frac{\mathsf{E}[s^2]}{\mathsf{E}[s]^2}\mathsf{E}[f_{i:K}^2] + \sum_{j=K-K_A+1, j\neq i}^{K} \mathsf{E}[f_{i:K} f_{j:K}]\right)}. && (14)
\end{aligned}$$

System capacity 1: low spectral efficiency

Increase K_A, lose in $(E_b/N_0)_{\text{min}}$, win in wideband slope.



For larger K you lose less! (solid $K = 10$, dashed $K = 1000$)

System capacity 1.1: low spectral efficiency

What is happening

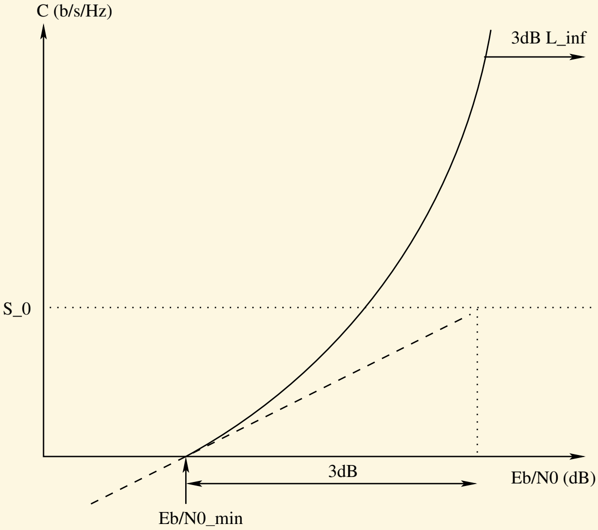
- For larger K, the difference of K_A largest f_k diminishes
- Second moment of y inversely proportional to K_A^2
- For large K_A:

$$\frac{2\mathsf{E}\left[\frac{1}{K_A}\sum_{k\in\mathscr{A}} d_k\right]^2}{\mathsf{E}\left[\left(\frac{1}{K_A}\sum_{k\in\mathscr{A}} d_k\right)^2\right]} \to \frac{2\mathsf{E}\left[\mathsf{E}\left[d\right]\right]^2}{\mathsf{E}\left[\mathsf{E}\left[d\right]^2\right]} = 2$$

Taking $K_A = \gamma K \to \infty$, $\gamma \leq 1$

- $(E_b/N_0)_{min} = \frac{\log(2)}{E[s]E_{\mathscr{A}}[f]}$
- $\mathscr{S}_0 = 2$

Reminder for dB penalty



Horizontal dB penalty

$$\mathscr{L}_\infty = \lim_{\mathsf{SNR}\to\infty}\left(\log_2 \mathsf{SNR} - \int_0^\infty \log_2(1+y\mathsf{SNR})\,dF_y(y)\right) \quad (15)$$

$$= -\mathsf{E}_y[\log_2(y)] \quad (16)$$

$$\leq -\mathsf{E}_s\left[\log_2\left(\frac{1}{K_A}\sum_{i=1}^{K_A} s_i\right)\right] \quad (17)$$

$$-\mathsf{E}_{f_{K-K_A+1:K}}[\log_2(f_{K-K_A+1:K})]\,. \quad (18)$$

System capacity 2.1: high spectral efficiency

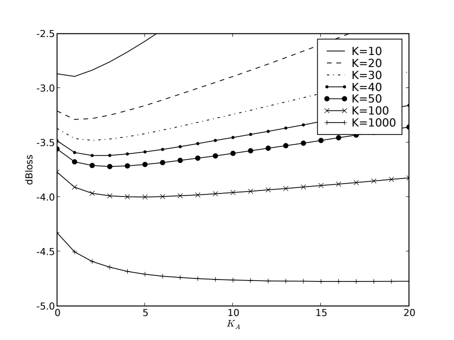
Larger K_A gives

- smaller penalty due to (17) – near-far gain (Jensen)
- larger penalty due to (18) – lost diversity gain.

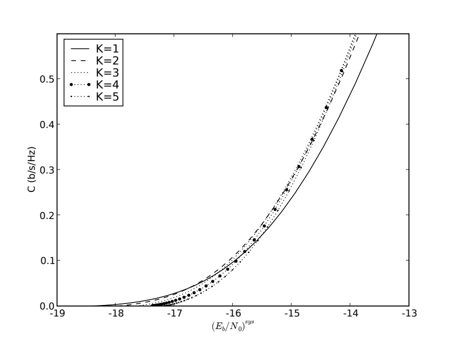
Also

- Near-far gain is bounded.
- Multiuser diversity loss is a function of user population size.
- There is a K-dependent optimal K_A.

System capacity 2: high spectral efficiency

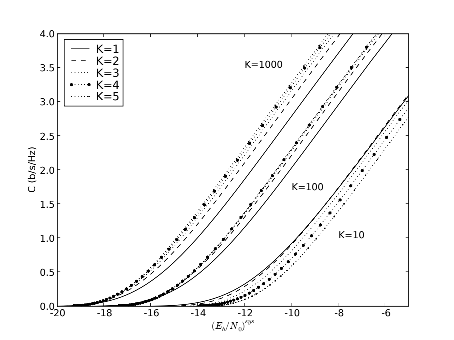


Comparison to PFS: I



Exact behavior at low spectral efficiency ($K = 100$).

Comparison to PFS: II



Already with 10 users scheduling 2 at a time is ok.

Summary

- Asymptotic tools can simplify your life.
- Minimum E_b/N_0, wideband slope and dB penalty useful parameters to analyze
- Scheduling multiple users with group-PFS
  - increases wideband slope
  - increases minimum E_b/N_0
  - decreases dB penalty with high enough K