Near-far gain in a multiuser diversity system

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NTNU
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2. Overview of PFS
3. Group scheduling
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Multiuser diversity and near-far

- Time varying channels
- Unbalance between channels
- Selection diversity over users
Multiuser diversity and near-far, examples

**Sum-rate greedy scheduler**
Schedule one user with strongest channel $\rightarrow$ time division.

**Hard-fair scheduler**
Schedule all users (invert channel, guarantee rate) $\rightarrow$ code division.
Motivation

Initial thought

- PFS is sum-rate suboptimal
  \[
  \text{(maximizes } \sum_{k=1}^{K} \log(E[R_k])\text{).}
  \]
- We should be able to improve by simultaneous scheduling

What shall we analyze

- Average spectral efficiency / sum rate.
- Focus on high and low spectral efficiency regimes (wideband regime)
The system

Single cell system – uplink (multiple access channel)

- $K$ users.
- Each user has fixed random pathloss $s_k$ (exponential).
- Each user has i.i.d. short term fading $f_k$ (Rayleigh).
- Channel gain of user $k$ is $d_k = s_k f_k$. 
At time $t$ PFS schedules the user $k$ with maximum metric

$$\frac{R_k(t)}{T_k(t)}$$

where

$$R_k(t) = \log_2 \left( 1 + \frac{d_k(t)}{N_0} \right)$$  \hspace{1cm} (1)$$

$$T_k(t + 1) = \lambda T_k(t) + (1 - \lambda)R_k(t)$$ \hspace{1cm} (2)$$

The forgetting factor $\lambda \in [0, 1)$

if $\lambda \to 0$, PFS is more round-robin
if $\lambda \to 1$, PFS is at its greediest
When $\lambda \to 1$ (infinite forgetting), PFS

- schedules the user with largest short term fading $f$,
- allocates rate according to channel state $d$,
- enforces orthogonal transmission (TDMA),
- provides no rate guarantee, and
- is (still) not sum rate optimal.
Group scheduling

Key idea

- Choose $K_A$ users with largest short term fading $f_k$
- Allocate each user the power $1/K_A$
- Allocate each user rate assuming superposition coding.

Answers we have

- System capacity behavior.
- Optimal number of simultaneous users.

Answers to find

- Can we formulate a simple recursive allocation cf. PFS?
- What did we just do to “fairness”?
**Group scheduling**

**Scheduling** \( k \in \mathcal{A} \)

- Received energy \( y = \frac{1}{K_A} \sum_{k \in \mathcal{A}} f_k s_k \).
- Sum rate \( R(y) = \log_2 (1 + y \text{SNR}) \).

**Average system capacity and system energy per bit**

\[
C = \int_0^\infty R(y) dF(y) \quad (3)
\]

\[
\left( \frac{E_b}{N_0} \right)_{sys} = \frac{\text{SNR}}{C} \quad (4)
\]

\( F(y) \) is the distribution of the \( K_A \) largest \( f_k \) (out of \( K \)) each multiplied with a random path loss \( s_k \).
System capacity

Low spectral efficiency approximation

\[
\left( \frac{E_b}{N_0} \right)_{\text{sys}} \bigg|_{\text{dB}} = \left( \frac{E_b}{N_0} \right)_{\text{min}} \bigg|_{\text{dB}} + \frac{3 \text{dB}}{L_0} C + o(C), \quad (5)
\]

High spectral efficiency approximation

\[
\left( \frac{E_b}{N_0} \right)_{\text{sys}} \bigg|_{\text{dB}} = \frac{3 \text{dB}}{L_\infty} C - 10 \log_{10} (C) + L_\infty 10 \log_{10} (2) + o(1), \quad (6)
\]
The same visually

\[ C (\text{b/s/Hz}) \]

\[ S_0 \]

\[ \text{Eb/N0}_\text{min} \]

\[ 3\text{dB} \]

\[ L_{\text{inf}} \]

\[ \text{Eb/N0} \text{ (dB)} \]
Minimum $E_b/N_0$

\[
\left( \frac{E_b}{N_0} \right)_{\text{min}} = \frac{\log(2)}{C'(0)}
\]

\[
= \frac{\log(2)}{\int_{0}^{\infty} \frac{y}{1+SNR_y} dF_y(y) \bigg|_{SNR=0}}
\]

\[
= \frac{\log(2)}{E[y]}
\]

\[
= \frac{\log(2)}{E[s] \frac{1}{K_A} \sum_{i=K-K_A+1}^{K} E[f_i;K]}.
\]
Wideband slope

\[ S_0 = \frac{2C' (0)^2}{-C'' (0)} \]  

\[ = 2 \left( \int_0^\infty \frac{y}{1+\text{SNR}} dF_y(y) \right)^2 \]  

\[ \Rightarrow \int_0^\infty \left( \frac{y}{1+\text{SNR}} \right)^2 dF_y(y) \text{ SNR} \to 0 \]  

\[ = 2 \mathbb{E} [y]^2 \]  

\[ \frac{\mathbb{E} [y^2]}{\mathbb{E} [y^2]} \]  

\[ = 2 \left( \sum_{i=K-K_A+1}^K \mathbb{E}[f_i^2] \right)^2 \]  

\[ = \sum_{i=K-K_A+1}^K \left( \frac{\mathbb{E}[s^2]}{\mathbb{E}[s]^2} \mathbb{E}[f_i^2] + \sum_{j=K-K_A+1,j \neq i}^K \mathbb{E}[f_i f_j \mid f_i^2] \right) \]  

\[ \text{ (11)} \]  

\[ \text{ (12)} \]  

\[ \text{ (13)} \]  

\[ \text{ (14)} \]
Increase $K_A$, lose in $(E_b/N_0)_{\text{min}}$, win in wideband slope.

For larger $K$ you lose less! (solid $K = 10$, dashed $K = 1000$)
System capacity 1.1: low spectral efficiency

What is happening

- For larger $K$, the difference of $K_A$ largest $f_k$ diminishes
- Second moment of $y$ inversely proportional to $K_A^2$
- For large $K_A$:

\[
2 \mathbb{E} \left[ \frac{1}{K_A} \sum_{k \in A} d_k \right]^2 \quad \frac{2 \mathbb{E} \left[ \mathbb{E} [d] \right]^2}{\mathbb{E} \left[ \mathbb{E} [d^2] \right]} = 2
\]

Taking $K_A = \gamma K \to \infty$, $\gamma \leq 1$

- $(E_b/N_0)_{\text{min}} = \frac{\log(2)}{E[s] E_{\mathcal{A}|f}}$
- $S_0 = 2$
Reminder for dB penalty

\[ C (\text{b/s/Hz}) \]

\[ S_0 \]

\[ \text{Eb/N0_min} \]

\[ \text{3dB L_{inf}} \]

\[ \text{3dB} \]

\[ \text{Eb/N0 (dB)} \]
\[ \mathcal{L}_\infty = \lim_{\text{SNR} \to \infty} \left( \log_2 \text{SNR} - \int_0^\infty \log_2 (1 + y\text{SNR}) \, dF_y(y) \right) \quad (15) \]

\[ = -\mathbb{E}_y [\log_2(y)] \quad (16) \]

\[ \leq -\mathbb{E}_s \left[ \log_2 \left( \frac{1}{K_A} \sum_{i=1}^{K_A} s_i \right) \right] \quad (17) \]

\[ -\mathbb{E}_{f_{K-K_A+1},K} [\log_2 (f_{K-K_A+1},K)] \cdot \quad (18) \]
Larger $K_A$ gives
- smaller penalty due to (17) – near-far gain (Jensen)
- larger penalty due to (18) – lost diversity gain.

Also
- Near-far gain is bounded.
- Multiuser diversity loss is a function of user population size.
- There is a $K$-dependent optimal $K_A$. 
System capacity 2: high spectral efficiency
Comparison to PFS: I

Exact behavior at low spectral efficiency ($K = 100$).
Already with 10 users scheduling 2 at a time is ok.
Asymptotic tools can simplify your life.
Minimum $E_b/N_0$, wideband slope and dB penalty useful parameters to analyze
Scheduling multiple users with group-PFS
  - increases wideband slope
  - increases minimum $E_b/N_0$
  - decreases dB penalty with high enough $K$