Capacity Maximizing Power Allocation for Interfering Wireless Links: A Distributed Approach

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Outline

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2. System Model
3. Centralized Optimal Power Allocation
4. Distributed Power Allocation
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1. Introduction - Motivation (1/3)

- Efficient utilization of limited spectral resources → Increase Reuse.
- Links operating on the same spectral resource are plagued by mutual interference.
- System designers push for more aggressive reuse → excessive interference → harms system capacity.
- To counter excessive interference → Power Control.
- Vary the transmit power in the network to achieve the desired goal.
- Classic work on SINR balancing [Zan92, Fos93].
1. Introduction - Rate Maximization (2/3)

• For best-effort data networks, maintaining a guaranteed SINR is not always required.
• The amount of data delivered is more relevant - focus on aggregate rate maximization.
• Basically optimize the transmit power to obtain system-wide gains.
• Centralized optimization can yield the maximum gain - distributed solution desirable.

• Considerations & Challenges for Distributed Control:
  – System Performance coupled across whole network.
  – How to predict/model behavior of other links (interference scenario)?
  – Self-organization/adaptation.
Figure 1: Centralized vs. Distributed Control.

Formulation applies to both types of networks.
2. System Model (1/2)

- \( N \) active transmit-receive pairs chosen by virtue of MAC protocol or scheduling policy.
- Focus is on one channel use - access slot.
- Common spectral reuse i.e. \( \text{reuse} = 1 \).
- Transmitter sends signal for its intended receiver only.
- Receiver however is interfered by all transmitters (Interference Channel).
- Practically can be ad-hoc network or cellular network (e.g. downlink with AP being transmitters)
2. System Model (2/2)

- Signal Model for receiver of link $n$:

$$Y_n = \sqrt{G_{n,n}} X_n + \sum_{i \neq n} \sqrt{G_{n,i}} X_i + Z_n,$$

- $X_n$ is the signal from transmitter $n$.
- $G_{n,i} \in \mathbb{R}^+$ is random channel gain between any arbitrary transmitter $i$ and receiver $n$ (assumed constant over access slot).
- $Z_n$ is additive white Gaussian noise

- Let $P_n = \mathbb{E}|X_n|^2$ and for simplicity $\mathbb{E}|Z_n|^2 = \sigma^2$. 
3. Centralized Optimal Power Allocation (1/4)

- Given complete system knowledge, how do we allocate power so as to maximize the system capacity?
- Let $0 \leq P_n \leq P_{\text{max}}$ be the transmit power of link $n$
- **Transmit power vector:**
  \[ P = [P_1 \ P_2 \ \cdots \ P_n \ \cdots \ P_N]. \]
- Feasible set of transmit power vectors is given by:
  \[ \Omega = \{ P \mid 0 \leq P_n \leq P_{\text{max}} \forall n = 1, \ldots, N \}. \]
3. Optimal Power Allocation (2/4)

- SINR

\[ \Gamma(P) = \frac{G_{n,n}P_n}{\sigma^2 + \sum_{i \neq n} G_{n,i}P_i} \]

- Objective Function: network capacity (bits/sec/Hz)

\[ C(P) \Delta \sum_{n=1}^{N} \log \left(1 + \Gamma(P)\right) \]  

(1)

- Optimal Power Allocation Problem:

\[ P^* = \arg \max_{P \in \Omega} C(P) \]  

(2)

where \( \Omega = \{P \mid 0 \leq P_n \leq P_{\text{max}} \quad \forall \quad n = 1, \ldots, N\} \)

- Difficult to solve due to the non-convexity of the problem.
3. Optimal Power Allocation (3/4)

However, for $N = 2$, we obtain interesting results...

**Theorem 1** *For the two-cell case, the sum throughput maximizing power allocation is binary [1]. Mathematically,*

$$\arg\max_{P \in \Omega} C(P) = \arg\max_{P \in \Omega^B} C(P)$$

*where $\Omega^B = \{P \mid P_{un} = 0 \text{ or } P_{un} = P_{\text{max}}\}$.*

**Proof:** see [2]

- Optimal power allocation vector is either $(P_{\text{max}}, P_{\text{max}})$, $(P_{\text{max}}, 0)$ or $(0, P_{\text{max}})$.
- Also valid for any $N$ in low SINR regime [2].
3. Optimal Power Allocation (4/4)

- Hence, in the two-cell case the transmit power range can be quantized to two values, either on or off, \textit{without} loss in capacity.

- Independent of user scheduling.

- Significant reduction in optimization search space.

- Unfortunately, does not hold for \( N > 2 \) (can be shown with a simple counter example)

- But, binary power allocation shown to be very close to the centralized geometric programming approach [2].

- Motivates adoption of Binary feasible set for \( N \) links.

Finding optimal power allocation vector still requires centralized control.
4. Distributed Power Allocation (1/8)

- Framework for Distributed Optimization:
  - assume statistical knowledge of unknown instantaneous information
  - optimize over what is known

- Network Capacity Maximization Under Statistical Knowledge.

- Set containing all network information: $\mathcal{G}$.

- Tx $n$ only knows $\mathcal{G}_n^{\text{local}}$.

- Unknown information Tx $n$: $\tilde{\mathcal{G}}_n = \mathcal{G} \setminus \mathcal{G}_n^{\text{local}}$.

- Optimization: $\max_{\mathcal{G}_n^{\text{local}}} \mathbb{E}_{\tilde{\mathcal{G}}_n}[f(\mathcal{G})]$, where $f(\mathcal{G})$ is objective function.

- In what follows we let $\mathcal{G}_n^{\text{local}} = \{G_{n,i} \ \forall \ i\}$. 
4. Distributed Power Allocation (2/8)

- Link $n$ maximizes expected network capacity,

$$\overline{c}_n(P) \triangleq \log_2 \left( 1 + \frac{G_{n,n}P_n}{\sigma^2 + \sum_{i \neq n} G_{n,i}P_i} \right) + \mathbb{E}_{G_n} \left\{ \log_2 \left( 1 + \frac{G_{m,m}P_m}{\sigma^2 + \sum_{i \neq m} G_{m,i}P_i} \right) \right\} $$

(3)

- Distributed power allocation problem under statistical knowledge can thus be written as

$$\text{for link } n \quad P^*_n = \left[ \arg \max_{P \in \Omega} \overline{c}_n(P) \right]_n $$

(4)

- Each link calculates whole power allocation vector but keeps only its respective power.

- However, difficult to analytically calculate expectation term.

We thus focus on the simpler case of $N = 2$...
4. Distributed Power Allocation (3/8)

The network capacity for $N = 2$ as a function of the transmit powers as

$$
\overline{c}(P_1, P_2) = \log_2 \left( 1 + \frac{G_{1,1}P_1}{\sigma_1^2 + G_{1,2}P_2} \right) + \mathbb{E} \left\{ \log_2 \left( 1 + \frac{G_{2,2}P_2}{\sigma_2^2 + G_{2,1}P_1} \right) \right\},
$$

where the expectation is taken over the other link channel gains, namely $G_{2,2}$ and $G_{2,1}$.

The problem now becomes

$$
P_i^* = \left[ \arg \max_{(P_1, P_2) \in \Omega} \overline{c}_i(P_1, P_2) \right]_i \quad \forall \ i = 1, 2
$$

(6)
4. Distributed Power Allocation (4/8)

Distributed Algorithm for 2 Link Network Capacity Maximization

- Optimality of Binary Power Allocation motivates adoption for distributed case.

- Rewrite distributed optimization problem (6) as

\[
P_i^* = \left[ \arg \max_{(P_1, P_2) \in \Omega^B} C_i(P_1, P_2) \right] \quad \forall \ i = 1, 2
\]  

(7)

- Search space significantly reduced

- Each link only considers 3 possibilities of the power allocation feasible set i.e. \((0, P_{\text{max}}), (P_{\text{max}}, 0)\) or \((P_{\text{max}}, P_{\text{max}})\).
4. Distributed Power Allocation (5/8)

- By simple manipulation of power allocation possibilities, link $i$ will be active if either [3]

$$\text{SINR}_i \geq 2^{[\overline{R}(0,1)-\overline{R}(1,1)]} - 1 \quad (8)$$

or

$$\text{SNR}_i \geq 2^{\overline{R}(0,1)} - 1 \quad (9)$$

where $\overline{R}(0, 1)$ and $\overline{R}(1, 1)$ are the expected capacities of link 2 under the respective power allocations.

- Due to symmetry, conditions link 2 can be expressed similarly.

- Call this algorithm Fully Distributed Power Allocation (FDPA).

- $\overline{R}(0, 1)$ and $\overline{R}(1, 1)$ need to be calculated.
4. Distributed Power Allocation (6/8)

Figure 2: 2 cells of radius $R$ at a distance $D$ from each other. A user lies at a random point $(x,y)$ drawn from a uniform distribution over the cartesian plane. $f(r, \theta) = f(x, y) |J(x, y)^{-1}| = \frac{r}{\pi R^2}$
4. Distributed Power Allocation (7/8)

- With no interference [4]

\[
\overline{R}(0, 1) = \frac{2}{\ln(2)R^2} \left[ \frac{\xi R^2}{4} - \frac{1}{4} \xi_2 F_1(-\frac{2}{\xi}, 1; 1 - \frac{2}{\xi}; -\frac{R^{-\xi}}{n})R^2 \right. \\
\left. + \frac{1}{2} \ln\left(\frac{R^{-\xi} + n}{n}\right)R^2 \right]
\]

where \( _2F_1 \) denotes the hypergeometric function, \( \xi \) is pathloss exponent and \( n \) is noise power.

- With interference [4]

\[
\overline{R}(1, 1) = \int_0^{2\pi} \int_0^R \log_2 \left( 1 + \frac{r^{-\xi}}{n + (r^2 + D^2 - 2rD \cos \theta)^{-\xi/2}} \right) f(r, \theta) dr d\theta
\]

Complicated closed form. Can be easily calculated numerically.

- Practically, \( \overline{R}(0, 1) \) and \( \overline{R}(1, 1) \) can be calculated offline for any channel model.
4. Distributed Power Allocation (8/8)

Capacity Enhancement with 1-bit Exchange [3]

- FDPA completely distributed i.e. no real-time information exchange from other links.

- Interesting to explore how information exchange (1-bit) can enhance performance.

- **1-Bit Distributed Power Allocation (1-BDPA):**
  1. Link 1 performs the optimization (7) and sends a signal (1-bit) to the other link to indicate whether it is active or not.
  2. Link 2 then performs the optimization (7) to calculate $P_2$ under the knowledge of $P_1$. 
5. Distributed Power Allocation and Scheduling

- Multi-user diversity exploited through user scheduling.

- For a cell to be active and thus contribute capacity to the system either of the conditions (8) and (9) should be satisfied.

- Thus, the user with the maximum SNR or SINR should be scheduled.

- Suppose there are $U_n$ users in cell $n$

\[
\max_{u_n \in [1, U_n]} \text{SINR}_n(u_n) \geq 2[\bar{R}(U_n)(0,1) - \bar{R}(U_n)(1,1)] - 1 \tag{10}
\]

or

\[
\max_{u_n \in [1, U_n]} \text{SNR}_n(u_n) \geq 2\bar{R}(U_n)(0,1) - 1 \tag{11}
\]

where $\bar{R}(U_n)(0,1)$ and $\bar{R}(U_n)(1,1)$ are the expected capacities based on employing the max-SNR and max-SINR scheduling policies.
6. Performance Results (1/4)

- Monte-Carlo simulation of 2 hexagonal cells.
- BS-user links based on distance path loss model including log-normal shadowing (variance: $\sigma^2$) plus fast-fading $\sim \mathcal{CN}(0, 1)$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating Frequency</td>
<td>1800 MHz</td>
</tr>
<tr>
<td>Cell Radii</td>
<td>1000 meters</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>10dB</td>
</tr>
<tr>
<td>Transmit Antenna Gain</td>
<td>16dB</td>
</tr>
<tr>
<td>Receive Antenna Gain</td>
<td>6dB</td>
</tr>
<tr>
<td>$P_{MAX}$</td>
<td>1W</td>
</tr>
</tbody>
</table>
Figure 3: Comparison of average network capacity for the distributed algorithm and 1-bit exchange approach with Optimal Power Allocation (\(d = \text{inter-BTS distance}\)).
Figure 4: Percentage Error of FDPA and 1-BDPA. Allowing 1-bit signaling reduces the number of errors made and thus 1-BDPA outperforms FDPA.
6. Performance Results (4/4)

![Figure 5: Power Allocation and Scheduling with U = 1 and U = 5](image)

Figure 5: Power Allocation and Scheduling with U = 1 and U = 5
7. Conclusions

- Formulated the Distributed Power Allocation problem for mutually interfering links.
- Proposed an optimization framework for Distributed Power allocation
- Distributed algorithm shown to offer gain as opposed to no power control.
- 1-bit information exchange enables us to exploit most of the gain offered by power control.
- User scheduling incorporated into distributed power allocation algorithm
8. Future Work Directions

- Explore the effect of different amount/kinds of local/non-local information.
- What knowledge gives the best capacity?
- What happens when network scales?
- Any ideas welcome...
References


