A New Approach to Exploiting Limited Feedback in Multi-user MIMO Channels

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Outline

- Problem statement
- Approaches to feedback reduction in MU-MIMO
- The feedback splitting idea
- Results
The downlink MU-MIMO channel

K users (user k has Mk antennas)

base station (N antennas)
The downlink MU-MIMO channel: Notations

We have:

- Let’s assume a group of $K \leq N$ users are selected by the downlink scheduler out of $U$ users.
- Let the $K$ users receive simultaneously from the base station.
- User $k$ has 1 receive antenna.
- Base has $N$ transmit antennas and peak power constraint $P$.
- Base transmits signal vector $X = \sum_k X_k$
- $X_k$ is signal intended to user $k$, with covariance $Q_k$.
- Power constraint ensured by $\sum_k \text{Tr}(Q_k) \leq P$.
- Channel between user $k$ and base is matrix $H_k$, of size $M_k \times N$.
- White noise with variance 1.
The downlink MU-MIMO signal model

Received signal model at user $k$:

$$y_k = H_k X + n_k \text{ where } X = \sum_k X_k$$  \hspace{1cm} (1)

Using the global downlink channel matrix:

$$H = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_K \end{bmatrix}$$  \hspace{1cm} (2)

We have the global receive vector for all users:

$$y = [y_1^T, \ldots, y_K^T]^T = H \sum_k X_k + n$$  \hspace{1cm} (3)

where $X_k$ is the signal vector designed to reach user $k$. 
Role of CSIT in MU-MIMO

Role of CSIT in downlink evidenced by capacity scaling analysis. With CSIT, it is found that [Hassibi05], with $M_k = M \forall k$:

$$\lim_{U \to \infty} \frac{E(R_{DPC})}{N \log \log(MU)} = 1$$

(4)

where $R_{DPC}$ is the sum rate achieved by dirty paper coding (optimal scheme).

Interpretation:

- CIST allows for transmit beamforming.
- With large $U$, the base can select and spatially multiplex the $N$ best users out of $U$ with negligible interference loss.
- Mobile antenna provide extra $M$ diversity factor.
- Multiplexing gain is not limited by single-antenna mobiles!
Role of CSIT in MU-MIMO (II)

Without CSIT, it is found that:

$$\lim_{U \to \infty} \frac{E(R_{DPC})}{\min(M,N) \log \text{SNR}} = 1$$

(5)

Interpretations:
- In the absence of CSIT, multiuser diversity gain vanishes
- multiplexing gain is limited to $\min(M,N)$.
- multiplexing gain vanishes if mobiles are equipped with single antenna.
Feedback reduction techniques

A panorama:

2. Quantizing the leading channel eigen directions (rather than the channel)
3. Eliminating users from feedback pool using Selective Multiuser Diversity (SMUD) [Gesbert et al.]
4. Dimension reduction (includes concept of random beamforming!) [Viswanath et al, Sharif et al.]
5. Exploiting redundancy (temporal, frequency) to reduce feedback close to rate of innovation [Avidor et al., Kountouris et al.]
6. Exploiting spatial statistics [Kountouris et al., Hammarwall et al.]
7. Using hybrid direction/gain information [Jindal et al.]
8. Can we do more??
New approach: "feedback splitting"

Key ideas behind feedback splitting:

- MU-MIMO schemes can be decomposed into scheduling and beamforming stages.
- Both stages require CSIT.
- Scheduling requires CSIT from $U \gg N$ users, but can live with coarse estimates.
- Beamforming to selected users requires CSIT from $\leq N$ users, but CSIT must be precise.
- Therefore the feedback requirement is clearly not the same for both stages.

Why not split the feedback load over the two stages?
Feedback Splitting Process: Stage I

- All $U$ users feedback their CSI $\rightarrow$ Channel estimate $\hat{H}_1 (U \times N)$
Feedback Splitting Process: Stage I

- The base station uses $\hat{H}_1$ to select users (set $\hat{A} \in \{1, \ldots, U\}, |\hat{A}| = N$)

$$\hat{A} = \arg \max_A \text{Zero-forcing Sum rate} = \arg \min_A \text{tr}(\hat{H}_{1,A} \hat{H}_1^H)^{-1}$$

\[ (6) \]
Feedback Splitting Process: Stage II

- Users in $\hat{A}$ feedback refined CSI $\rightarrow$ Channel estimate $\hat{H}_{2,\hat{A}} (N \times N)$
Performance metric

- The base station designs the ZF precoding matrix $\hat{W}_{ZF}$:

$$\hat{W}_{ZF} = \frac{\hat{H}_{2,\hat{A}}^\dagger}{\sqrt{\text{tr}((\hat{H}_{2,\hat{A}}\hat{H}_{2,\hat{A}}^H)^{-1})}}$$ (7)

- The received signal:

$$y = \sqrt{P}H_{\hat{A}}\frac{\hat{H}_{2,\hat{A}}^\dagger}{\sqrt{\text{tr}((\hat{H}_{2,\hat{A}}\hat{H}_{2,\hat{A}}^H)^{-1})}}s + n$$ (8)

- Performance metric ($H_{\hat{A}} = \hat{H}_{2,\hat{A}} + E_{2,\hat{A}}$):

$$SR_{ZF-Q2} = \sum_{i=1}^{N} \log_2(1 + SINR_{\hat{A}_i})$$ (9)

where

$$SINR_{\hat{A}_i} = \frac{P}{\text{tr}((\hat{H}_{2,\hat{A}}\hat{H}_{2,\hat{A}}^H)^{-1}) + P\|(E_{2,\hat{A}}\hat{H}_{2,\hat{A}}^\dagger)^i\|^2}$$ (10)
Feedback split model

Let $0 \leq \alpha \leq 1$ be the split factor:

- Let $B_{\text{total}}$ denote the total number of bits available for feedback
- $B_1 = \alpha B_{\text{total}}$ bits dedicated to the scheduling
- $B_2 = (1 - \alpha)B_{\text{total}}$ bits dedicated to beamforming matrix design
- A user selected in second phase refines his initial $B_1/U$-bit feedback with $B_2/N$-bit feedback

- Achievable distortion at each stage:
  \begin{align*}
  \sigma_{e_1}^2 &= 2^{-b_1/N} = 2^{-\alpha B_{\text{total}}/(U \times N)} \quad \text{(11)} \\
  \sigma_{e_2}^2 &= 2^{-(b_1+b_2)/N} = 2^{-\frac{B_{\text{total}}}{N} \left( \frac{\alpha}{U} + \frac{1-\alpha}{N} \right)} \quad \text{(12)}
  \end{align*}
**Extreme cases**

1. $\alpha = 0$: No user selection, $\sigma_e^2 = 2 \frac{B_{\text{total}}}{N^2}$

   Using statistics of the minimum eigenvalue of a Wishart matrix, we bound average sum rate

   $$NeE_1 \left( \frac{N^2}{Pc_0} \right) < \overline{SR}_{ZF-Q,0} < N \left[ eE_1 \left( \frac{N}{Pc_0} \right) + \gamma_{EM} \right]$$

   (13)

2. $\alpha = 1$: No refinement, $\sigma_e^2 = 2 \frac{B_{\text{total}}}{N \times U}$

   Average sum rate bounded as follows

   $$N \sum_{k=1}^{\left\lfloor \frac{U}{N_t} \right\rfloor} (-1)^{k+1} \binom{N^2}{k} eE_1 \left( \frac{kN^2}{Pc_0} \right) < \overline{SR}_{ZF-Q,1} < N \left[ \gamma_{EM} + \log \left( 1 + P \frac{c_0}{N} H_{\left( \frac{U}{N_t} \right)} \right) \right]$$

   (14)

   $$\Rightarrow c_0 = \frac{1-\sigma_e^2}{1+P\sigma_e^2}$$ quantifies power loss with respect to perfect channel knowledge.
Extreme cases Illustrated

Sum rate for $N = 2$ base antennas, $U = 20$ single-antenna users, $B_{\text{total}} = 80$ bits:
Feedback split optimization

The optimal $\alpha$ is that which maximizes the average sum rate

$$\alpha_{opt} = \arg \max_{\alpha} SR_{ZF-Q2}$$  \hspace{1cm} (15)

$SR_{ZF-Q2}$ bounded as follows:

$$N \mathbb{E}_{\lambda_2, \min} \log \left( 1 + \frac{P \lambda_{2, \min}}{N(1 + P \sigma_{e_2}^2)} \right) < SR_{ZF-Q2} < N \left[ \gamma_{EM} + \mathbb{E}_{\lambda_2, \min} \log \left( 1 + \frac{P \lambda_{2, \min}}{1 + P \sigma_{e_2}^2} \right) \right]$$  \hspace{1cm} (16)

But the distribution of $\lambda_{2, \min}$ is difficult to obtain $\Rightarrow$ Find a heuristic
Feedback split optimization (cont.)

Intuition:

- When full CSIT is available, sum rate should approximately scale as $N \log \log U$
- In the $\alpha \in \{0, 1\}$ cases, $c_0$ came out as a power loss factor with respect to the perfect CSIT case.

$\Rightarrow$ Introduce a similar power loss factor for intermediate cases. This would be a combination of:

- $PL_1$: Power Loss in MUD due to first stage quantization error
- $PL_2$: Power Loss due to remaining error in CSIT after refinement

Lemma: $\alpha_{opt}$ is approximated by the following solution:

$$\alpha_{opt} \approx \arg \max_{\alpha \in [0,1]} PL$$

where

$$PL = PL_1 + PL_2 = \frac{1 - \sigma^2_{e_1}}{1 + P\sigma^2_{e_2}} + \frac{\sigma^2_{e_1} - \sigma^2_{e_2}}{\log U(1 + P\sigma^2_{e_2})}$$

(17)
Sum rate performance

Sum rate for $N = 2$ base antennas, $U = 30$ single-antenna users, $B_{\text{total}} = 120$ bits

![Graph showing the relationship between SNR and sum rate for different scenarios.](chart)
Sum rate performance (Actual quantization)

Sum rate for $N = 2$ base antennas, $U = 30$ single-antenna users, $B_{total} = 120$ bits
Conclusion and Future Work

What we did:

- We introduced a feedback rate splitting approach to optimize performance in a multi-user MIMO system.
- We found a heuristic choice of the splitting factor for the Rayleigh fading scenario.

Future Work:

- Generalize to more complex channel models (adaptive approach?)
- Generalize to more complex systems (cooperating base stations, MIMO-OFDM?)
Thank you for your attention!