TECHNISCHE UNIVERSITÄT DRESDEN

'Friedrich List' Faculty of Transport and Traffic Sciences Institute of Traffic Communications Engineering

Pdf based Blind Source Separation for Digital Communication Systems

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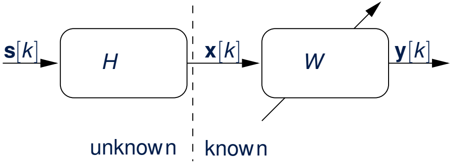
Outline

Mixing and demixing



$$\mathbf{x}[k] = H\mathbf{s}[k] \tag{1}$$
$$\mathbf{y}[k] = W\mathbf{x}[k] \tag{2}$$
$$= WH\mathbf{s}[k] \tag{3}$$

Sources are separated if:

$$WH = \mathbf{PSI} \longleftrightarrow W = \mathbf{PS}H^{-1} \tag{4}$$

Assuption: Sources are statistically independent

Stochastic independency

Definition:

$$p(y_1 y_2) = p_1(y_1) p_2(y_2) \tag{5}$$

$$E\{h_1(y_1) h_2(y_2)\} = E\{h_1(y_1)\} E\{h_2(y_2)\}. \tag{6}$$

Weak form is uncorrelatedness:

$$E\{y_1 y_2\} - E\{y_1\} E\{y_2\} = 0 \tag{7}$$

BUT uncorrelatedness does not imply independency!
Example: 4 discrete valued pairs (y_1, y_2) with $p = 1/4$: $(0, 1), (0, -1), (1, 0), (-1, 0)$

$$E\{y_1^2 y_2^2\} = 0 \neq 1/4 = E\{y_1^2\} E\{y_2^2\} \tag{8}$$

Iterative Inversion Approach

Optimization criterion: Kullback-Leibler-Divergence

$$\Psi_{MI}(W) = D\left(p_{\mathbf{Y}}(\mathbf{y}) \middle\| \prod_{i=1}^{M} p_{Y_i}(y_i)\right) \tag{9}$$

Used Algorithm:

Cruces, S.; Cichocki, A.; Ribas, L.C.:An Iterative Inversion Approach to Blind Source Separation; IEEE Trans. on Neural Networks, 2000, 11, 1423-1437

Cruces' iterative scheme:

$$W^{(n+1)} = W^{(n)} - \mu^{(n)}\left(R_{f(\mathbf{y})g(\mathbf{y})}(W^{(n)}) - \mathbf{I}\right)W^{(n)} \tag{10}$$

$$\mu^{(n)} = \frac{\eta}{1 + \eta\|R_{fg}^{(n)}\|}. \tag{11}$$

Exact Nonlinearities

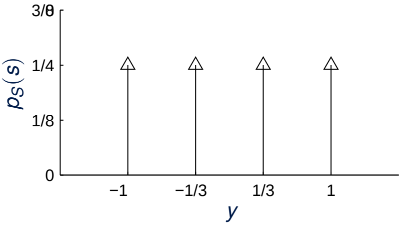
Correlation function R_{fg} is an approximation of $R_{\psi\mathbf{y}}$ with

$$\psi(y) = \begin{pmatrix} -\frac{d\ln(p_{\hat{S}_1}(y_1))}{dy_1} & \dots & -\frac{d\ln(p_{\hat{S}_N}(y_N))}{dy_N} \end{pmatrix}^T. \quad (12)$$

Reason: Probability density function $p_{\hat{S}_1}$ is normaly unknown

BUT in digital communication systems pdf is given by used modulation

Example: Pdf of 4-PAM



Discrete pdf:

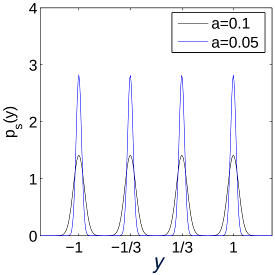
$$p_S(y) = \frac{1}{4} \cdot \sum_{i=1}^{4} \delta\left(y - \frac{2}{3}i + \frac{5}{3}\right) \quad (13)$$

Nonlinearity ψ:

$$\psi(y_i) = -\frac{d \ln p_S(y_i)}{dy_i} = -\frac{p_S'(y_i)}{p_S(y_i)} = \text{n.d.} \quad (14)$$

Denominator of ψ is almost everywhere ZERO
$\hookrightarrow$ Exact pdf can not be used!

Approximated pdf of 4-PAM signals I



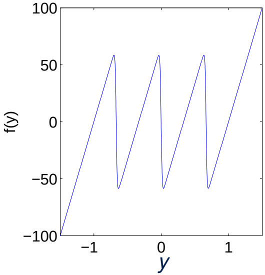
Approximation by **Gaussian distribution**:

$$p_S(y) = \frac{1}{a\sqrt{\pi}} \sum_{i=1}^{4} \exp\left(-\frac{(y - 2/3 \cdot i + 5/3)^2}{a^2}\right) \quad (15)$$

Resulting Nonlinearity ψ:

$$\psi(y) = -\frac{p_S'(y)}{p_S(y)} = \frac{\sum_{i=1}^{4} \frac{2(y-2/3\cdot i+5/3)}{a^2} \exp\left(-\frac{(y-2/3\cdot i+5/3)^2}{a^2}\right)}{\sum_{i=1}^{4} \exp\left(-\frac{(y-2/3\cdot i+5/3)^2}{a^2}\right)} \quad (16)$$

Approximated pdf of 4-PAM signals I



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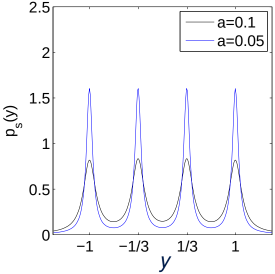
Approximated pdf of 4-PAM signals II



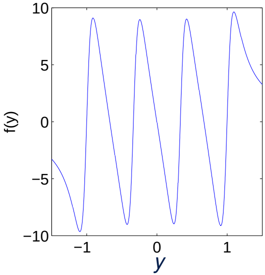
Approximation by **Cauchy distribution**:

$$p_S(y) = \frac{1}{\pi}\sum_{i=1}^{4}\frac{1}{a^2+(y-2/3\cdot i+5/3)^2} \quad (17)$$

Resulting Nonlinearity ψ:

$$\psi(y) = \frac{p'_S(y)}{p_S(y)} = \frac{\sum\limits_{i=1}^{4}\frac{2(y-2/3\cdot i+5/3)}{(a^2+(y-2/3\cdot i+5/3)^2)^2}}{\sum\limits_{i=1}^{4}\frac{1}{a^2+(y-2/3\cdot i+5/3)^2}} \quad (18)$$

Approximated pdf of 4-PAM signals II



Approximation by **Cauchy distribution**:

$$p_S(y) = \frac{1}{\pi}\sum_{i=1}^{4}\frac{1}{a^2 + (y - 2/3 \cdot i + 5/3)^2} \quad (17)$$

Resulting Nonlinearity ψ:

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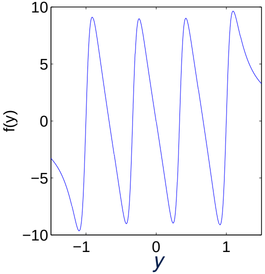
Approximated pdf of 4-PAM signals III

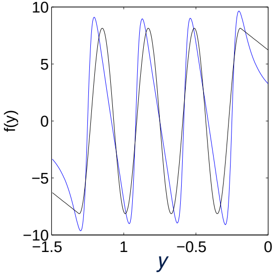


Approximation of Cauchy-NL by sine function

Nonlinearity ψ:

$$\psi(y) = \begin{cases} -\frac{0{,}5y+1{,}373}{a} & \text{for } y < -1,12 \\ -\frac{0{,}815}{a}\sin(3,136\pi y) & \text{for } -1,12 \leq y \leq 1,12 \\ -\frac{0{,}5y-1{,}373}{a} & \text{for } y > 1,12. \end{cases} \tag{19}$$

Approximated pdf of 4-PAM signals III



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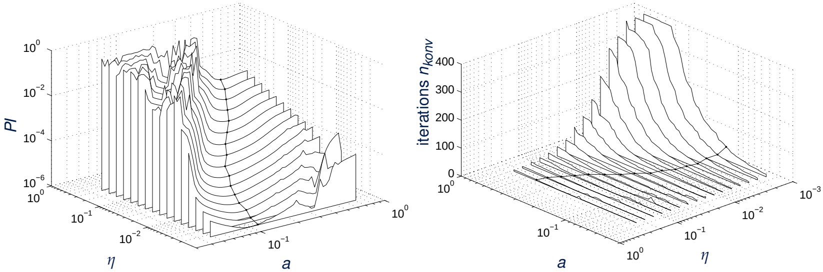
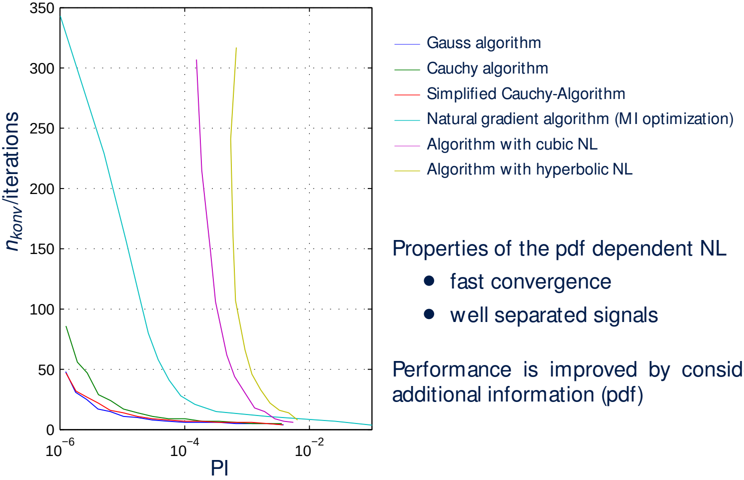
Optimisation of the slope *a*

Example: Gaussian disribution



Equal results for Cauchy and simplified Cauchy NL

Separation capability

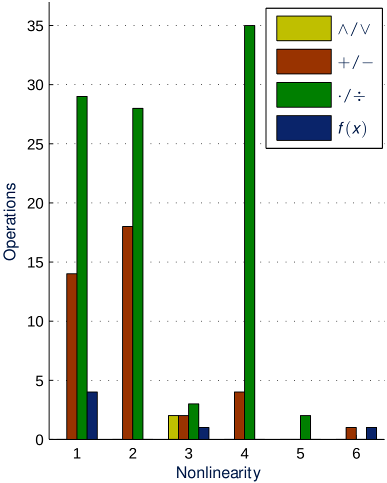


Properties of the pdf dependent NL

- fast convergence
- well separated signals

Performance is improved by considering additional information (pdf)

Calculation effort

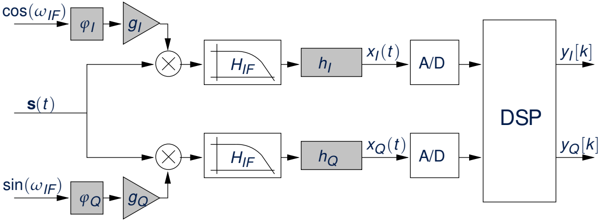


1 - Gauss algorithm
2 - Cauchy algorithm
3 - Simplified Cauchy-Algorithm
4 - Natural gradient algorithm (MI optimization)
5 - Algorithm with cubic NL
6 - Algorithm with hyperbolic NL

Properties of the algorithms:

- high calculation effort
- simplification leads to reduction of calculation effort

Model of IQ-Imbalance

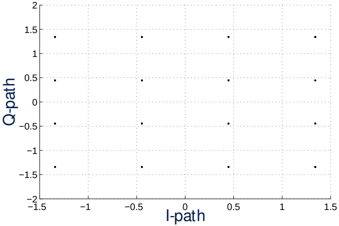


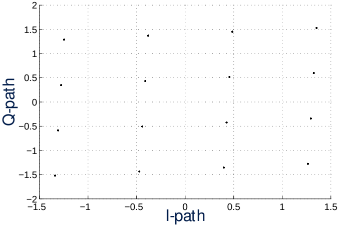
- Mixture of I- and Q-path by error of phase of the LO
- Mixing matrix for ideal filter functions:

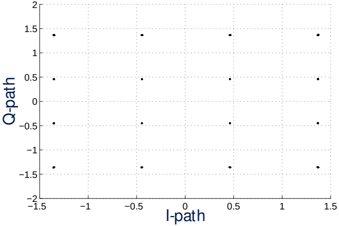
$$H = \begin{pmatrix} g_I \cos(\varphi_I) & g_I \sin(\varphi_I) \\ g_Q \sin(\varphi_Q) & g_Q \cos(\varphi_Q) \end{pmatrix} \tag{20}$$

Separation results

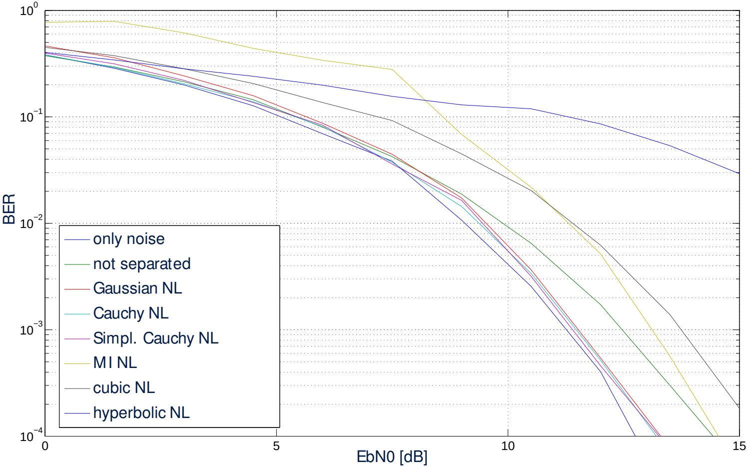
Phase and amplitude error: $g_I = 0.97$ $\varphi_I = 2^\circ$ $g_Q = 1.05$ $\varphi_Q = 5^\circ$





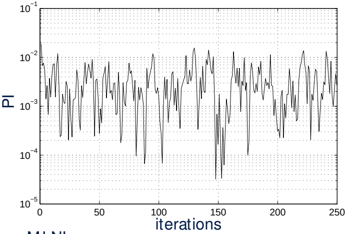


Bit Error Rate

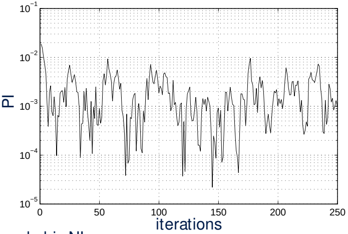


Convergence

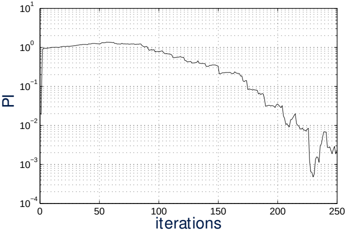
Cauchy NL



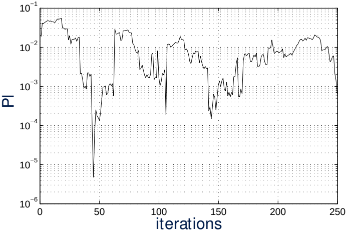
Simplified Cauchy NL



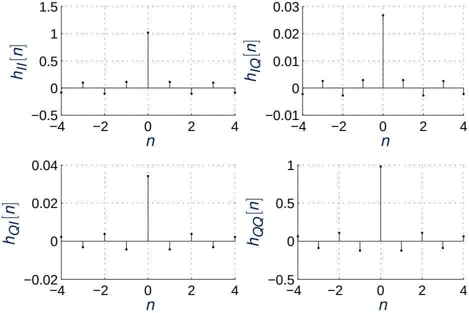
MI NL



kubic NL



Convolutive Mixture



- Temporal mixture caused by filter inaccuracies
- Temporal separation is handled equal to spatial separation

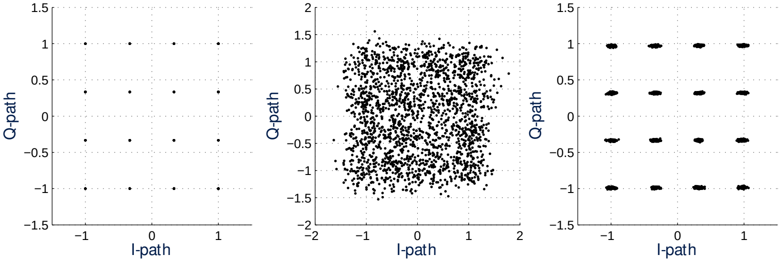
Problem: Expansion of temporal dependency (inversion FIR $\Rightarrow$ IIR)

Solution: Reduction to significant filter length L_w

Separation results

Phase and amplitude error:

$g_I = 1,02 \qquad \varphi_I = 2^\circ \qquad g_Q = 0,98 \qquad \varphi_Q = 1,5^\circ$



- Temporal inaccuracies introduce strong interference
- Deconvolution with simplified Cauchy NL reduces these interference

Conclusions

Results

- Blind cancelation of IQ-Imbalance
- Increasing performance by pdf dependent nonlinearities
- Reduction of calculation effort by simplification
- Extension to convolutive mixture

Perspectives

- Performance analyse of deconvolution (BER)
- Comparision to non blind algorithms
- Noise reduction by assuming additional source

Thank you for your attention!