Spectral Efficiency of Cognitive Radio Systems

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RF Spectrum occupation from 1.39 to 5.923 GHz

Figure: Mid-Band.

Figure: High-Band.
Why Cognitive Radio?

- In some locations and/or at some times of the day, 70 percent of the allocated spectrum may be sitting idle.
- The FCC has recently recommended that significantly greater spectral efficiency could be realized by deploying wireless devices that can coexist with the licensed users.

Cognitive Radio Overview

- A new class of radios was defined by the term *cognitive radio*
- Several definitions (and variations) of Cognitive Radio exist:
  1. Mitola\(^2\) - "Cognitive radio signifies a radio that employs model based reasoning to achieve a specified level of competence in radio related domains".
  2. FCC - "A cognitive radio (CR) is a radio that can change its transmitter parameters based on interaction with the environment in which it operates".
- Such devices must be able to:
  1. *sense* the spectral environment over a wide bandwidth,
  2. *detect* the presence/absence of legacy users (primary users),
  3. *adapt* the parameters of their communication scheme only if the communication does not interfere with primary users.

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1. The System Model
   - The Cognitive Radio Scenario,
   - Sensing.

2. Performance Analysis
   - spectral efficiency analysis,
   - asymptotic performance.

3. Simulation results
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Part I
System model
The System model

Figure: Two-user cognitive radio TDD-uplink system in a wideband/multiband context.
The cognitive radio scenario

- We investigate the idea of using cognitive radio to reuse locally unused spectrum to increase the total system capacity,
- We consider a multiband/wideband system in which users wish to communicate to the base station, subject to mutual interference,
- We assume that each user knows only his channel and the unused spectrum through adequate sensing,
- We impose the constraint that users successively transmit over available bands through proper water filling:

\[ P_i = \left( \frac{1}{\gamma_0} - \frac{N_0}{|h_i|^2} \right)^+ \]

- The cognitive user will listen to the channel and, if sensed idle, will transmit during the voids.
The cognitive radio scenario

Sensing

Figure: One primary user and two cognitive users in a system with $N = 8$ sub-bands.
The received signal at user $i$ can therefore be written as:

$$y_i^i(k) = \begin{cases} 
  c_{i-1,i}^i(k)\sqrt{P_{i-1}^i S_{i-1}^i(k)} + n_{i-1}^i(k), & \text{if } P_{i-1}^i \neq 0 \\
  n_{i-1}^i(k), & \text{otherwise}
\end{cases}$$

Possible ways to detect the presence/absence of a primary user:

1. power detection,
2. radar detection (Neyman Pearson test)
Part II

Performance Analysis
The capacity per band of user $l$ given a number of sub-bands $N$ is:

$$C_{l,N} = \frac{1}{\text{card}(\Omega_l)} \sum_{i \in \Omega_l} \log_2 \left( 1 + \frac{P_i^l \mid h_i^l \mid^2}{\sigma^2} \right)$$  \hspace{1cm} (1)$$

Where $\Omega_l$ represents the set of the remaining idle sub-bands sensed by user $l$.

The optimal power allocation which maximizes the transmission rate of user $l$ is:

$$P_i^l = \left( \frac{1}{\gamma_0} - \frac{N_0}{\mid h_i^l \mid^2} \right)^+$$
The spectral efficiency per band of user $l$ is given by:

$$\Phi_{l,N} = \frac{1}{N} \sum_{i \in \Omega_l} \log_2 \left( 1 + \frac{P_i |h_i|^2}{\sigma^2} \right)$$ (2)

By multiplying and dividing (2) by $\text{card}(\Omega_l)$, we obtain:

$$\Phi_{l,N} = \begin{cases} 
\Delta_{1,N} \cdot C_{1,N}, & \text{if } l = 1 \\
\Delta_{l,N} \cdot C_{l-1,N}, & \text{for } l \in [2, L] 
\end{cases}$$ (3)

We define $\Delta_{l,N}$ as the **band factor gain** of user $l$ for $N$ sub-bands, namely:

$$\Delta_{l,N} \triangleq \frac{\text{card}(\Omega_l)}{N}, \text{ for } l \in [1, L]$$ (4)
The sum spectral efficiency of a system with $N$ sub-bands per user is given by:

$$\Phi_{sum,N} = \sum_{l=1}^{L} \Phi_{l,N}$$
Asymptotic Performance

- Devices are assumed to operate in a wide-band context (i.e. $N \to \infty$).

- The instantaneous capacity of user $l$ for a finite number of sub-bands in (1) becomes:

$$C_{l,\infty} = \int_{0}^{\infty} \log_2 \left( 1 + \frac{P_l(t) \cdot t}{\sigma^2} \right) \cdot e^{-t} dt, \quad \text{for} \quad l \in [1, L]$$

- Similarly to our approach for finite $N$, we define the band factor gain $\Delta$ as the fraction of the band sensed idle from user $l$ to user $l + 1$ over the total bandwidth $W$:

$$\Delta \triangleq \frac{\Delta f}{W}, \quad \text{for} \quad l \in [2, L]$$
The asymptotic spectral efficiency of user $l$ is given by:

$$\Phi_{l,\infty} = \begin{cases} 
C_{1,\infty}, & \text{if } l = 1 \\
\Delta_{\infty} \cdot C_{l-1,\infty}, & \text{for } l \in [2, L] 
\end{cases}$$  

(6)

Where $\Delta_{\infty}$ is given by:

$$\Delta_{\infty} = 1 - \exp(-\gamma_0 \sigma^2)$$

The overall asymptotic sum spectral efficiency for a system with $L$ users is therefore:

$$\Phi_{\text{sum,}\infty} = \frac{1 - \Delta_{\infty}^L}{1 - \Delta_{\infty}} \cdot C_{1,\infty} \geq 1$$  

(7)
Part III

Simulation Results
Figure: Comparison between theoretical expression of the sum spectral efficiency and simulated one for $L = 5$ and $N=32$. 
Figure: Sum spectral efficiency gains of the system with 5 users.
Simulation Results

Figure: The maximum number of users for different number of sub-bands, $N$. 
Simulation Results

Thank You for attention!

Any Questions?