Exploiting Multiuser Diversity in MIMO Broadcast Channels with Limited Feedback

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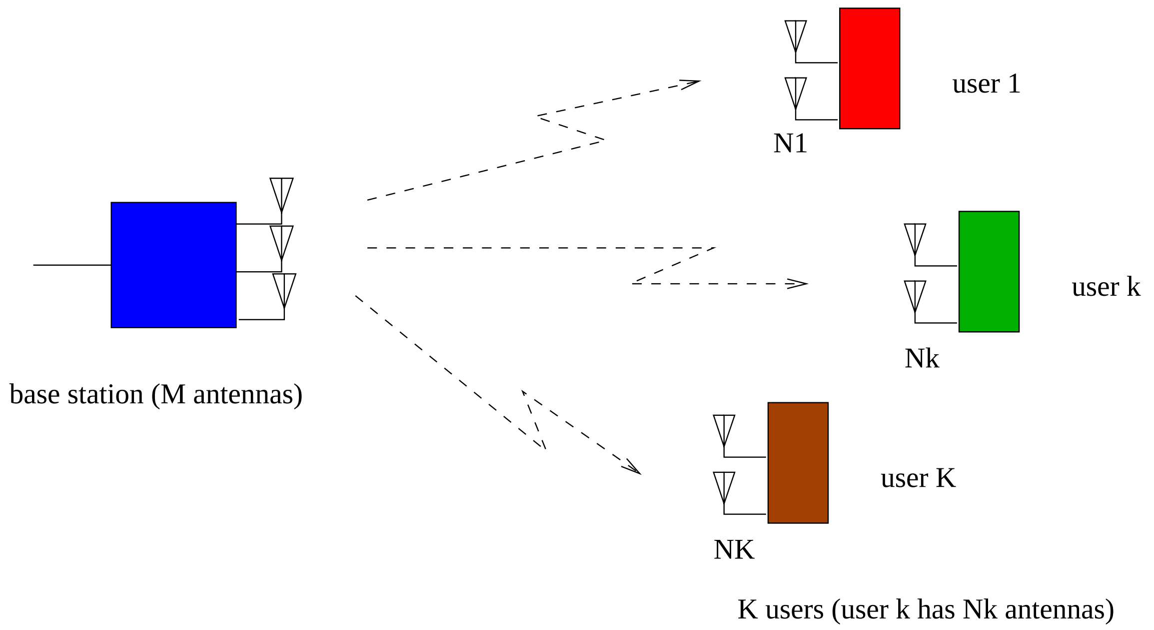
Context: downlink MU MIMO channels



- BS or AP equipped with M transmit antennas, and K active terminals. Each user k has N_k antennas.
- Active users: set of users simultaneously downloading data during one given scheduling window (subset of connected users).
- Users might have different QoS constraints/demands (data rate, delay).

What does MU Information Theory tell us?

- In MIMO broadcast channels (BC), the capacity can be boosted by means of SDMA (exploiting the spatial multiplexing capability of Tx antennas and transmit to multiple users simultaneously).
- Dirty Paper Coding (DPC) is capacity-achieving, but
  - *difficult* to implement in practice.
  - requires perfect Channel State Information at the Tx/Rx (CSIT/R).

  $\longrightarrow$ *practical* & *low-complexity* downlink transmission techniques are of interest!
- Downlink linear precoding, although suboptimal, is shown to achieve a large portion of DPC capacity, if combined with efficient *user selection* $\longrightarrow$ reduced complexity PHY layer.

The cardinal role of CSIT

- The capacity gain of multiuser MIMO systems seems to remain highly dependent on the channel state information available at the transmitter (CSIT).
  - if a BS with M transmit antennas communicating with K single-antenna mobiles has perfect CSIT, the multiplexing gain = $\min(M, K)$.
  - if there is a complete lack of CSIT, the multiplexing gain collapses to one!!
- However, full/perfect CSIT is unrealistic ...

Can we benefit from MUDiv and spatial multiplexing gains under limited feedback rate constraint ?

Effect of Limited Feedback in MIMO BC

In point-to-point systems:

- even a few feedback bits are sufficient for near-optimal performance.
- the level of CSIT only affects the SNR-offset; it does not affect the slope of the capacity vs. SNR curve (multiplexing gain).

However, in MIMO Broadcast Channels ...

- The level of CSIT critically affects the multiplexing gain of the MIMO downlink channel.
- It was recently shown that *in order to achieve full multiplexing gain* the number of FB bits/user must increase approximately linearly with the number of Tx antennas and the average SNR (in dB).

Feedback link design challenges

- The need for channel information is a limiting factor in multiuser MIMO systems.
- Such information must be fed back at the Tx, but the amount of feedback should be kept *minimal*.
- What kind and portion of partial CSIT is necessary and sufficient ?!
  - to identify "good" users to be scheduled.
  - to design efficient beamforming techniques (amplify the received signal and reduce the multiuser interference).
  - to achieve near optimal capacity growth.

System Model

- Multiple antenna broadcast channel with M transmit antennas and K single-antenna receivers (with $K \geq M$).
- At the k-th mobile the received signal is given by

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k, \qquad k = 1, \ldots, K \tag{1}$$

Assumptions:

- We consider an i.i.d. block Rayleigh flat fading channel, where each user has perfect knowledge of its own channel $\mathbf{h}_k$ (CSIR).
- The transmitted signal is subject to an average transmit power constraint $\mathbb{E}\{\|\mathbf{x}\|^2\} = P$.
- Homogeneous network where all users have the same average signal-to-noise ratio (SNR).

CSIT feedback model

- Consider now that the feedback channel is divided into 2 types of information:
  - Channel Direction Information (CDI)
  - Channel Quality Information (CQI)

Motivation

- CDI can be used to achieve full multiplexing gain (with proper feedback load scaling) when $K \leq M$.
- When $K > M$, CDI cannot exploit multiuser diversity gain $\rightarrow$ *an efficient CQI metric is required*.

However, the feedback rate is finite $\rightarrow$ loss of degrees of freedom (MUDiv and MUX gain).

What type of CQI metric allows us to still benefit from the gains promised by MU MIMO with limited CSIT ?

CDI Finite Rate Feedback Model

- Quantization codebook $\mathcal{V}_k = \{\mathbf{v}_{k1}, \mathbf{v}_{k2}, \ldots, \mathbf{v}_{kN}\}$ containing $N = 2^B$ unit norm vectors (known to both the k-th Rx and Tx).
- At each timeslot, the k-th mobile, based on its current channel realization $\mathbf{h}_k$, determines the vector that maximizes

$$\hat{\mathbf{h}}_k = \mathbf{v}_{kn} = \arg \max_{\mathbf{v}_{ki} \in \mathcal{V}_k} |\bar{\mathbf{h}}_k^H \mathbf{v}_{ki}|^2 = \arg \max_{\mathbf{v}_{ki} \in \mathcal{V}_k} \cos^2(\angle(\bar{\mathbf{h}}_k, \mathbf{v}_{ki})) \tag{2}$$

where $\bar{\mathbf{h}}_k = \mathbf{h}_k / \|\mathbf{h}_k\|$.

- Each user sends the corresponding quantization index n back to the transmitter through an error-free, and zero-delay feedback channel using $B = \lceil \log_2 N \rceil$ bits.

Zero Forcing (ZF) Beamforming (1/2)

- Let $\mathcal{G} = \{1, \cdots, K\}$ be the set of indices of all K users. Let $\mathcal{S} \in \mathcal{G}$, be one such group of $|\mathcal{S}| = \mathcal{M} \leq M$ users selected for transmission at a given time slot.
- The signal model is given by

$$\mathbf{y}(\mathcal{S}) = \mathbf{H}(\mathcal{S})\mathbf{W}(\mathcal{S})\mathbf{P}\mathbf{s}(\mathcal{S}) + \mathbf{n} \tag{3}$$

where $\mathbf{H}(\mathcal{S})$, $\mathbf{W}(\mathcal{S})$, $\mathbf{s}(\mathcal{S})$, $\mathbf{y}(\mathcal{S})$ are the concatenated channel vectors, beamforming vectors, uncorrelated data symbols and received signals respectively. $\mathbf{P}$ is a diagonal power allocation matrix with equal entries.

- We use ZF beamforming on the quantized channel directions as a multiuser transmission strategy:

$$\mathbf{W}(\mathcal{S}) = \hat{\mathbf{H}}(\mathcal{S})^H(\hat{\mathbf{H}}(\mathcal{S})\hat{\mathbf{H}}(\mathcal{S})^H)^{-1}\mathbf{\Lambda} \tag{4}$$

where $\hat{\mathbf{H}}(\mathcal{S}) = \left[\hat{\mathbf{h}}_{k_1}\ \hat{\mathbf{h}}_{k_2} \ldots \hat{\mathbf{h}}_{k_\mathcal{M}}\right]^H$ with $\{k_i\}_{i=1}^{\mathcal{M}} \in \mathcal{S}$.

Zero Forcing (ZF) Beamforming (2/2)

- The SINR at the k-th receiver is

$$SINR_k = \frac{P_k |\mathbf{h}_k^H \mathbf{w}_k|^2}{\sum\limits_{j \in \mathcal{S}-\{k\}} P_j |\mathbf{h}_k^H \mathbf{w}_j|^2 + 1} \tag{5}$$

  where $\sum_{i \in \mathcal{S}} P_i = P$ in order to satisfy the power constraint on the transmitted signal.
  For simplicity, equal power allocation is considered, i.e. $P_i = \frac{P}{\mathcal{M}}, \forall i \in \mathcal{S}$.

- The expected sum rate, assuming Gaussian inputs, is equal to

$$\mathcal{R} = \mathbb{E}\left\{ \sum_{k \in \mathcal{S}} \log\left(1 + SINR_k\right) \right\} \tag{6}$$

CQI Finite Rate Model

Efficient Feedback Metrics exploiting Multiuser Diversity

Metric I: Upper Bound on SINR

Bounding the expected interference

- Let $\phi_k = \angle(\hat{\mathbf{h}}_k, \bar{\mathbf{h}}_k)$ be the angle between the normalized channel vector and the quantized channel direction.
- We consider that each user provides information on its effective channel (SINR) by feeding back the following scalar metric

$$\xi_k^{UB} = \frac{P\left\|\mathbf{h}_k\right\|^2 \cos^2\phi_k}{P\left\|\mathbf{h}_k\right\|^2 \sin^2\phi_k + M} \tag{7}$$

- This metric encapsulates information on the channel gain as well as the CDI quantization error ($\sin^2\phi_k$).
- It can be interpreted as an upper bound (UB) on the received SINR_k (equal power allocation) $\longrightarrow$ an estimate of the multiuser interference <u>*at the mobile side*</u> (without cooperation).

Metric II: Lower Bound on SINR (1/3)

Bounding the actual multiuser interference

- Let $\cos\theta_k = \left|\bar{\mathbf{h}}_k^H \mathbf{w}_k\right|$, $\mathbf{\Psi}_k(\mathcal{S}) = \sum_{j\in S, j\neq k} \mathbf{w}_j \mathbf{w}_j^H$, the operator $\lambda_{max}\{\cdot\}$, which returns the largest eigenvalue, and $\mathbf{U}_k \in \mathbb{C}^{M\times(M-1)}$ an orthonormal basis spanning the null space of $\mathbf{w}_k$.
- *Theorem 1: Given a set $\mathcal{S}$ of unit-norm BF vectors, an upper bound on the interference over the normalized channel $\bar{I}_k(\mathcal{S}) = \sum_{j\in\mathcal{S}, j\neq k} \left|\bar{\mathbf{h}}_k^H \mathbf{w}_j\right|^2$ is given by*

$$\bar{I}_k(\mathcal{S}) \leq \cos^2\theta_k \alpha_k(\mathcal{S}) + \sin^2\theta_k \beta_k(\mathcal{S}) + 2\sin\theta_k\cos\theta_k\gamma_k(\mathcal{S}) \quad (8)$$

where

$$\begin{cases} \alpha_k(\mathcal{S}) = \mathbf{w}_k^H \mathbf{\Psi}_k(\mathcal{S})\mathbf{w}_k \\ \beta_k(\mathcal{S}) = \lambda_{max}\left\{\mathbf{U}_k^H \mathbf{\Psi}_k(\mathcal{S})\mathbf{U}_k\right\} \\ \gamma_k(\mathcal{S}) = \left\|\mathbf{U}_k^H \mathbf{\Psi}_k(\mathcal{S})\mathbf{w}_k\right\| \end{cases} \quad (9)$$

Metric II: Lower Bound on SINR (2/3)

- We now impose an ϵ-orthogonality constraint between $\hat{\mathbf{h}}_k$ and $\epsilon_{ZF} = \max_{i,j\in\mathcal{S}} \left|\mathbf{w}_i^H \mathbf{w}_j\right|$ (worst-case orthogonality under ZFBF).
- *Lemma 1: The orthogonality of the set of M normalized ZFBF vectors (ϵ_{ZF}) and alignment with the normalized channel ($\cos\theta_k$) are bounded as:*

$$\epsilon_{ZF} \leq \vartheta \text{ and } \cos\theta_k \geq \frac{\left|\cos\phi_k - \sqrt{\vartheta}\right|}{1+\vartheta} \text{ with } \vartheta = \frac{\epsilon}{1-(M-1)\epsilon}$$

- *Theorem 2: Given an ϵ-orthogonal set $\mathcal{S}$, with $|\mathcal{S}| = M$, a system that performs ZFBF can guarantee the following SINR for the k-th user*

$$SINR_k^{ZF} \geq \frac{P\left\|\mathbf{h}_k\right\|^2 \cos^2\theta_k}{P\left\|\mathbf{h}_k\right\|^2 \overline{I}_{UB_k} + M} \tag{10}$$

where $\overline{I}_{UB_k} = \cos^2\theta_k \ (M-1)\epsilon_{ZF}^2 + \sin^2\theta_k \ [1+(M-2)\epsilon_{ZF}]$
$+2\sin\theta_k\cos\theta_k\,(M-1)\epsilon_{ZF}$

with $\epsilon_{ZF} = \vartheta$ *and* $\cos\theta_k = \frac{\left|\cos\phi_k - \sqrt{\vartheta}\right|}{1+\vartheta}$ *where* $\vartheta = \frac{\epsilon}{1-(M-1)\epsilon}$

Metric II: Lower Bound on SINR (3/3)

- Motivated by the above results, we propose that each user provides information on its SINR lower bound (LB) and reports the following scalar metric

$$\xi_k^{LB} = \frac{\frac{P}{(1+\vartheta)^2} \left\| \mathbf{h}_k \right\|^2 (\cos\phi_k - \sqrt{\vartheta})^2}{P \left\| \mathbf{h}_k \right\|^2 \overline{I}_{UB_k} + M} \tag{11}$$

Metric III: Decoupling the CQI into two scalars (1/2)

- Main drawback of the above metrics:
  they estimate the SINR by assuming $|\mathcal{S}| = \mathcal{M} = M$.

- However, in extreme regimes (high SNR, low number of users), it is often better to transmit to $\mathcal{M} < M$ users.

- We decompose the CQI feedback by letting each user feed back the following two scalar values:
  - the alignment $\cos^2 \phi_k$
  - the channel norm, $\|\mathbf{h}_k\|^2$

Metric III: Decoupling the CQI into two scalars (2/2)

Now, everything can be done at base station ...

- The transmitter selects the user based on the following lower bound on the received SINR

$$\xi_k^{LBd} = \frac{P \left\| \mathbf{h}_k \right\|^2 \rho_k^2}{P \left\| \mathbf{h}_k \right\|^2 \bar{I}_{UBd_k} + \mathcal{M}} \tag{12}$$

  where $\bar{I}_{UBd_k} = \rho_k^2 \alpha_k(\mathcal{S}) + \left(1 - \rho_k^2\right) \beta_k(\mathcal{S}) + 2\rho_k \sqrt{1-\rho_k^2}\gamma_k(\mathcal{S})$ can be explicitly calculated at the transmitter using (9), and $\rho_k^2 = \cos^2(\phi_k + \angle(\hat{\mathbf{h}}_k, \mathbf{w}_k))$.

- Assuming $\epsilon \to 0$, then $\bar{\text{I}}_{UBd_k} \to \sin^2 \phi_k \to$ a more refined metric can be used

$$\xi_k^{UBd} = \frac{P \left\| \mathbf{h}_k \right\|^2 \rho_k^2}{P \left\| \mathbf{h}_k \right\|^2 \sin^2 \phi_k + \mathcal{M}} \tag{13}$$

User Selection Schemes (1/2)

Greedy - Semi-orthogonal US [Yoo et al.'06]

- Define ϵ as the maximum allowed non-orthogonality (maximum correlation) between $\hat{\mathbf{h}}_k$ (set *a priori*).

Step 0 Set $\mathcal{S} = \emptyset$, $\mathcal{Q}^0 = \{1, \ldots, K\}$

Select the first user: $k_1 = \arg\max_{k \in \mathcal{Q}^0} \xi_k$

For $i = 1, 2, \ldots, M-1$ repeat

Step 1 Define user set $\mathcal{Q}^i = \left\{1 \leq k \leq K : |\hat{\mathbf{h}}_k^H \hat{\mathbf{h}}_{k_j}| \leq \epsilon, 1 \leq j \leq i\right\}$

Step 2 $k_{i+1} = \arg\max_{k \in \mathcal{Q}^i} \xi_k$

Step 3 $\mathcal{S} = \mathcal{S} \cup k_{i+1}$

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where $\mathcal{R}(\mathcal{S}_i) = \sum_{k \in \mathcal{S}_i} \log_2(1 + \xi_k)$, with ξ_k being either: ξ_k^{UB}, ξ_k^{LB}, ξ_k^{UBd} or ξ_k^{LBd}

User Selection Schemes (2/2)

Greedy - User Selection

- We extend the greedy user selection algorithm of [Dimic et al.'05] for the case of limited feedback.

Step 0 Set $\mathcal{S}_0 = \emptyset$, $\mathcal{S} \subseteq \mathcal{G}$ and $\mathcal{R}(\mathcal{S}_0) = \emptyset$

For $i = 1, 2, \ldots, M$ repeat

Step 1 $\quad k_i = \arg \max_{k \notin \mathcal{S}_{i-1}} \mathcal{R}(\mathcal{S}_{i-1} \cup \{k\})$

Step 2 $\quad$ if $\mathcal{R}(\mathcal{S}_{i-1} \cup \{k\}) < \mathcal{R}(\mathcal{S}_{i-1})$

set $\mathcal{S} = \mathcal{S}_{i-i}$ and break

else if $i = M$ then $\mathcal{S} = \mathcal{G}$ and break

else set $\mathcal{S}_i = \mathcal{S}_{i-1} \cup \{k\}$ and go to **Step 1**

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where $\mathcal{R}(\mathcal{S}_i) = \sum_{k \in \mathcal{S}_i} \log_2(1 + \xi_k)$, with ξ_k being either: ξ_k^{UB}, ξ_k^{LB}, ξ_k^{UBd} or ξ_k^{LBd}

Performance analysis (1/4)

Asymptotic sum rate in the large K regime

- We analyze the sum-rate performance $\mathcal{R}$ of a system using the Greedy-SUS algorithm in conjunction with metric II at large K and M fixed.
- The sum rate is lower bounded by:

$$\mathcal{R} \geq M\left(1 - \Pr\left\{|\mathcal{S}| \neq M\right\}\right)\log_2\left(1 + \min_{k\in\mathcal{S}} \xi_k^{LB}\right) \tag{14}$$

- Choosing $\epsilon = \frac{1}{\log K}$, so that $\lim_{K\to\infty}\left|\mathcal{Q}^i\right| = \infty$ and $\lim_{K\to\infty}\epsilon = 0$, then $\xi_k^{LB} \overset{K\to\infty}{\to} \xi_k^{UB}$. Denoting $\beta = 2^{-B}\left(P/M\right)^{M-1}$, we can show:
- *Theorem 3:* The sum rate $\mathcal{R}$ converges to the optimum capacity of full CSI case $\mathcal{R}_{opt}$, for $K \to \infty$, i.e.,

$$\lim_{K\to\infty}\left(\mathcal{R}_{opt} - \mathcal{R}\right) = \lim_{K\to\infty}\left[M\log_2\frac{1+\frac{P}{M}\log K}{1+\frac{P}{M}\log\left(K/\beta\right)}\right] = 0 \tag{15}$$

with probability one.

Performance analysis (2/4)

Sum rate in the interference-limited region

- We characterize the sum-rate performance of our scheme in the high-power regime ($P \to \infty$).
- *Theorem 4:* The sum rate of the proposed scheme at high SNR with finite B and K is upper bounded by

$$\mathcal{R} \leq \frac{M}{M-1}\left(B + \frac{log_2 e}{\kappa_{max}} H_K\right) \quad (16)$$

where $H_K = \sum_{k=1}^{K} \frac{1}{k}$ is the harmonic number and $\kappa_{max} = \max_{i=1,\ldots,M} \kappa_{i-1}$.

$\kappa_i = \left|\mathcal{Q}^i\right| / K \approx I_{\epsilon^2}(i, M-i)$ (regularized incomplete beta function) captures the multiuser diversity gain reduction due to greedy scheduling.

Performance analysis (3/4)

Sum rate in the interference-limited region

- The system becomes interference-limited $\rightarrow$ bounded sum rate at high SNR (even for arbitrary large but finite B and K).
- As $\partial\mathcal{R}/\partial M < 0 \rightarrow \mathcal{R}$ is monotonically decreasing with M $\rightarrow M = 1$ at high SNR (switch from SDMA to TDMA).
- For large K, we have $\lim_{K\rightarrow\infty} H_K = \log K + \gamma$
  (γ: Euler-Mascheroni constant)
- The sum rate with $P \rightarrow \infty$ and $K \rightarrow \infty$ exhibits logarithmic growth with $K \rightarrow$ multiuser diversity compensates for the loss in degrees of freedom.

Performance analysis (4/4)

Sum rate in the low-power regime

- When $P \to 0$, $\xi_k^{LB} = \frac{P}{M(1+\vartheta)^2} \|\mathbf{h}_k\|^2 (\cos\phi_k - \sqrt{\vartheta})^2 \leq \frac{P}{M} \|\mathbf{h}_k\|^2 \cos^2\phi_k$.
- *Lemma 2*: The distribution of $X = \|\mathbf{h}_k\|^2 \cos^2\phi_k$ is given by

$$f_X(x) = (1-\delta) \sum_{k=0}^{\infty} \frac{\zeta_k x^{k+M-1} e^{-x/(1-\delta)}}{(1-\delta)^{k+M} \Gamma(k+M)} u(x) \tag{17}$$

where $u(\cdot)$ is the unit step function, $\delta = 2^{-\frac{B}{M-1}}$, and ζ_k is obtained recursively by

$$\zeta_0 = 1 \;\text{ and }\; \zeta_{k+1} = \frac{1}{k+1} \sum_{i=1}^{k+1} \left(\delta^i\right) \zeta_{k+1-i}, k = 0, 1, 2, \ldots \tag{18}$$

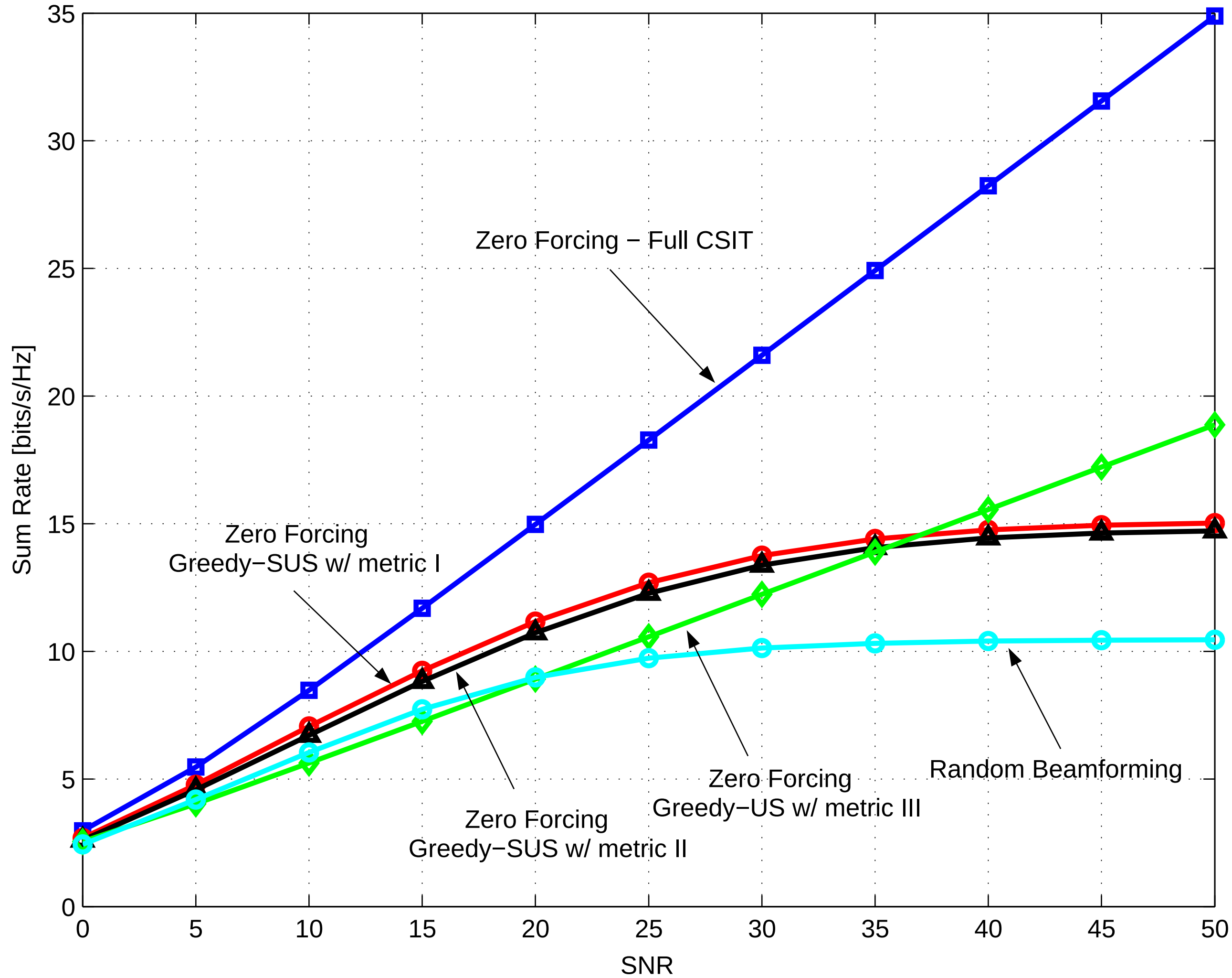
- *Proposition*: The r.v X stochastically dominates $\tilde{X}$ ($F_X(x) \leq F_{\tilde{X}}(x)$), whose CDF is given by

$$F_{\tilde{X}}(x) = \int_0^x (1 - e^{-x/(1-\delta)})^{M-1} dx \tag{19}$$

- For finite B, the sum rate is lower bounded as

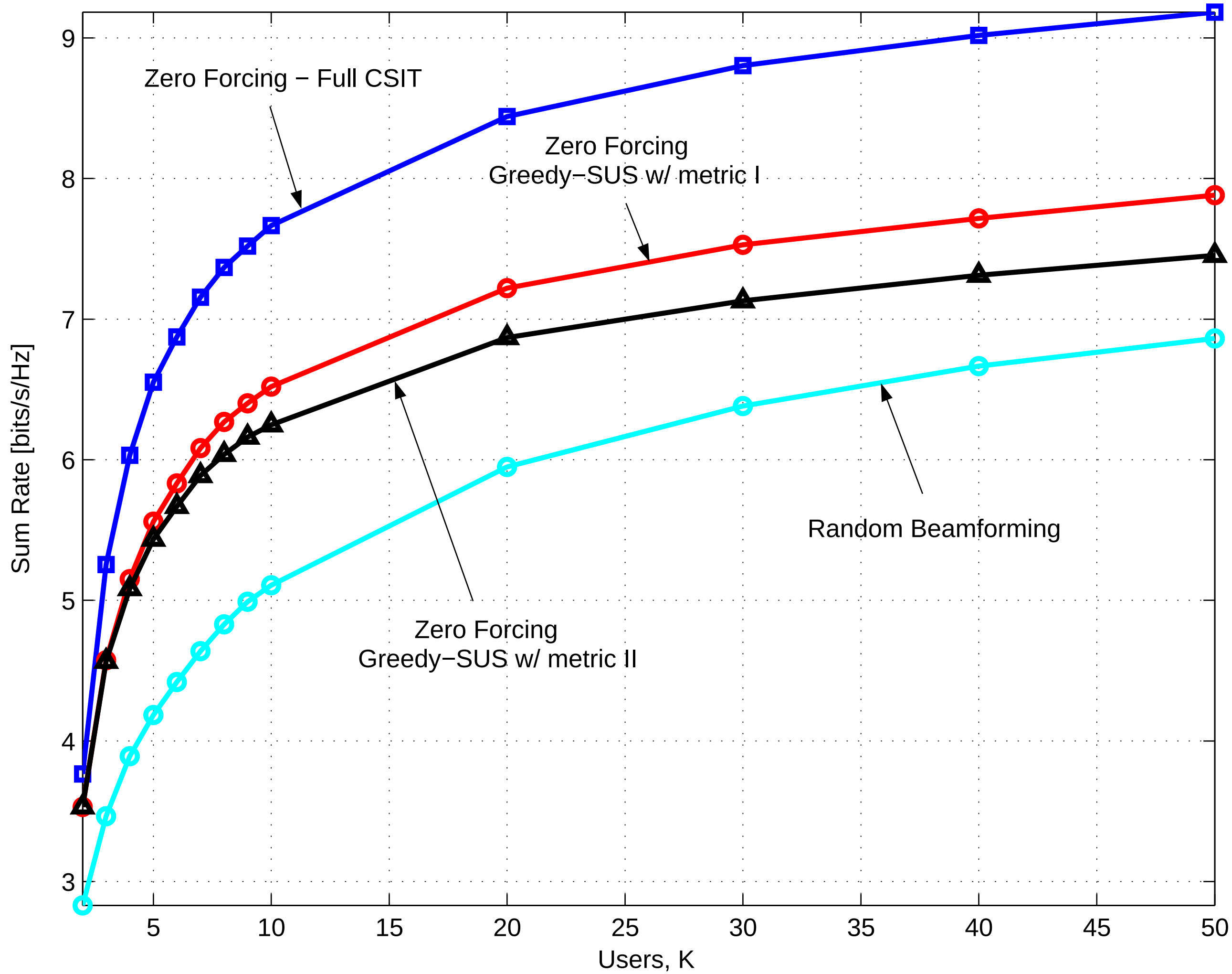
$$\mathcal{R} \geq \sum_{i=1}^{M} \int_0^\infty \log_2\left(1 + \frac{P}{M}x\right) dF_{\tilde{X}}^{\mathcal{K}_i}(x) \approx \log_2 e \frac{P}{M} \sum_{i=1}^{M} \int_0^\infty x dF_{\tilde{X}}^{\mathcal{K}_i}(x) \tag{20}$$

Numerical Results (1)



Sum rate vs. the average SNR for $B = 4$ bits, $M = 2$ Tx antennas and $K = 20$ users.

Numerical Results (2)



Sum rate as a function of the number of users for $B = 4$ bits, $M = 2$ Tx antennas and $SNR = 10$ dB.

In brief ...

- We studied a multiple antenna broadcast channel in which partial CSIT is conveyed via a limited rate feedback channel.
- Several scalar feedback metrics have been proposed, which provide an efficient estimate of the received SINR at the receiver side.
- These metrics, combined with greedy scheduling and ZFBF on the quantized channel directions, can achieve a significant fraction of full CSI capacity by means of multiuser diversity.
- We derived upper bounds on the instantaneous MU interference that allows us to analytically predict the worst case interference and SINR in a system employing ZF on the quantized channel directions.
- A multi-mode switching scheme from MU to SU transmission is proposed. This scheme exhibits a linear sum-rate growth at high SNR.

References

[Yoo et al.'06] T. Yoo and A. Goldsmith, 'On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming,' in *IEEE JSAC*, vol. 24(3), March 2006.

[Dimic et al.'05] G. Dimic and N.D. Sidiropoulos, 'On Downlink Beamforming with Greedy User Selection: Performance Analysis and a Simple New Algorithm,' in *IEEE Trans. Signal Processing*, , vol. 53(10), Oct. 2005.