Resource allocation in multicell wireless networks: Some capacity scaling laws

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Outline

- Network model and objectives
- Distributed resource allocation in multicell networks
- Scaling laws for network capacity
- Performance, conclusions, future work
Cellular or Adhoc Networks?
Basic signal model

In cell $n$, the received signal $Y_{un}$ at user $u_n$ is given by

$$Y_{un} = \alpha_{u_n,n} X_{un} + \sum_{i \neq n}^{N} \alpha_{u_n,i} X_{ui} + Z_{un},$$

where $X_{un}$ is the message-carrying signal from the serving AP, subject to a peak power constraint $P_{max}$. $\sum_{i \neq n}^{N} \alpha_{u_n,i} X_{ui}$ is the interference and $Z_{un}$ is the additive noise.
Goals and limitations of wireless networks

- Key goal is to increase spectrum efficiency
- Should operate with aggressive reuse factors
- Spectral reuse gives interference
- Interference solved today by static or semi-dynamic channel allocation
- This solution does not scale easily and is inefficient.

Two kinds of solutions:
- Signal processing (interference canceling, MIMO, etc.)
- Smart resource allocation
Learning from Multi-user MIMO

- One base can serve multiple users as long as there is a decent SNR
Multicell MU-MIMO

- The right thing to do!
- Base antennas are like distributed MIMO arrays
- But Base NEEDS accurate channel information.
- Centralized control hard to avoid (in general)
- Ongoing research (SPAWC07, project with FT ..).
Multicell resource allocation

Motivations:

- When network grows large, distributed MIMO is too complex, or multicell routing of data not desirable.
- Multicell R.A. addresses the allocation of spectral resources (time, frequency, codes, power) to optimize system capacity
- Here we focus on scheduling and power control
- Fundamentally exploits the variability (diversity) of channel in all dimensions

Multicell R.A. can be:

- Centralized
- Distributed
Centralized resource allocation
Definitions for scheduling and power control

Definition 1: A **scheduling vector** $U$ for a given resource slot contains the set of users simultaneously scheduled across all cells:

$$U = [u_1 \ u_2 \ \cdots \ u_n \ \cdots \ u_N]$$

where $[U]_n = u_n$. Noting that $1 \leq u_n \leq U_n$,

Definition 2: A **transmit power vector** $P$ for a given resource slot contains the transmit power values used by each AP to communicate with its respective user:

$$P = [P_{u_1} \ P_{u_2} \ \cdots \ P_{u_n} \ \cdots \ P_{u_N}]$$

where $[P]_n = P_{u_n} = \mathbb{E}|X_{u_n}|^2$. Due to the peak power constraint $0 \leq P_{u_n} \leq P_{\text{max}}$. 
Optimal scheduling and power control (centralized)

The SINR for the user selected in cell $n$ is

$$\Gamma([U]_n, P) = \frac{G_{u_n,n} P_{u_n}}{\sigma^2 + \sum_{i \neq n} G_{u_n,i} P_{u_i}},$$

(1)

The system capacity is

$$\mathcal{C}(U, P) \triangleq \frac{1}{N} \sum_{n=1}^{N} \log \left(1 + \Gamma([U]_n, P)\right).$$

(2)

The optimal resource allocation problem is

$$([U]^*, [P]^*) = \arg \max_{P \in \Omega} \mathcal{C}(U, P),$$

(3)
But we want...distributed resource allocation
Towards distributed resource allocation

- We want to maximize system capacity, by having each cell making an independent decision (power, user).
- Problem in general impossible...but
- Key intuition: max sum rate scheduling tends to increase perceived SINR in each cell
- If there are many users to choose from, one may pick users less sensitive to interference.
- Let us examine the gap in capacity due to interference, in case of many users.
A bounding approach

We study two bounds on capacity:

- Upper bound obtained with no interference
- Lower bound obtained with full powered interference

In two network scenarios:

- All users have same average received power (located on circle around the base)
- Users uniformly located in the cell
Upper bound on capacity

Assuming no interference:

\[ C(U^*, P^*) \leq C^{ub} = \frac{1}{N} \sum_{n=1}^{N} \log \left( 1 + \Gamma^{ub}_n \right). \]  \hspace{1cm} (4)

where the upper bound on SINR is given by:

\[ \Gamma^{ub}_n = \max_{u_n=1..U} \left\{ G_{u_n,n} \right\} P_{max} / \sigma^2 \]  \hspace{1cm} (5)

The corresponding scheduler is the max SNR scheduler: Fully distributed

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Lower bound on capacity

Assuming all interferers transmit at $P_{max}$:

$$\mathcal{C}(U^*, P^*) \geq \mathcal{C}^{lb} = \mathcal{C}(U_{FP}^*, P_{max})$$  \hspace{1cm} (6)

where $U_{FP}^*$ denotes the optimal scheduling vector assuming full power everywhere, defined by

$$[U_{FP}^*]_n = \arg \max_{U \in \Upsilon} \Gamma^{lb}_n = \frac{\{G_{u_n,n}\} P_{max}}{\sigma^2 + \sum_{i \neq n}^N G_{u_n,i} P_{max}}$$  \hspace{1cm} (7)

The corresponding scheduler is the max SINR scheduler: Also fully distributed
**Capacity scaling with many users** ($U \rightarrow \infty$)

In the interference-free case (using extreme value theory):

**Lemma:** Let $G_{u_n,n} = \gamma_{u_n,n}|h_{u_n,n}|^2$, $u_n = 1..U, n = 1..N$, where $\gamma_{u_n,n} = \gamma_n$. Assume $|h_{u_n,n}|^2$ is Chi-square distributed with 2 degrees of freedom ($\chi^2(2)$) (i.e. $h_{u_n,n}$ is a unit-variance complex normal random variable). Assume the $|h_{u_n,n}|^2$ are i.i.d across users. Then for fixed $N$ and $U$ asymptotically large, the upper bound on the SINR in cell $n$ scales like

$$\Gamma_n^{ub} \approx \frac{P_{max} \gamma_n}{\sigma^2} \log U$$  \hspace{1cm} (8)

**Theorem:** For fixed $N$ and $U$ asymptotically large, the average of the upper bound on the network capacity scales like

$$E(C^{ub}) \approx \log \log U$$  \hspace{1cm} (9)
Capacity scaling with many users \( (U \to \infty) \)

In the full powered interference case (using extreme value theory):

**Lemma:** Let \( G_{u_n,i} = \gamma_{u_n,i} |h_{u_n,i}|^2 \), \( u_n = 1..U, n = 1..N \), where \( \gamma_{u_n,n} = \gamma_n \), \( \gamma_{u_n,i} = \beta_{d_{n,i}}^{-\epsilon} \) for \( i \neq n \). Assume \( |h_{u_n,i}|^2 \) is Chi-square distributed with 2 degrees of freedom \( (\chi^2(2)) \). Assume the \( |h_{u_n,i}|^2 \) are i.i.d across users, cells. Then for fixed \( N \) and \( U \) asymptotically large, the lower bound on the SINR in cell \( n \) scales like

\[
\Gamma_n^{lb} \approx \frac{P_{\max} \gamma_n}{\sigma^2} \log U 
\]  \hspace{1cm} (10)

**Theorem** Then for fixed \( N \) and \( U \) asymptotically large, the average of the lower bound on the network capacity scales like

\[
E(C^{lb}) \approx \log \log U 
\]  \hspace{1cm} (11)
**Capacity scaling with many users** \( (U \rightarrow \infty) \)

- Upper bounds and lower bounds have same growth rates!
- Interference creates vanishing loss for large number of users
- Physically, the scheduler looks for users **shielded from interference and large SNR**
- When number of users is large, **interference becomes small compared with noise.**

Average capacity with optimum power control and scheduling:

\[
E(C(U^*, P^*)) \approx \log \log U
\]  

(12)
Capacity scaling for symmetric network

Scaling of upper and lower bounds of capacity, versus $U$ for a symmetric network ($N = 3$)
Capacity scaling for non-symmetric network

Users are located randomly in the cell. Users closer to the base have better SNR...

**Theorem:** The upper bound on capacity will behave like:

\[ E(C^{ub}) \approx \frac{\epsilon}{2} \log U \quad \text{for large } U \]  

(13)

**Theorem:** The lower bound on capacity will behave like:

\[ E(C^{lb}) \approx \frac{\epsilon}{2} \log U \quad \text{for large } U \]  

(14)
Capacity scaling for asymmetric network

Scaling of upper and lower bounds of capacity, versus \( U \) for a non-symmetric network \( (N = 3) \)
Conclusions

- Asymptotic case (in nbr of users) reveals simple structure of the resource allocation problem
- Scheduling makes price paid in interference almost negligible
- Dense networks can operate with reuse one, if right scheduler is used!
- Two problems: System is unfair and large number of users needed.

Other approaches for making capacity-maximizing resource allocation distributed:
- Game theoretic approaches (however pessimistic for operated network)
- Discretizing power control (powerful approach: see Saad Kiani’s prez next week)