Ultra Wideband Systems: Information theoretic considerations

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Eurerecom Weekly Seminar
• UWB channel systems challenges
• UWB channel capacity and signaling
  – Duty-cycled DSSS
  – Channel Division Multiple Access Technique: ChDMA
• UWB channel modeling using information theoretic arguments
  – Subspace Analysis Using Akaike Information theoretic Criteria (AIC)
  – Model selection using AIC
• Conclusions & Perspectives
Motivations

• Third generation wireless systems and beyond (3G and 4G)
• Compatibility with existing systems.
• UWB capacity issues.
• UWB Applications:
  - Cable replacement
  - Location Based Services
  - Cognitive Radio...
Ultra Wideband Communications Challenges

Telatar & Tse (00) show that: With spread spectrum signals over multipath channels, the data rate is inversely proportional to the number of channel paths.

– Direct sequence spread spectrum with no duty cycle has zero throughput in the limit, if the number of channel paths increases with bandwidth.
– The channel uncertainty versus the bandwidth has to be assessed.
Subspace Analysis using Information Theoretic Criteria

Wax and Kailath (1985) presented a new approach for estimating the number of signals in multichannel time-series and frequency-series, based on AIC (Akaike Information Criterion) and MDL (Minimum Description Length). Let \( \mathbf{y}(t) = [y_1(t), y_2(t), \ldots, y_N(t)] \) be a set of candidate covariance matrices, \( \mathbf{R}(\mathbf{h}) \), with rank \( k \).

Consider a set of candidate covariance matrices, \( \mathbf{R}(\mathbf{h}) \), with rank \( k \) and \( \mathbf{z}(t) \) is the noise variance.

\[
\mathbf{z}(t) + \mathbf{H}^\dagger \mathbf{R}(\mathbf{h}) \mathbf{H} \mathbf{z}(t) - \gamma \mathbf{I} = \sum_{i=1}^{\gamma} \mathbf{R}(i)
\]

where \( \lambda_i \) is the \( i \)-th eigenvalue, \( \mathbf{\psi}_i \) is the \( i \)-th eigenvector and \( \sigma^2 \) is the noise variance.
Information Theoretic Criteria (1)

Akaike proposed to select the model which gives the minimum AIC, defined by:

$$AIC = -2 \log (f(h|\hat{\theta}_k)) + 2k$$

Both Schwartz’s and Rissanen’s approaches yield to the same criterion, given by:

$$MDL = -\log (f(h|\hat{\theta}_k)) + \frac{1}{2} k \log (N)$$

where $\hat{\theta}_k$ is the maximum likelihood estimate of the parameter vector $\theta_k$ and $k$ is the number of freely adjustable parameters in $\theta_k$.

With $\theta_k = (\lambda_1, \ldots, \lambda_k, \sigma^2, \psi_1, \ldots, \psi_k)$.

\[
\frac{1}{(1 + \frac{\gamma - d z}{\gamma}) (N)^{\theta \log}}
\]

\[
\text{Information Theoretic Criteria (2)}
\]

The MDL function is given as follows:

\[
\text{MDL}(k) = \frac{\gamma}{(\gamma - d) N} \left( \frac{\eta}{(\gamma - d)^{\gamma}} \sum_{d=1}^{\gamma - d} \frac{\gamma - d}{1} \right) b \log N = \gamma \log(\gamma) \text{MDL}
\]

The AIC is given as follows:

\[
\frac{1}{(y - d)(d+1)} \left( \frac{\eta}{(y - d)^{y}} \sum_{d=1}^{\gamma - d} \frac{\gamma - d}{1} \right) b \log N = \gamma \log(\gamma) \text{AIC}
\]

\[
(\gamma)\gamma = \gamma \frac{1 + \gamma - d}{d} \sum_{d=1}^{\gamma - d} \frac{\gamma - d}{1}
\]

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\]

\[
(\gamma)\gamma = \gamma \frac{1 + \gamma - d}{d} \sum_{d=1}^{\gamma - d} \frac{\gamma - d}{1}
\]
Estimation of the degrees of freedom

The number of degrees of freedom, mainly the number of significant eigenvalues, is determined as the value of \( k \in \{ 0, 1, \ldots, p - 1 \} \) which minimizes the value of (1) or (2).

In this work, the number of DoF represents the number of unitary-dimension independent channels that constitute an UWB channel.
The number of DoF doesn't scale linearly with the bandwidth.

Figure 1: The number of UWB channel DoF for LOS setting.

- Figure 1: The number of UWB channel DoF for LOS setting.

- Results
Schuster and Bolcskei (06) used Akaike’s Information-Theoretic Criteria to determine suitable distributions for UWB channel impulse response taps. The goal of the model selection procedure is to choose the distribution that minimizes the discrepancy among all members of the candidate set.

Denote the unknown cumulative distribution function (CDF) of the operating model by \( F \), and the set of all CDFs by \( \mathcal{M} \). A parametric candidate family \( \mathcal{G} \) is the subset of \( \mathcal{M} \), with \( \mathcal{G} = \left\{ G_{\Theta} \mid \Theta \in \Theta \right\} \), with \( \Theta \in \Theta \subseteq \mathbb{R}^U \), where \( U \) is the number of parameters. The parametric candidate family \( \mathcal{G} \) is the subset of \( \mathcal{M} \), with each CDF parametrized by the \( U \)-dimensional vector \( \Theta \). The goal of the model selection procedure is to choose the distribution that minimizes the discrepancy among all members of the candidate set.
Model selection using AIC (2)

AIC is an approximately unbiased estimator of the expected Kullback-Liebler (KL) distance:

$$\frac{\sum_{i=1}^{N} \log g_j(\mathbf{x}_n)}{2} + \left( u \mathbf{x} \right)^{\hat{\Theta}_j}$$

where

$$\hat{\Theta}_j = \arg \cdot \max \Theta_j \in \mathcal{T}_j$$

The Akaike weights are given by:

$$\omega_j = \frac{1}{\sum_{i=1}^J \exp \left(-\frac{1}{2} D_j \sum_{i=1}^J \exp \left(-\frac{1}{2} D_i \right) \right)}$$

with

$$D_j = AIC_j - \min_{i}AIC_i$$

and $i \in J$.

with $D_j \in \mathcal{T}_j$ and $i \in J$.
Model selection using AIC: Results

Model selection using AIC show that Rayleigh, Rice and Weibull distributions exhibit a good fit to the measurement data.

Figure 1: PDF and Akaike weights for measurement Campaign 1

Figure 2: PDF and Akaike weights for measurement Campaign II
Maximum entropy approach to UWB channel modeling

Idea: Given a set of measurements, we try to find the best process model under some constraints.

The entropy rate of a stationary Gaussian stochastic process can be expressed as:

\[ h(\chi) = \frac{1}{2} \log 2\pi e + \frac{1}{4} \int_{-\pi}^{\pi} \log S(\lambda) d\lambda \]
Burg’s Maximum Entropy Theorem:

The maximum entropy rate stochastic process $X$ satisfies the constraints

$$E \sum_{d=1}^{\gamma} a_d X_d X_d^* = \alpha_k, \forall k = 0, 1, \ldots, p,$$

for all $i$. The $Z_i$ are i.i.d. $N(0, \sigma^2)$. The vectors $a_1, a_2, \ldots, a_p, \sigma^2$ are chosen to satisfy eqn. (4).

$$z_i = \sum_{d=1}^{\gamma} a_d X_d X_d^* = X_i$$

is the $p$th order Gauss-Markov process of the form

$$X_i = -\sum_{k=1}^{p} a_k X_{i-k} + Z_i$$

The maximum entropy rate stochastic process $X_i$ satisfying the constraints is of the form

$$X_i = \mathcal{E} \left[ \sum_{d=1}^{\gamma} a_d X_d X_d^* \right]$$
These equations exactly resemble the Yule-Walker equations. Therefore, we can solve for the parameters of the processes from the covariances using fast algorithms such as the Levinson and the Durbin recursion. 

\begin{equation}
R(l) = l, 2, \ldots, p.
\end{equation}

\begin{equation}
\begin{align*}
\sigma^2 \sum_{d=1}^{\alpha} R^{\alpha} r^{\alpha} - & = (l) \mathcal{H} \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 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Maximum entropy approach to UWB channel modeling

Spectrum estimation

After the coefficients $a_1, a_2, \ldots, a_p$ have been calculated from the covariances, the spectrum of the maximum entropy process is seen to be

$$S_l = \sigma^2 \left| 1 + \sum_{k=1}^{p} a_k e^{-ikl} \right|^2$$

(8)

This is the maximum entropy spectral density subject to the constraints $R(0), R(1), R(2), \ldots$.
Figure 2: Entropy variation with respect to the bandwidth.

Maximum entropy analysis shows that the channel information doesn’t increase so much with increasing the bandwidth.
• The PDP shape can be reproduced with a limited AR model order.

Figure 3: Estimated Power Delay Spectrum with 500MHz Bandwidth and 6GHz Bandwidth.
Golay (49) showed that this capacity, with non-fading channels, can be approached by on-off keying (pulse position modulation) with very low duty cycle.

Kennedy (69) proved this for flat fading channels using FSK signals with duty cycle transmissions.

Telatar & Tse (00) extended the proof for multipath channels with any number of paths.

Summary: FSK with duty cycle achieves AWGN capacity for any number of paths.
UWB systems capacity and signaling (2)

• Médard & Gallager (02) show that direct sequence spread spectrum with no duty cycle approaches zero data rate in the limit of infinite bandwidth and with a high number of paths.

• Telatar & Tse (00) show that with spread spectrum signals over multipath channels, the data rate is inversely proportional to the number of channel paths, the data rate is penalized when the receiver has to estimate the channel.

Why? The data rate is penalized when the receiver has to estimate the channel.

Question: How to perform multiple access in the infinite bandwidth case?

Summary: Direct sequence spread spectrum with no duty cycle has zero throughput in the limit, if the number of channel paths increases with bandwidth.
Due to the channel uncertainty that communication systems face, a necessary condition for a communication system to achieve the AWGN channel capacity in the limit of infinite bandwidth is that the channel estimation in the limit is perfect.

\[ SNR_{est} = \lim_{L \to \infty} \frac{2P \theta}{L} \]

Where \( L \) is the number of independent resolvable paths, \( T_c \) is the coherence time and \( \theta \) is the duty cycle parameter. This requires [Porat & Tse]:

\[ \frac{7 \theta^0 N}{2T_c} \to \infty \]

and

[\text{UWB systems capacity and signaling (3)}]

Due to the channel uncertainty that communication systems face, a necessary condition for a communication system to

\[ \infty \quad SNR_{est} \quad 2 \]

\[ = \quad \frac{7 \theta^0 N}{2T_c} \to \infty \]

Where \( L \) is the number of independent resolvable paths, \( T_c \) is the coherence time and \( \theta \) is the duty cycle parameter.
• Porrat, Tse & Nacro (06) showed that duty-cycled DSSS systems achieve the wideband capacity as long as the number of independently faded resolvable paths increases sub-linearly with the bandwidth while duty-cycled PPM signaling achieves the wideband capacity only if the number of paths increases sub-logarithmically.

Why? Because PPM is an orthogonal modulation, so the rate increases only logarithmically with the bandwidth whereas the rate of DSSS systems increases linearly.
UWB systems capacity and signaling: ChDMA principle

Idea

ChDMA address the problem of multiple access in decentralized networks (ad-hoc UWB) and non-cooperative transmissions.
Each user sends a very modulating peaky signal every $T_d$.

(UWB, $T_d$ is typically about 15 ns whereas $T_c$ about 100 µs)

Channels can act as codes if they have enough independent entries (similar to CDMA).

The system uses very low duty cycles and is flexible from an ad-network perspective (no code allocation).
We consider a time invariant channel $c(k)$ of user $k$ given by:

$$\sum_{l=1}^{L(k)} \lambda_l \delta(\tau - \tau_l(k))$$

(1)

The number of paths is the same, each user operates in the same environment, operating at the same bandwidth.

- All users are in the same environment, operating at the same bandwidth.
- The number of paths is the same ($L(k)$).

where $\lambda_l$ and $\tau_l$ represent respectively the gain and the delay of the $l$-th multipath.
Furthermore, because of the pulse signal employed for the transmission of the symbols on the environment, the channel vector length is given by the ratio between the temporal resolution ($T_r$) and the symbol period ($T_s$).

The discrete channel matrix $H$ is given by the concatenation of the discrete channel vector of each user as shown in the following:

$$H = \left[ h_1 \, h_2 \, \ldots \, h_K \right]$$

(12)

where $\mathcal{H}$ is the transmit filter.

The discrete channel matrix of the $k$-th user is given by

$$\left( h_k \right)_\mathcal{H} = \left( \sum_{l=1}^{L} \lambda_k \cdot g_{\left( \tau_0 - \tau_k l \right)} \right)$$

(11)

where $g$ is the transmit filter.
The mutual information per dimension (known as spectral efficiency),

\[ N \mathbf{I}_c^0 = (H \mathbf{uu}) \mathbb{E} \]

\[ H \mathbf{HH} + N \mathbf{I}_c^0 = (H \mathbf{XX}) \mathbb{E} \]

In the case of Gaussian independent entries, since

\[
\left( H \mathbf{HH} \frac{2^\nu}{\mathbb{I}} + N \mathbf{I} \right) \frac{N}{\mathbb{I}} = \gamma
\]

with optimum receiver, is:

\[
\log \det \left( \mathbb{I}_N + \frac{N^2}{\mathbb{I}} \right)
\]
For the matched filter (MF) and the MMSE receivers:

\[ \gamma = 1 \frac{\log_2 (1 + \text{SINR}_i)}{N \sum_{j=1}^{K} \frac{|h_j h_j^H|}{|\sum_{j=1}^{K} h_j h_j^H|}} \]

with:

\[ \text{SINR}_{MF} = \frac{|\sum_{j=1}^{K} h_j h_j^H|}{|\sum_{j=1}^{K} h_j h_j^H|} \]

\[ \text{SINR}_{MMSE} = \frac{|\sum_{j=1}^{K} h_j h_j^H|}{|\sum_{j=1}^{K} h_j h_j^H|} \]

where \( H^i \) is a \( N \times (K - 1) \) matrix which contains all time response vectors \( h_j \) for all \( j \neq i \).

With BPSK signaling, the mutual information becomes:

\[
\gamma = 1 - \sum_{i=1}^{N} \frac{1}{2} \int_{-\infty}^{\infty} e^{-v^2/2} e^{-v(2\text{SINR}_i - 2\sqrt{\text{SINR}_i} v)} dv,
\]

\[\text{(15)}\]

where \( \text{SINR}_i \) for MF and MMSE receivers are already defined before.
How many users can simultaneously communicate?

The number of degrees of freedom of the wideband channel. The number \( L \) has already been characterized before as the only one that matters in the number of users since the system depends on it.

\[
\left( B^0 \frac{N^2}{\lambda d} + I \right) \log_2 \sum_{I} \frac{N}{B} = \left( H \frac{B^0 N}{d} + N I \right) \log_2 \det \left( I_N + \frac{P N_0 B}{\lambda^2} \right)
\]

There is a limit on the number of users since the system depends only on \( L \).

For \( L \gg 1 \) (and supposing the non-zero eigenvalues equal to \( \lambda \)) with \( B \):
The CDMA-based case has been simulated in an ideal AWGN channel.

With asynchronism, the CDMA gain is not very significant with respect to ChDMA for the MMSE and Optimal receiver and may be reduced or become even worst in frequency selective fading.

With BPSK signaling, the results are the same.

The optimum results, for both CDMA and ChDMA, are with $T_s$ equal to channel delay spread.

ChDMA versus Duty-Cycled DSSS

• Peaky modulation
• Spread spectrum like signaling due to UWB channel high temporal resolution
• Location dependent signatures: No code allocation, security issues, decentralized architecture...
• Information Theoretic Arguments are used to address UWB channel diversity, modeling and entropy.

• Duty-cycled DSSS systems achieve the wideband capacity as long as the number independently faded resolvable paths increases sub-linear with the bandwidth while duty cycled PPM signaling achieves the wideband capacity only if the number of paths increases sub-logarithmically.

• A New Multiple Access Scheme has been devised which is flexible with no constraint in terms of code acquisition.

• Initial results for ChDMA show very good separation capability and interference reduction due to the wideband nature of the channel especially for the asynchronous mode.

• Further studies are being conducted to assess the optimal channel diversity, modeling and entropy.
Ultra Wideband Systems: Information theoretic considerations

References


THANK YOU!

Questions?