Lossy Transmission over Slow Fading Channels:

A Comparison of Transmission Strategies and

Code Constructions

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Outline of the Presentation

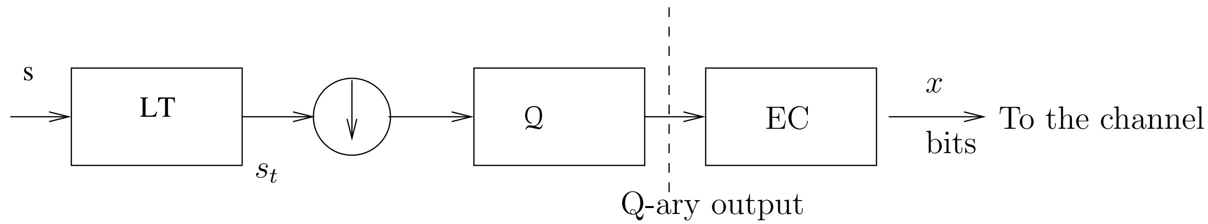
- Motivations and goals
- General problem ⇒ transmission over fading channels with fidelity criteria, successive refinement of information
- Optimization of transmission techniques
  - **Superposition**
  - **Progressive**
  - **Hybrid Digital / Analog**
- Results and comparisons for Rayleigh fading – Path Loss
- Preliminary results of code constructions
- Conclusions

Motivations and Goals

- Transmission of analog sources over the wireless link $\Rightarrow$ **End to End Distortion** instead of **BER**
- Optimality of **analog transmission** only for lucky cases $\Rightarrow$ interest in **improving spectral efficiency**, $\eta = \frac{W_s}{W_c}$
- Spectral efficiency: source bandwidth is given by nature, channel bandwidth is very expensive.
- We suppose $\eta > 1 \Rightarrow$ the source needs to be compressed
- **Separated approach** vs **joint approach**: what are the weaknesses and the advantages?
- Successive refinement of information $\Rightarrow$ graceful degradation of performances

Weakness and Advantages

– SEPARATED APPROACH



- LT: Linear Transformation, Fourier or Wavelet subband decomposition
- $\downarrow$: decimation
- $\mathcal{Q}$: Quantization (Scalar or Vector)
- EC: lossless Entropy Coder $\Rightarrow$ Implemented by **adaptive arithmetic coding**.

Because of EC: Not robust to residual channel errors, main problem:

propagation of errors

Weaknesses and Advantages (cont'd)

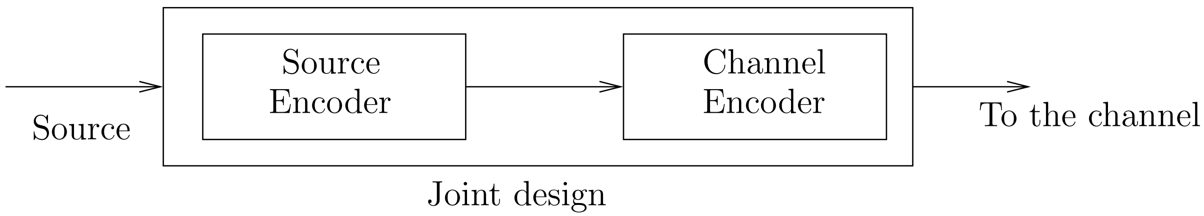
- They require **very strong constraints** on channel codes performances
- Complete separation between source operations and channel operations ⇒ **independent design of channel/source codes**

Result ⇒ pick the best source encoder and the best channel code!

Lot of work done in this direction in order to protect good source encoder like Jpeg2000

Weakness and Advantages (cont'd)

– JOINT APPROACH

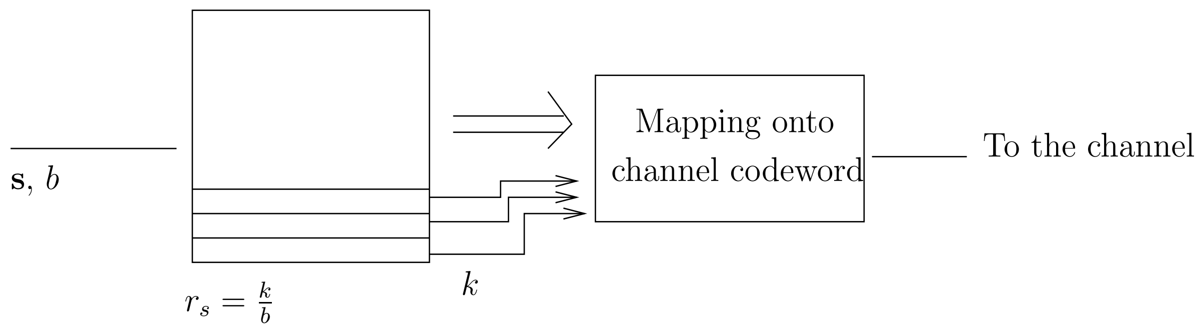


The channel code is designed to take into account the source encoder

- **Mild conditions** on channel code performances
- **Dependency** between channel coders and source coders

Result ⇒ we can approach the limit

System Model: Superposition and Progressive schemes



- i.i.d Gaussian source with bandwidth W_s
- Channel bandwidth W_c under **end to end quadratic distortion**
- Ideal successive refinement source encoder
- Each layer of information conveys r_s bits/source symbol
- Each layer is mapped into a channel codeword and transmitted over the BF-AWGN channel.

System Model (cont'd)

Consider a block-fading AWGN channel (BF-AWGN) described by

$$y_t = hx_t + z_t, \quad t = 1, \ldots, n$$

- h is the block fading coefficient, $a = |h|^2$ fading power gain, with *continuous* pdf $f_A(z)$ and cdf $F_A(z)$.
- $z_t \sim \mathcal{N}_{\mathbb{C}}(0, N_0)$ is complex circularly-symmetric AWGN.
- Input constraint $\mathbb{E}[|x|^2] \leq E_s$, and we define $\Gamma = E_s/N_0$

The BF-AWGN $\Rightarrow$ equivalent to the Gaussian broadcast channel if the transmitter is not informed about the value of the fading gain a but of the statistic

[Shamai-Steiner,2003]: Broadcast approach for the compound channel $\Rightarrow$ maximization of the average rate vs power profile.

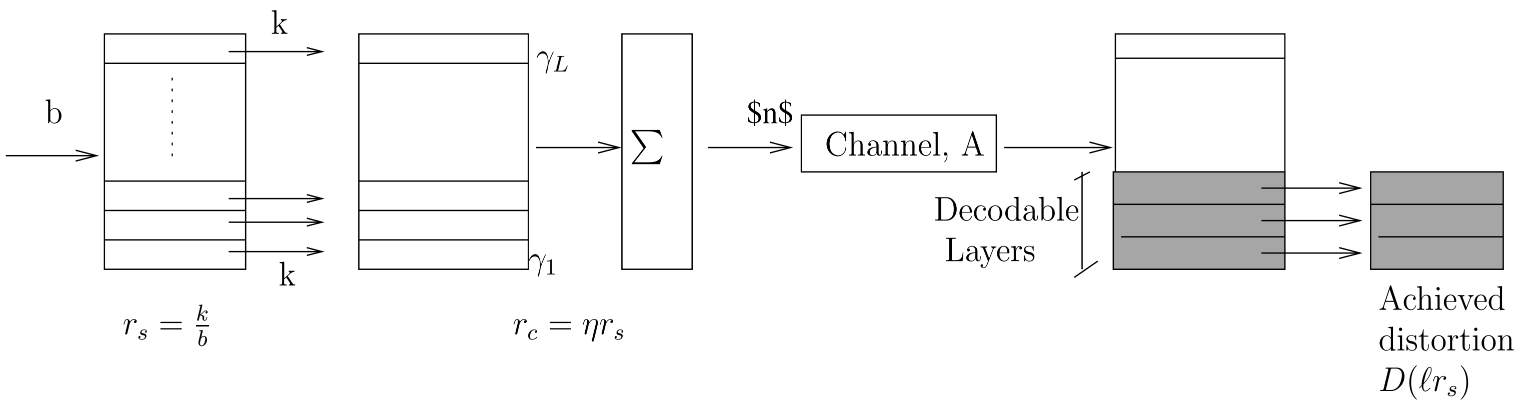
Goals ...

Analyze and optimize two fully digital strategies: **Superposition, Progressive Transmission** and compare them with a **Hybrid Digital/Analog schemes**. Performances in terms of average/ instantaneous end-to-end distortion vs channel SNR

- The optimization is done by minimizing the average distortion with a transmitted power constraint $\Rightarrow$ algorithms
- Compare these strategies with the fully separated approach and the Shannon bound
- Construction of codes for the best scenario

Superposition Approach

It maps each layer onto one channel codeword with a rate r_c, the codewords are summed and sent trough the fading channel



Algorithm for computing the optimal solution as well as for handling non-ideal channel codes and successive refinement source codes

Superposition Approach

- L layers, the number of source layers coincides with the number of superposition channel coding layers
- Each layer has source coding rate r_s bit/source symbol and channel coding rate r_c bit/channel uses, $\eta = r_c / r_s$
- Define $D_\ell \triangleq D(\ell r_s) = \mathbb{E}\left[d\left(\mathbf{s} - \hat{\mathbf{s}}_\ell\right)\right]$, $(D_0 = 1)$ is the $\ell-$th layer distortion, where ℓ is the number of layers successively decoded
- $\mathcal{C}'$ is identified by the rate SNR-threshold pair (r_c, τ)
- The transmitted superposition codeword is given by $\mathbf{x} = \sum_{\ell=1}^{L} \sqrt{\gamma_\ell} \mathbf{c}'_\ell$, γ_ℓ are the power levels and $\mathbf{c}'_\ell$ are the codewords of $\mathcal{C}'$ associated to level ℓ

Superposition Approach

Define the fading gain thresholds $0 < a_1 < \cdots < a_L$, $a_{L+1} = \infty$ such that layers up to ℓ can be decoded if $A \in [a_\ell, a_{\ell+1})$.

- Condition for successive decodability until level ℓ $\frac{a_\ell \gamma_\ell}{1+a_\ell \sum_{j=\ell+1}^{L} \gamma_j} \geq \tau$
- Average distortion
  $D_{\text{av}}(r_s, \gamma_\ell) = F_A(a_1) + \sum_{\ell=1}^{L} D_\ell \left(F_A(a_{\ell+1}) - F_A(a_\ell)\right)$
- Power profile as a function of the fading thresholds

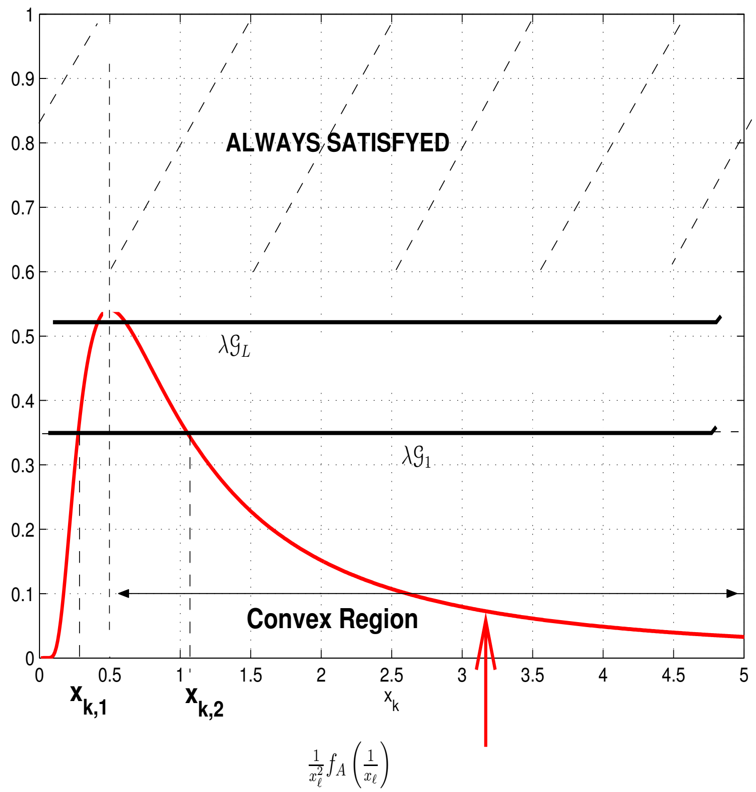
$$\gamma_\ell = \frac{\tau}{a_\ell} + \frac{\tau^2}{a_{\ell+1}} + \sum_{j=\ell+2}^{L} \frac{\tau^2}{a_j} (1+\tau)^{j-\ell-1}$$

- Problem: $\min_{r_s}\ \min_{a_\ell} D_{av}(a_\ell)$ subject to $\sum_{i=1}^{L} \gamma_\ell = \Gamma$

$$\Rightarrow \quad f_A(a_\ell) a_\ell^2 \leq \frac{\lambda \tau (1+\tau)^{\ell-1}}{D_{\ell-1} - D_\ell} = \lambda \mathcal{G}_\ell$$

Superposition Approach

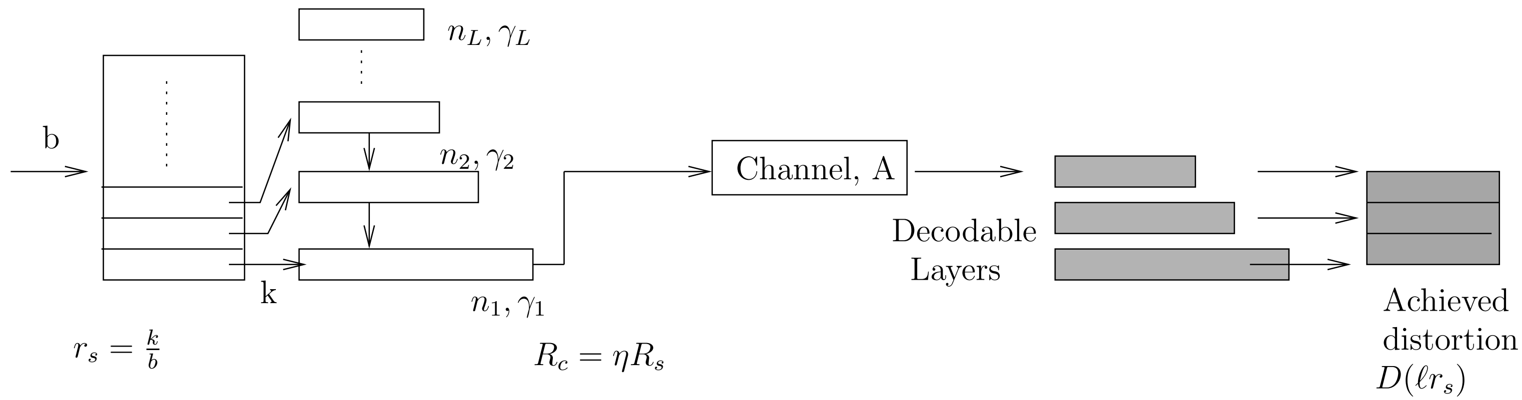
Rayleigh fading, fixed r_s, the set of solution is given by



Condition for optimality

$$\Rightarrow \quad f_A\left(\frac{1}{x_\ell}\right)\left(\frac{1}{x_\ell}\right)^2 \leq \lambda \mathcal{G}_\ell$$

Progressive Transmission



- The multilevel quantizer provides L levels of information bits each providing k bits.
- Each level is mapped into a channel codeword with rate $r_i = \frac{b}{n_i}$
- The codewords are sent using TDMA.
- The spectral efficiency is the same as for the superposition approach

Progressive Transmission

- Fix the spectral efficiency $\Rightarrow$ Code rate constraint $\sum_{i=1}^{L} \frac{1}{r_i} = \frac{1}{\eta r_s}$
- Call γ_ℓ the energy per channel symbol for the $\ell-$th codeword, $\Rightarrow$ Power constraint $\sum_{\ell=1}^{L} \frac{\gamma_\ell}{r_\ell} = \Gamma$
- Define a set of fading thresholds $0 < a_1 < a_2 \cdots < a_{L+1} = \infty$ such that codewords up ℓ can be correctly decoded if $A \in (a_\ell, a_{\ell+1})$
- Let $r_\ell = \log_2(1 + a_\ell \gamma_\ell)$ and call $x_\ell = \frac{1}{r_\ell}$ and $y_\ell = \frac{\gamma_\ell}{r_\ell}$.

Progressive Transmission

Minimization problem:

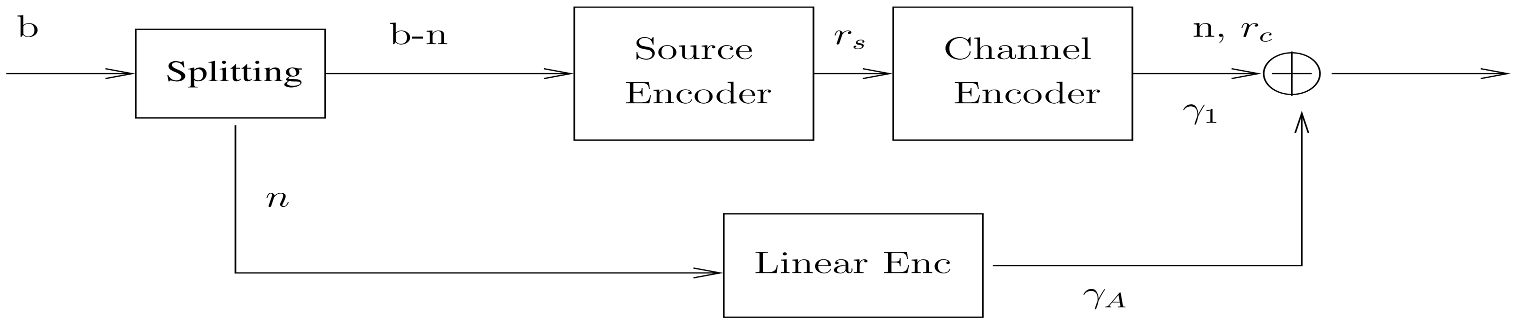
$\min_{r_s}\ \min_{\mathbf{x},\mathbf{y}} D_{av}(\mathbf{x},\mathbf{y})$ subject to $\sum_{i=1}^{L} x_\ell = \frac{1}{\eta r_s},;\quad \sum_{i=1}^{L} y_\ell = \frac{\Gamma}{\eta r_s}$

- $D_{\text{av}}(r_s, a_i) = F_A(a_1) + \sum_{\ell=1}^{L} D_\ell \left(F_A(a_{\ell+1}) - F_A(a_\ell)\right)$
- Lagrange Minimization of D_{av}, $\mu = \frac{\rho}{\lambda}$, Khun Tucker's conditions

$$\frac{\partial \Phi}{\partial x_\ell} = \Delta D_\ell e^{-\frac{(2^{1/x_\ell}-1)x_\ell}{y_\ell}} \left(\frac{\left(2^{1/x_\ell}-1\right)x_\ell - 2^{1/x_\ell}\ln 2}{x_\ell y_\ell} \right) + \lambda \geq 0$$

$$\frac{\partial \Phi}{\partial y_\ell} = \Delta D_\ell e^{-\frac{(2^{1/x_\ell}-1)x_\ell}{y_\ell}} \left(-\frac{x_\ell \left(2^{1/x_\ell}-1\right)}{y_\ell^2} \right) + \rho \geq 0$$

Hybrid Digital/Analog (HDA) Scheme



Phamdo[02]: **Nearly robust** HDA scheme based on bandwidth and power splitting between linear encoder and digital part ⇒ particular choice of γ_1, γ_A.

if $SNR > SNR^*$ digital part $\rightarrow D_1 = 2^{-2r_s}$ otherwise $D_0 = 1 \Rightarrow$ Optimization of the SNR threshold SNR^* to minimize D_{av}

Hybrid Digital/Analog (HDA) Scheme (cont'd)

- One digital layer ⇒ one fading level. Power constraint ⇒ $\gamma_1 + \gamma_A = \Gamma$
- The channel code is defined by (r_c, τ), s.t. $\tau = 2^{(\eta-1)r_s} - 1$
- Analog layer = noise ⇒ decodability if $\frac{a_1\gamma_1}{1+\gamma_A a_1} \geq \tau$

$$D_{\text{av}}(a_1) = \frac{\eta-1}{\eta}\left(F_A(a_1) + D_1(1 - F_A(a_1))\right) + \frac{1}{\eta}\left[\int_0^{a_1}\left(1 - \frac{a\gamma_A}{1+a\Gamma}\right) f_A(a)da + \int_{a_1}^{\infty}\frac{f_A(a)da}{1+a\gamma_A}\right]$$

Problem: $\min_{r_s}\ \min_{a_1} D_{\text{av}}(a_1)$ ⇒ find a_1 s.t. $\frac{\partial D_{av}}{\partial a_1} = 0$

Comparison

- Shannon's limit $\Rightarrow$ $D_{Sh} = \frac{1}{(1+a\Gamma)^{2/\eta}}$
- Fully separated approach with one digital layer
  - Problem: $\min_{r_s} D_{\text{sep}}$

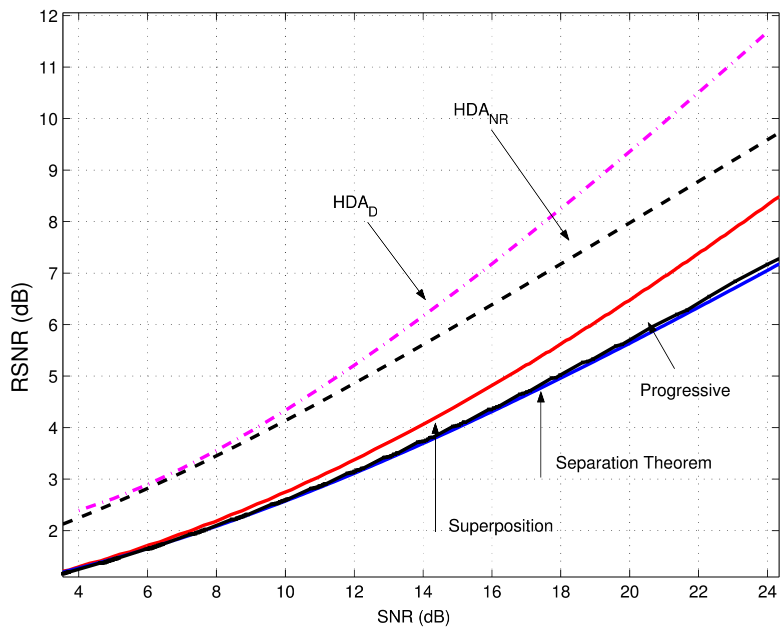
$$D_{\text{sep}} = F_A\left(\frac{2^{\eta r_s}-1}{\Gamma}\right) + 2^{-2r_s}\left[1 - F_A\left(\frac{2^{\eta r_s}-1}{\Gamma}\right)\right]$$

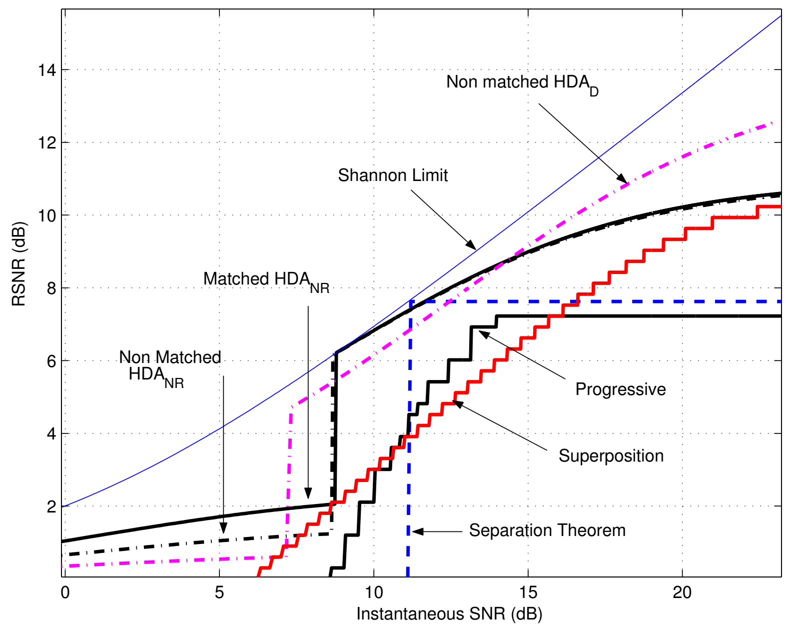
  - Rayleigh fading $\Rightarrow$ optimal r_s is the solution of

$$\left[\frac{\eta}{\Gamma}2^{r_s\eta} - \frac{1}{2^{2r_s}}\left(2 + \frac{\eta}{\Gamma}2^{r_s\eta}\right) = 0\right]$$

Results

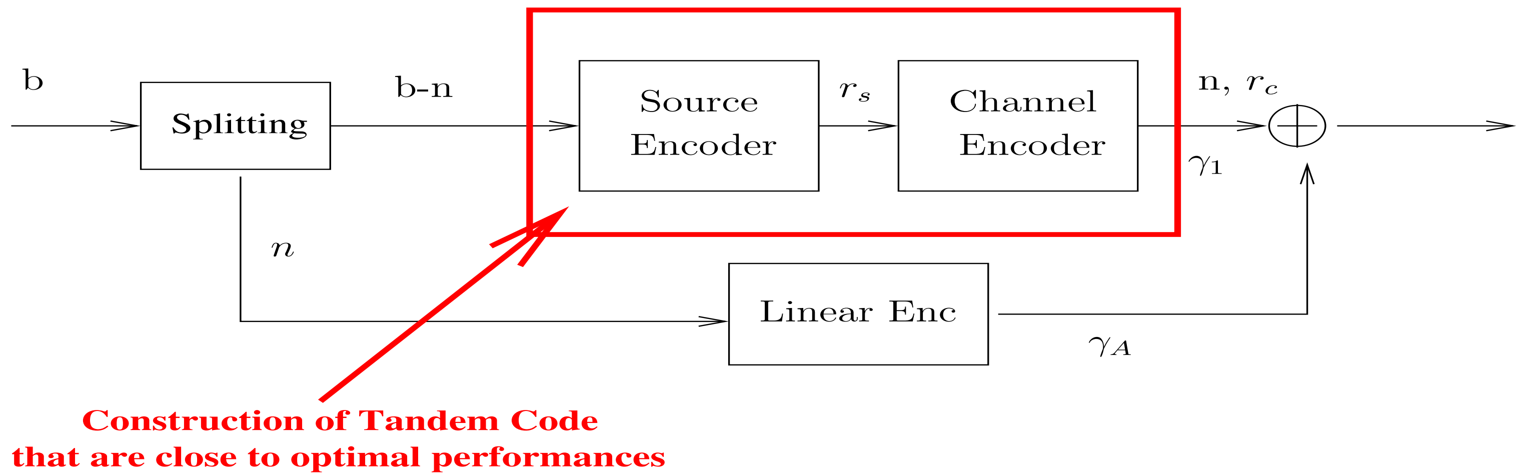
Performances for Superposition, Progressive and HDA schemes, $\Gamma = 20\text{dB}$





Code Construction

Best scheme ⇒ Optimized HDA to achieve minimum distortion.



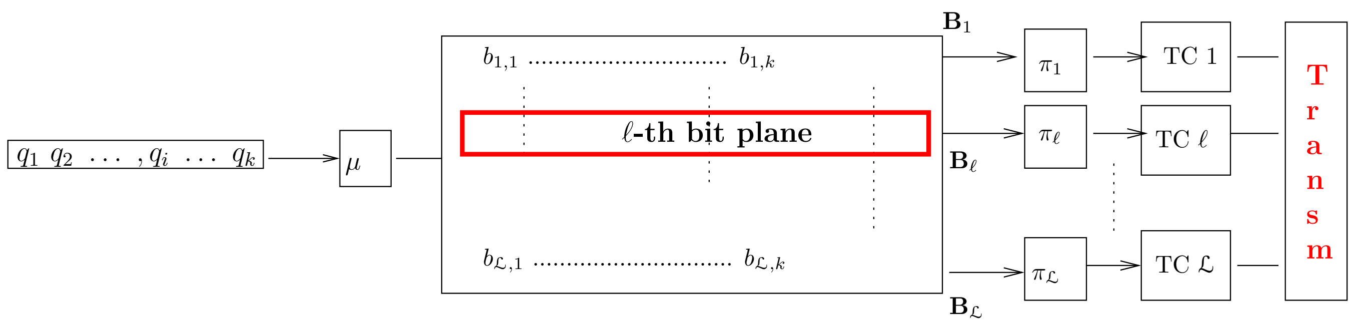
⇒ Use of **Entropy Constrained Scalar Quantizer** mapped on **Turbo Codes**

Entropy Constrained Scalar Quantization (ECSQ)

- $\mathcal{T}_i = (t_{i-1}, t_i)$ i-th quantization bin
- $\hat{s}_i$ i-th reconstruction level
- Consider the output entropy as a measure of rate
  $H(Q) = -\sum_{i=1}^{M} p_i \log_2 p_i \ \ (p_i = P(s \in \mathcal{T}_i)\ )$
- Problem: $\min D_{av}(t_i, \hat{s}_i)$, subject to $H(Q) \leq R$

Solution: $\log \frac{p_{i+1}}{p_i} = \lambda\,(\hat{s}_{i+1} - \hat{s}_i)\,(\hat{s}_{i+1} + \hat{s}_i + 2t_i)$, $\hat{s}_i = \mathbb{E}\,[s \,|\, s \in \mathcal{T}_i]$

Compression and Protection via Linear Codes



- The output of SQ is an index $q \in (0, \dots, \mathcal{Q}-1)$, with entropy $H(q)$
- Consider a mapping $\mu : \mathbb{Z}_\mathcal{Q} \to \mathbb{F}_2^\mathcal{L}$, s.t. $\mu(q) = (\mu_1(q), \dots, \mu_\mathcal{L}(q))$
- ℓ-th bitplane $\mathbf{B}_\ell = \mu_\ell(\mathbf{q})$
- Transmission by bitplanes

Compression and Protection via Linear Codes (cont'd)

- Linear codes provide compression
- Consider a linear systematic code with rate $R = k/n$, $\mathbf{c} = \mathbf{sG} = [\mathbf{sI} \,|\, \mathbf{sP}]$, where $p_i = P(s_i = 1)$ **side information**
- Compressed sequence is $\mathbf{x} = \mathbf{sP} \Rightarrow$ the parity part
- The channel codeword pass through a channel with parameter ρ $\Rightarrow \mathbf{y} = \mathbf{x} + \mathbf{n}$
- Decoder side: parity part $\Rightarrow \rho$, systematic part $\Rightarrow$ extrinsic channel p_i.

Compression and Protection via Turbo Codes (cont'd)

- A-priori probability for each level

$$P_\ell(b_{1:\ell-1}) = P(\mu_\ell(q) = 1 | \mu_{\ell-1}(q) = b_{\ell-1}, \ldots, \mu_1(q) = b_1)$$

- Conditional entropy at level ℓ

$$H_\ell = \sum_{b_{1:\ell-1}} \sum_{q \in \mathbb{Z}_Q : \mu_1^{\ell-1}(q) = b_{1:\ell-1}} P(q) h(P_\ell(b_{1:\ell-1}))$$

- Call $R_\ell = k/n_\ell$ the rate at level ℓ, $m_\ell = k(\frac{1}{R_\ell} - 1)$
- Channel rate $k/n \leq C$, Compression ratio $m_\ell/n \geq H_\ell$

$$k/m_\ell \leq C/H_\ell \quad \Rightarrow \quad R_\ell \leq \frac{C}{H_\ell + C}$$

- Spectral efficiency $\eta = \frac{k}{\sum_1^{\mathcal{L}} m_\ell} \leq \frac{C}{H(q)}$

Compression and Protection via Turbo Codes (cont'd)

Reconstruction: TC_i provides soft information $\text{APP}_i(q)$

$$\Downarrow$$

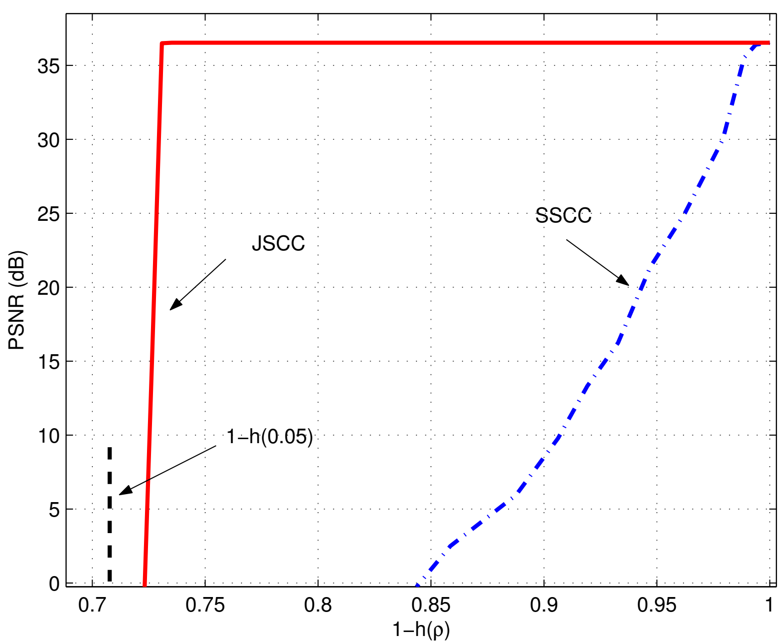
ℓ-th level reconstruction $\hat{q}_\ell = \sum_{q \in \mathbb{Z}_Q} \mu^{-1}(q)\mathbf{APP}_\ell(q)$

- This easily can be adapted to progressive transmission of information by defining an embedded mapping
- Example of result for a particular DPCM based source encoder.
  JSSC: DPCM $+\,\mu(.) +$ TC_ℓ,
  SSCC: DPCM + Arithmetic encoder + TC

Example of advantage

BSC channel $\rho = 0.05$, $PSNR = 10 \log_{10} \frac{255^2}{D}$

ℓ	$\bar{H}_\ell$	R_ℓ
1	0.7554	0.4858
2	0.1564	0.8202
3	0.4285	0.6248
4	0.8388	0.4597
5	0.1967	0.7839
6	0.0998	0.8773



Conclusions

- We have studied the problem of reliable transmission of analog sources aver a BF-AWGN channel
- Successive refinement of information is highly desirable for some broadcast applications
- Analysis of fully digital transmission strategies and hybrid digital/analog schemes $\Rightarrow$ Optimization to get minimum average distortion
- Best result given by HDA $\Rightarrow$ construction of tandem encoder
- Arithmetic code is not robust to channel errors
- Compression and protection via Turbo Codes yields big improvements