The Case for Non-cooperative Multihoming of Users to Access Points in IEEE 802.11 WLANs

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The IEEE 802.11 Cell

- Users compete for channel use: no fixed throughput.
- Throughput depends on number of users, PHY rates and frame sizes.
The Single Cell

• Call the cell $s$. Let the frame size be $L_i$ bits and PHY rate used be $C^s_i$ bits per slot.

• Transmission overhead $T_o$ and Collision overhead $T_c$. Uplink throughput is given by

$$\theta(i, n) = \frac{\beta e^{-n\beta L_i}}{1 + n\beta e^{-n\beta} \left( T_o - T_c + \frac{1}{n} \sum_{i=1}^{n} \frac{L_i}{C^s_i} \right) + \left( 1 - e^{-n\beta} \right) T_c}.$$
For large $n$ the uplink throughput of all users using $(L_i, C_{s_i})$ simplifies to

$$\tau(\alpha_i) = \frac{\alpha_i L_q}{\kappa + \sum_{j=1}^{Q} \frac{\alpha_j L_j}{C_{s_j}}}$$

where $\alpha_q(n) = m_q/n$ and $\kappa$ is a constant.
Multihoming

- Users have one NIC each.
- Can split traffic probabilistically.
• Three Independent Channels.
• Multihoming based on location and hardware.
Fluid Model

• A class $q$ of users have the same APs available and use the same values of $(L_q, C^s_q)$.

• We would like to consider the system deterministically and hence use a fluid model with infinitesimal users – a class is associated with a real valued mass $d_q$.

• The ratio in which the masses of are divided gives the probabilities of association.

• Ex: if 3 units of a class associate to one AP and 1 unit to another AP, then it means that the mixed strategy being played is $[\frac{3}{4}, \frac{1}{4}]$. 
Throughput

- Let there be $d_q$ users of class $q$. Let a fraction $x^s_q$ be connected to AP $s$. Total number of users connected to AP $s$ is $n^s = \sum_{q=1}^{Q} d_q x^s_q$.

- Scaling considered: $d_q = n \hat{d}_q$.

- $n$ can be interpreted as the sum of all demand.

- Throughput per unit mass for users of class $q$ in cell $s$:

$$T^s_q(y^s) \triangleq \frac{L_q}{\kappa \sum_{j=1}^{Q} y^s_j + \sum_{j=1}^{Q} y^s_j w^s_j},$$

where $y^s_q \triangleq \hat{d}_q x^s_q$ and $w^s_q \triangleq \frac{L^s}{C^s_q}$.
Population Games

• A population game $F$, with $Q$ non-atomic populations is defined by a mass $\tilde{d}_q$ and a strategy set for each population.

• A strategy distribution $y$ (or the state) is the way the players partition themselves into the different strategies available to them, i.e., $y = \{y_1, y_2, ..., y_Q\}$, where each $y_i$ is a vector.

• The marginal payoff per unit mass of class $q$ in cell $i$ when the system state is $y$ is denoted by $F_q^i(y)$. 
A state $\hat{y}$ is a **Wardrop equilibrium** if:

- for any population $q$, all strategies being used by the members of $q$ yield the same marginal payoff
- the marginal payoff that would be obtained by members of $q$ is lower for all strategies not used by population $q$.

Once you get there you stay there.
Selfish Dynamics

• Ever infinitesimal player would like to maximize his own payoff.
• The system state is the strategy distribution being played, and the payoff is a function of state.
• User strategies change with time as they adapt to the state – evolution.

How do I maximize my payoff?
Replicator Dynamics

• Successful strategies replicate, while unsuccessful ones die out – **Darwinism**.
• Measure of success of strategy $s$ is $\dot{y}_q^s / y_q^s$
• Replicator equation

$$\dot{y}_q^s = y_q^s \left( F_q^s(y) - \frac{1}{\tilde{d}_q} \sum_{i=1}^{S_q} y_q^i F_q^i(y) \right)$$

**Fitness of $s$**

**Average fitness**
Brown von Neumann Nash Dynamics

- Define

\[ \gamma_q^s = \max \left\{ F_q^s(y) - \frac{1}{\bar{d}_q} \sum_{i=1}^{S_q} y_q^i F_q^i(y), 0 \right\} \]

- BNN equation

\[ \dot{y}_q^s = \bar{d}_q \gamma_q^s - y_q^s \sum_{j=1}^{S_q} \gamma_j^s. \]

Increase proportional to excess payoff of \( s \)

Decrease proportional to sum of excess payoffs
Positive Correlation

• The dynamics $\dot{y} = V(y)$ are said to be positively correlated if

$$\sum_{k=1}^{Q} \sum_{i=1}^{S_k} F^i_k(y) V^i_k(y) > 0 \text{ whenever } V(y) \neq 0$$

• If $V(y)$ satisfies PC, then all Wardrop equilibria of $F$ are the stationary points of $\dot{y} = V(y)$. 
Both Replicator and BNN dynamics (and a combination thereof) are positively correlated.

\[ \sum_{k=1}^{Q} \sum_{i=1}^{S_k} F_k^i(y) V_k^i(y) \]

\[ = \sum_{k=1}^{Q} \sum_{i=1}^{S_k} F_k^i(y) y_k^i \left( F_k^i(y) - \frac{1}{\hat{d}_k} \sum_{j=1}^{S_k} y_k^j F_k^j(y) \right) \]

\[ = \sum_{k=1}^{Q} \hat{d}_k \left( \frac{1}{\hat{d}_k} \sum_{i=1}^{S_k} y_k^i \left( F_k^i(y) \right)^2 - \left( \frac{\sum_{i=1}^{S_k} y_k^i F_k^i(y)}{\hat{d}_k} \right)^2 \right) \]

Jensen
Potential Games

• We call $F$ a potential game if there exists a $C^1$ function $T : \mathcal{Y} \rightarrow \mathbb{R}$ such that

$$\frac{\partial T}{\partial y_q^i}(y) = F_q^i(y)$$

for all $y \in \mathcal{Y}$, $i \in \mathcal{S}_q$ and $q \in \mathcal{Q}$

• Follows that if $F$ is a potential game and the dynamics are PC, then the potential function acts as a Lyapunov function for $\dot{y} = V(y)$. 
Potential Function
What have we got?

• If we can find a potential function, then the dynamics will converge.
• The stationary point is either a Wardrop equilibrium or a boundary point of the strategy space.
• If we characterize the stationary point we will know how good or bad the equilibrium happens to be.
Association as a Population Game

- Each class corresponds to a population of users.
- The strategy distribution consists of dividing the masses amongst the available access points.
- Deterministic equivalent of a mixed strategy.
- We must define the payoff function.

- Users could follow replicator, BNN or a combination of both.
The service provider has to transfer traffic to an upstream provider.

We assume that the cost borne (throughput units) by the ISP is the total traffic generated by the users, i.e., it is equal to the system throughput

\[ T(y) \triangleq \sum_{k=1}^{S} \sum_{i=1}^{Q} \tau_{i}^{k}(y) = \sum_{k=1}^{S} \sum_{i=1}^{Q} y_{i}^{k} T_{i}^{k}(y) \]

The ISP would like to recover this cost, while maximizing the throughput.
Shared Resource

• The users compete for channel use not for throughput.

• Throughput depends on location and hardware → some users will make more efficient use and get higher throughput.

• Pricing should be based on the fraction of time that the channel is held by a user.
Channel Occupancy

• Ratio of time occupied by users of class $q$ to total time by all users:

$$\delta^s_q(y) \triangleq \frac{\kappa + w^s_q}{\kappa \sum_{j=1}^{Q} y^s_j + \sum_{j=1}^{Q} y^s_j w^s_j}$$

• Lower the occupancy $\rightarrow$ higher the efficiency
Cost Price Mechanism

• We find the cost in units of throughput.
• Cost of maintaining a unit mass of users of class $q$ is given by

\[ C^s_q(y) \triangleq \delta^s_q(y) \sum_{i=1}^{Q} \tau^s_i(y) \]

• Price charged is “proportionally” fair.

Occupancy \rightarrow \text{Total Throughput}
Why “Cost Price”? 

\[
\sum_{i=1}^{Q} C_i^s(y) y_i^s = \sum_{i=1}^{Q} \left( \delta_q^s(y) \sum_{j=1}^{Q} \tau_j^s(y) y_i^s \right)
\]

\[
= \sum_{j=1}^{Q} \tau_j^s(y) \sum_{i=1}^{Q} \delta_q^s(y) y_i^s
\]

\[
= \sum_{j=1}^{Q} \tau_j^s(y) \sum_{i=1}^{Q} \frac{(\kappa + w_q^s) y_i^s}{\kappa \sum_{j=1}^{Q} y_j^s + \sum_{j=1}^{Q} y_j^s w_j^s}
\]

\[
= \sum_{j=1}^{Q} \tau_j^s(y) \text{ Cost Price}
\]
User Payoff

- User payoff is the throughput obtained minus the cost

\[ F_q^s(y) \triangleq T_q^s(y) - C_q^s(y) \]

- Users take actions which would yield maximum payoffs.
Potential Function

- Total throughput acts as a potential function.

\[
\frac{\partial T(y)}{\partial y^s_q} = \frac{\partial}{\partial y^s_q} \sum_{k=1}^{S} \sum_{i=1}^{Q} y^k_i T^k_i(y)
\]

\[
= \frac{\partial}{\partial y^s_q} \sum_{k=1}^{S} \sum_{i=1}^{Q} \frac{y^k_i L_i}{\kappa \sum_{j=1}^{Q} y_j^k + \sum_{j=1}^{Q} y_j^k w_j^k}
\]

\[
T^s_q(y) = T^s_q(y) - \delta^s_q(y) \sum_{i=1}^{Q} \tau^s_i(y)
\]

\[
= F^s_q(y),
\]

Total Throughput

Occupancy

Throughput
Example: Non-Unique Maxima

- Only one class of users.
- Payoff per unit mass is zero in all cells.
- All states are Wardrop equilibria.
- Potential function is

\[ \sum_{k=1}^{S_q} \frac{L_q}{\kappa + w_q^k} \]

regardless of \( y \).
The Price of Anarchy

• We know that the state converges to a stationary point of the dynamics.
• What does such a point look like, i.e., what is the effect that selfish multihoming has on system throughput?
• Most work on selfish routing predicts that the system performance suffers due to selfish decision making — the price of anarchy.
Stationary point

- For both replicator and BNN dynamics the stationary point is

\[ F_q^S(\hat{y}) = \frac{1}{\hat{a}_q} \sum_{i=1}^{S_q} \hat{y}_q^i F_q^i(\hat{y}) \]

or

\[ \hat{y}_q^S = 0 \]

- Look like the Khun-Tucker first order conditions of an optimization problem.
• The Lagrange dual corresponding to the above

\[
\min_{\lambda} \max_y \left( \sum_{k=1}^{S} \sum_{i=1}^{Q} y_{i}^{k} T_{i}^{k}(y) - \sum_{i=1}^{Q} \lambda_i \left( \sum_{j=1}^{S_i} y_{i}^{j} - \tilde{d}_{i} \right) \right)
\]

Primal \leadsto Constraints

• It yields Khun-Tucker first order conditions with

\[
\lambda_i = \frac{1}{\tilde{d}_q} \sum_{q=1}^{S_q} \tilde{y}_q^i F_q^i(\hat{y})
\]
Primal Problem

The primal problem is

$$\max_y \left( \sum_{k=1}^{S} \sum_{i=1}^{Q} y_i^k T_i^k(y) \right)$$

Subject to the constraints

$$\sum_{j=1}^{S_i} y_{ij} = \tilde{d}_i \quad \forall i \in \{1, 2, \ldots, Q\}$$

System Throughput!
We have just shown...

• The solution of the primal problem is a state which maximizes throughput.
• Thus, under the cost price mechanism, selfish multihoming comes at zero cost – **anarchy is free!**
Economics of Multihoming

• Suppose that the ISP is not a disinterested player, but wants to make a profit.
• Is it in its interest to permit multihoming?
• Will efficiency still be greater when users are charged more than the “cost price”? Why should I allow multihoming?
Why should the ISP Allow Multihoming?

- Let the ISP charge more than the cost price – compare profits under unihoming and multihoming.
- Assume that users of class \( q \) would connect to AP \( s \) only if \( P^s \leq \Lambda^q \) (i.e., demand follows a threshold law).
- The user payoff \( F^s_q(y) \triangleq T^s_q(y) - C^s_q(y) - P^s \)
- The profit to the ISP is

\[
\rho_{\text{multi}}(P) \triangleq \sum_{j=1}^{S} P^j \sum_{i=1}^{Q} y^j_i
\]
Two Results

• The profit under multihoming is at least that achievable when unihoming – follows from the fact that total user mass in the system is the same for a given set of prices.

• The system throughput when multihoming is permitted is at least that of when it is not – follows from an argument similar to the primal-dual solution.
Conclusion

• Sought to make a convincing case to allow multihoming in WLANs.
• Constructed a fluid model of user populations.
• Showed that under the cost price mechanism, selfish dynamics actually maximize the system throughput.
• Economics from the ISPs perspective.
• Would multihoming be efficient in the Internet as a whole?
Bye!