Joint User Pairing and Resource Allocation for Multiuser SC-FDMA Transmission

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Outline

1. Introduction
2. System Model
3. Equalizers for SC-FDMA
5. Algorithms for Time / Frequency Domain Resource Allocation
6. Numerical Results
7. Conclusions
Basic Idea

System with

- $N_{\text{user}}$ users
- One transmit antenna per user
- One base station with $N_u$ antennas
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Considered Scenarios

User Pairing/Grouping:

- $N_u$ users transmit on the same time and frequency resource
- $N_u = 2 \Rightarrow$ user pairing
- $N_u > 2 \Rightarrow$ user grouping

Pairing/Grouping Criteria:

- Capacity
- BER

Resource Allocation:

- TDMA scheme: transmit in subsequent time slots, using entire bandwidth
- FDMA scheme: transmit in same time slot, on different frequency chunks
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**SC-FDMA Transmitter**

**Figure:** SC-FDMA transmitter structure for user $i$ in pair $o$.

- **$W$:** $M$-point DFT matrix
- **$K_o$:** subcarrier mapping matrix for group $o \in \{1, \ldots, O\}$
- **$V$:** $N$-point DFT matrix
- **$P_{in}$:** cyclic prefix insertion matrix
**System Model**

**SC-FDMA Receiver**

\[ n_l[\kappa] \rightarrow \text{CP removal} P_{out} \rightarrow N{-}\text{DFT} V \rightarrow Y_{l,1}[\mu] \]

\[ n_{NR}[\kappa] \rightarrow \text{CP removal} P_{out} \rightarrow N{-}\text{DFT} V \rightarrow Y_{NR,1}[\mu] \]

**Figure:** SC-FDMA BS receiver structure.

- \( n_l[\kappa] \): spatially and temporally white Gaussian noise at receive antenna \( l \)
- \( P_{out} \): cyclic prefix removal matrix
- \( V \): \( N \)-point DFT matrix
- \( K_o \): subcarrier mapping matrix for group \( o \in \{1, \ldots, O\} \)
V-MIMO Channel

Signal at the $l$th receive antenna:

$$r_{cl}[\kappa] = \sum_{i=1}^{N_{tx}} \sum_{\lambda=0}^{q_h} h_{l,i}[\lambda] \ b_{ci}[\kappa - \lambda] + n_l[\kappa]$$

$h_{l,i}[\lambda]$: discrete-time subchannel impulse response characterizes the transmission from user $i$ to the $l$th receive antenna including transmit and receiver input filtering

$\lambda \in \{0, 1, \ldots, q_h\}$

$q_h$: channel order

$N_{tx}$: number of users being transmitted within one subframe ($N_u$ for TDMA scheme, $N_{users}$ for FDMA scheme)
V-MIMO Channel

Signal at the $l$th receive antenna in matrix-vector notation:

\[
\mathbf{r}_{cl} = \sum_{i=1}^{N_{tx}} \mathbf{H}_{l,i} \mathbf{b}_{ci} + \mathbf{n}_l,
\]

$\mathbf{H}_{l,i}$: channel convolution matrix for transmission from user $i$ to receive antenna $l$
**Linear MMSE Equalizer for SC-FDMA**

- Filter matrix $F_o$ for each pair $o$ in frequency domain
- Error variance of the $p$th user within pair $o$:
  \[
  \sigma^2_{e_{u(o,p)}} = \frac{1}{M} \sum_{\mu=0}^{M-1} \Sigma_{pp}^{2} [\nu_{\text{offset}} + \nu_o + \mu]
  \]
- $\Sigma_{pp}^{2} [\nu]$: error variance on $\nu$th subcarrier for user $p$
Filter vectors $f_{o,\gamma}$ and $g_{o,\eta}$ for each user $\gamma$ and $\eta$ of pair $o$

Equalization order depends on SINR after equalization

$$\text{SINR}_{\text{biased, } p \{ o \}} = \frac{\sigma_{a_{u(o,p)}}^2}{\sigma_{e_{u(o,p)}}^2}$$

with

$$\sigma_{e_{u(o,p)}}^2 = \sigma_{a_{u(o,p)}}^2 - f_{o,p}^H \varphi_{y_o} a_{u(o,p)}[0]$$
Random Grouping and Resource Allocation

- Random grouping
- Random allocation of users to resources

⇒ Huge potential is wasted
Criteria for User Grouping and Resource Allocation

Capacity Grouping

Capacity of group $o$:

$$C_o = \frac{1}{M} \sum_{\mu=0}^{M-1} \log \det \left( \mathbf{I}_{NR} + \frac{\text{SNR}}{N_u} \mathbf{H}_o [\nu_{\text{offset}} + \nu_o + \mu] \right)$$

$$\times \mathbf{H}_o^H [\nu_{\text{offset}} + \nu_o + \mu]$$

Aim: find a mapping function $u(o, p)$ that maximizes the sum of the capacities of all individual groups:

$$C_{\text{sum}} = \sum_{o=1}^{O} C_o.$$
Approximation for BER of an uncoded transmission:

$$\text{BER} \approx \frac{N_{\text{min}}}{\text{ld}(M)} Q\left(\sqrt{d_{\text{min}}^2 \frac{E_b}{N_0}}\right),$$

- $N_{\text{min}}$: average number of nearest neighbors of a signal point of the modulation alphabet
- $M$: size of the modulation alphabet
- $Q(\cdot)$: complementary Gaussian error integral
- $d_{\text{min}}^2$: normalized minimum squared Euclidean distance
- $= 3 \text{ ld}(M)/(M - 1)$ for QAM constellations
Approximation for BER of an uncoded transmission:

\[ BER \approx \frac{N_{\min}}{\log_2(M)} Q \left( \sqrt{d_{\min}^2 \frac{E_b}{N_0}} \right), \]

BER for each user \( p \) within the group \( o \) can be approximated by

\[ BER_{u(o,p)} \approx \frac{N_{\min}}{\log_2(M)} Q \left( \sqrt{\zeta \text{SINR}_{ub,u(o,p)} G_c} \right), \]

\[ \zeta = \frac{3}{R (M-1)} \]

\( G_c \): gain of the channel code
Criteria for User Grouping and Resource Allocation

Bit Error Rate Grouping II

- Average BER for group $o$

$$\text{BER}\{o\} = \frac{1}{N_u} \sum_{p=1}^{N_u} \text{BER}_{u(o,p)},$$

- Average BER for all $N_{\text{user}}$ users is

$$\text{BER}_{\text{total}} = \frac{N_u}{N_{\text{user}}} \sum_{o=1}^{O} \text{BER}\{o\},$$
Two-dimensional optimization problem

- Hungarian Algorithm (HA) finds solution fast
- HA needs polynomial time vs. double factorial for full search
- Symmetric cost matrix with entries for each pair
Combined optimization of resource allocation and user pairing ⇒ three-dimensional optimization problem

A three-dimensional assignment problem is in general \( \mathcal{NP} \)-hard

Task: find that mapping function \( u(o, p) \), that either maximizes the sum capacity or minimizes the average BER for all users

Each value of \( o \in \{1, 2, \ldots, O\} \) stands for the number of the considered frequency chunk
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FD Resource Allocation – Full Search

- Compute sum capacity ($C_{\text{sum}}$) or BER for all possible groups ($\text{BER}_{\text{total}}$) and group/chunk combinations $u(o,p)$

- Choose combination with the maximum sum capacity/minimum average BER

- If the frequency bandwidth is divided into $O = N_{\text{user}}/N_u$ disjoint chunks the total number of combinations is

$$\Omega_{\text{FS}} = \binom{N_{\text{user}}}{N_u} \cdot \binom{N_{\text{user}} - N_u}{N_u} \cdot \ldots \cdot \binom{2 \cdot N_u}{N_u} \cdot \binom{N_u}{N_u}$$

$$= \frac{N_{\text{user}}!}{(N_u!)^{N_{\text{user}}/N_u}}$$
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**FD Resource Allocation – Hungarian Algorithm**

Before HA

<table>
<thead>
<tr>
<th>user 1</th>
<th>user 3</th>
<th>...</th>
<th>user $N_{\text{user}} - 1$</th>
</tr>
</thead>
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<tr>
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<td>user 4</td>
<td></td>
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After HA

<table>
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<tr>
<th>user $N_{\text{user}} - 1$</th>
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<th>...</th>
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<tr>
<td>user $N_{\text{user}}$</td>
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**Figure:** Example for result of HA for the optimization of the resource allocation of pairs.

HA finds the best assignment of $O$ given user groups to $O$ chunks.
FD Resource Allocation – Hungarian Algorithm

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An exchange of users of different groups can be realized by the binary switching algorithm (BSA).

Based on
- Simply try to exchange users between different groups
- Compute the resulting cost/weight function $BER_{\text{total}}/C_{\text{sum}}$
- Perform the exchange which yields the maximum decrease/increase of the cost/weight function of all considered trials

To limit complexity of the BSA ⇒ confine the number of users that can be switched simultaneously to $N_{\text{sw}}$

Number of total switches for $N_{\text{sw}} = 1$

$$O_{\text{BSA}} = N_u \cdot (N_{\text{user}} - N_u) + N_u \cdot (N_{\text{user}} - 2N_u) + \ldots + N_u \cdot N_u$$

$$= \frac{1}{2} \left( N_{\text{user}}^2 - N_u \cdot N_{\text{user}} \right).$$
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= \frac{1}{2}(N_{\text{user}}^2 - N_u \cdot N_{\text{user}}).
\]
FD Resource Allocation – Binary Switching Algorithm II

BSA with user 1

user 1  user 3  ...  user $N_{\text{user}} - 1$
user 2  user 4  ...  user $N_{\text{user}}$

BSA with user 2

user 1  user 3  ...  user $N_{\text{user}} - 1$
user 2  user 4  ...  user $N_{\text{user}}$

BSA with user 3

user 1  user 3  ...  user $N_{\text{user}} - 1$
user 2  user 4  ...  user $N_{\text{user}}$

RBs
### FD Resource Allocation – Binary Switching Algorithm II

#### BSA with user 1

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...
FD Resource Allocation – Combined HA+BSA I

- **Initialization**: random groups assigned to random chunks
- **First step**: use HA to find optimum allocation of given groups to chunks
  *BUT* HA does not exchange users between chunks
- **Second step**: BSA exchanges $N_{sw} = 1$ users between chunks
- **Iterations**: repeat first and second step for $N_{it}$ iterations
- **Convergence**: the algorithm is guaranteed in principle
  *BUT* resulting solution might not be the optimum one
Initialization: random groups assigned to random chunks

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Example for HA+BSA for the pairing and resource allocation of $N_{\text{user}} = 4$ users and $N_u = 2$ users per group.
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Example for HA+BSA for the pairing and resource allocation of $N_{\text{user}} = 4$ users and $N_u = 2$ users per group.
Number of computations of the capacity/BER for the HA+BSA with $N_{it}$ iterations

\[ Q_{HA+BSA} = N_{it} \cdot \left( \frac{1}{N_u^2} N_{user}^2 + (N_{user}^2 - N_u \cdot N_{user}) \right) \]

\[ = N_{it} \cdot \left( \left( \frac{1}{N_u^2} + 1 \right) N_{user}^2 - N_u \cdot N_{user} \right). \]
Complexity $Q$ of the full search and of the HA+BSA for $N_u = 4$ and $N_{it} \in \{1, 2, 5, 10, 20\}$, respectively.
BLER versus $\bar{E}_b/N_0$ for TD resource allocation, $N_{\text{user}} = 20$, $N_u = 2$, different pairing criteria, 10 MHz bandwidth.
Numerical Results

BLER versus $N_{\text{user}}$ for TD resource allocation, $\bar{E}_b/N_0 = 8$ dB, $N_u = 2$, different pairing criteria, MMSE LE, 10 MHz bandwidth.
Numerical Results

Numerical Results III

BLER versus $\bar{E}_b/N_0$ for FD res. alloc., $N_{user} = 10$, $N_u = 2$, $N_{it} = 20$, different pairing criteria, MMSE LE, 10 MHz bandwidth.
Numerical Results

Numerical Results IV

BLER versus $\bar{E}_b/N_0$ for FD resource allocation, $N_{\text{user}} = 8$, $N_u = 4$, $N_{\text{it}} = 10$, MMSE LE, 20 MHz bandwidth.
Numerical Results

Numerical Results V

BLER versus $\bar{E}_b/N_0$ for FD resource allocation, $N_{\text{user}} = 16$, $N_u = 4$, MMSE LE, 20 MHz bandwidth.
Numerical Results VI

BLER versus $\overline{E}_b/N_0$ for FD resource allocation, $N_{user} = 20$, $N_u = 2$, BER grouping, MMSE LE, 20 MHz bandwidth.
Random user grouping and resource allocations wastes huge potential

Hungarian Algorithm finds optimum solution for two dimensional time domain resource allocation and user pairing problem

Full search finds optimum solution for three dimensional frequency domain resource allocation and user grouping problem with extremely high complexity

Proposed iterative HA+BSA finds close to optimum solution for frequency domain resource allocation and user grouping problem at much lower complexity
Random user grouping and resource allocations wastes huge potential

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Conclusions

Future Work

- Lower bound for capacity pairing (orthogonality in random Gaussian channels, famous balls-and-bins problem)
- Power allocation not yet implemented
- Rate adaptation not considered
- $4 \times 4$ V-MIMO with two transmit antennas per mobile
- $\Rightarrow$ Use of network simulator necessary
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