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Joint User Pairing and Resource Allocation for Multiuser SC-FDMA Transmission

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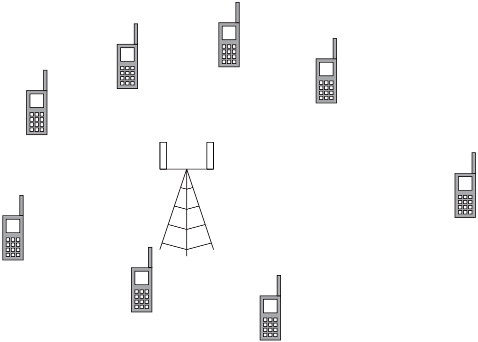
Outline

1. Introduction
2. System Model
3. Equalizers for SC-FDMA
4. Criteria for User Grouping and Resource Allocation
5. Algorithms for Time / Frequency Domain Resource Allocation
6. Numerical Results
7. Conclusions

Basic Idea

System with

- N_{user} users
- One transmit antenna per user
- One base station with N_{u} antennas



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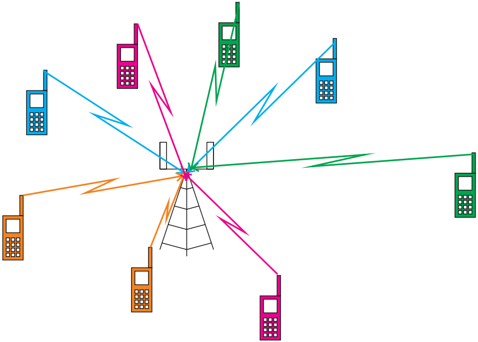

Considered Scenarios

- User Pairing/Grouping:
  - N_u users transmit on the same time and frequency resource
  - $N_\mathrm{u} = 2 \Rightarrow$ user pairing
  - $N_\mathrm{u} > 2 \Rightarrow$ user grouping
- Pairing/Grouping Criteria:
  - Capacity
  - BER
- Resource Allocation:
  - TDMA scheme: transmit in subsequent time slots, using entire bandwidth
  - FDMA scheme: transmit in same time slot, on different frequency chunks

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SC-FDMA Transmitter

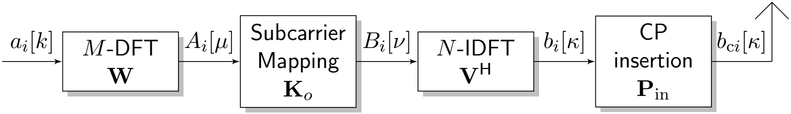


Figure: SC-FDMA transmitter structure for user i in pair o.

$\mathbf{W}$: M-point DFT matrix
$\mathbf{K}_o$: subcarrier mapping matrix for group $o \in \{1, \ldots, O\}$
$\mathbf{V}$: N-point DFT matrix
$\mathbf{P}_{\mathrm{in}}$: cyclic prefix insertion matrix

SC-FDMA Receiver

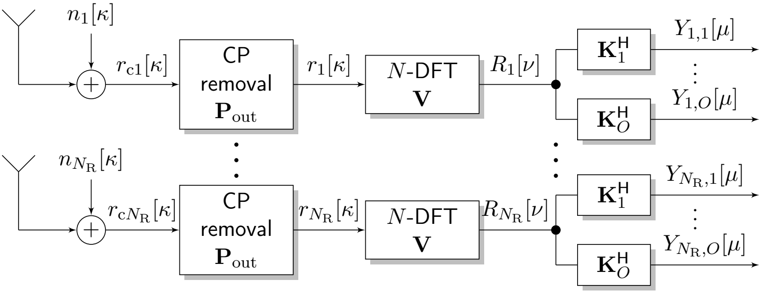


Figure: SC-FDMA BS receiver structure.

$n_l[\kappa]$: spatially and temporally white Gaussian noise at receive antenna l
$\mathbf{P}_{\text{out}}$: cyclic prefix removal matrix
$\mathbf{V}$: N-point DFT matrix
$\mathbf{K}_o$: subcarrier mapping matrix for group $o \in \{1, \ldots, O\}$

V-MIMO Channel

Signal at the lth receive antenna:

$$r_{cl}[\kappa] = \sum_{i=1}^{N_{tx}} \sum_{\lambda=0}^{q_h} h_{l,i}[\lambda]\, b_{ci}[\kappa - \lambda] + n_l[\kappa]$$

$h_{l,i}[\lambda]$: discrete-time subchannel impulse response characterizes the transmission from user i to the lth receive antenna including transmit and receiver input filtering

λ $\in \{0,\ 1,\ \ldots,\ q_h\}$

q_h: channel order

N_{tx}: number of users being transmitted within one subframe (N_u for TDMA scheme, N_{users} for FDMA scheme)

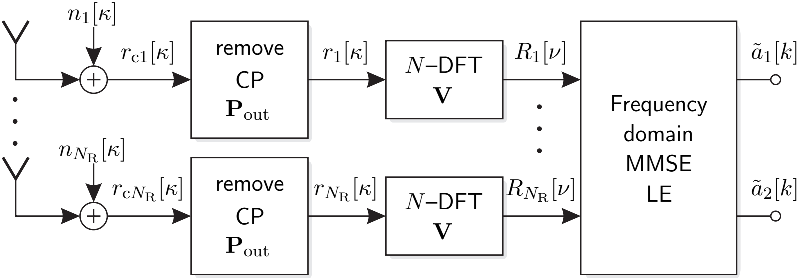
V-MIMO Channel

Signal at the lth receive antenna in matrix-vector notation:

$$\mathbf{r}_{\mathrm{c}l} = \sum_{i=1}^{N_{\mathrm{tx}}} \mathbf{H}_{l,i}\, \mathbf{b}_{\mathrm{c}i} + \mathbf{n}_l,$$

$\mathbf{H}_{l,i}$: channel convolution matrix for transmission from user i to receive antenna l

Linear MMSE Equalizer for SC-FDMA



- Filter matrix $\mathbf{F}_o$ for each pair o in frequency domain
- Error variance of the pth user within pair o:

$$\sigma^2_{e_{u(o,p)}} = \frac{1}{M} \sum_{\mu=0}^{M-1} \Sigma^2_{pp}[\nu_{\text{offset}} + \nu_o + \mu]$$

- $\Sigma^2_{pp}[\nu]$: error variance on νth subcarrier for user p

SIC Equalizer for SC-FDMA

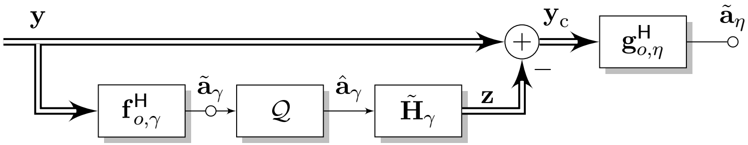


Figure: SC-FDMA SIC equalizer structure for two users.

- Filter vectors $\mathbf{f}_{o,\gamma}$ and $\mathbf{g}_{o,\eta}$ for each user γ and η of pair o
- Equalization order depends on SINR after equalization

$$\mathrm{SINR}_{\mathrm{biased},p}\{o\} = \sigma^2_{a_{u(o,p)}} / \sigma^2_{e_{u(o,p)}}$$

with

$$\sigma^2_{e_{u(o,p)}} = \sigma^2_{a_{u(o,p)}} - \mathbf{f}_{o,p}^{\mathsf{H}}\, \boldsymbol{\varphi}_{\mathbf{y}_o a_{u(o,p)}}[0]$$

Random Grouping and Resource Allocation

- Random grouping
- Random allocation of users to resources

$\Rightarrow$ Huge potential is wasted

Capacity Grouping

Capacity of group o:

$$C_o = \frac{1}{M} \sum_{\mu=0}^{M-1} \mathrm{ld} \det \Big(\mathbf{I}_{N_\mathrm{R}} + \frac{\mathrm{SNR}}{N_\mathrm{u}} \, \mathbf{H}_o[\nu_\mathrm{offset} + \nu_o + \mu] \times \mathbf{H}_o^\mathsf{H}[\nu_\mathrm{offset} + \nu_o + \mu] \Big)$$

Aim: find a mapping function $u(o, p)$ that maximizes the sum of the capacities of all individual groups:

$$C_\mathrm{sum} = \sum_{o=1}^{O} C_o.$$

Bit Error Rate Grouping I

- Approximation for BER of an uncoded transmission:

$$\mathrm{BER} \approx \frac{N_{\min}}{\mathrm{ld}(\mathcal{M})} \mathrm{Q}\left(\sqrt{d^2_{\min} \frac{E_\mathrm{b}}{\mathcal{N}_0}}\right),$$

$N_{\min}$: average number of nearest neighbors of a signal point of the modulation alphabet

$\mathcal{M}$: size of the modulation alphabet

$\mathrm{Q}(\cdot)$: complementary Gaussian error integral

$d^2_{\min}$: normalized minimum squared Euclidean distance $= 3\,\mathrm{ld}(\mathcal{M})/(\mathcal{M}-1)$ for QAM constellations

Bit Error Rate Grouping I

- Approximation for BER of an uncoded transmission:

$$\mathrm{BER} \approx \frac{N_{\min}}{\mathrm{ld}(\mathcal{M})} \mathrm{Q}\left(\sqrt{d_{\min}^2 \frac{E_{\mathrm{b}}}{\mathcal{N}_0}}\right),$$

- BER for each user p within the group o can be approximated by

$$\mathrm{BER}_{u(o,p)} \approx \frac{N_{\min}}{\mathrm{ld}(\mathcal{M})} \mathrm{Q}\left(\sqrt{\zeta \; \mathrm{SINR}_{\mathrm{ub},u(o,p)} \; G_{\mathrm{c}}}\right),$$

$\zeta \quad = \frac{3}{R\,(\mathcal{M}-1)}$

G_{c}: gain of the channel code

Bit Error Rate Grouping II

- Average BER for group o

$$\mathrm{BER}\{o\} = 1/N_{\mathrm{u}} \sum_{p=1}^{N_{\mathrm{u}}} \mathrm{BER}_{u(o,p)},$$

- Average BER for all N_{user} users is

$$\mathrm{BER}_{\mathrm{total}} = \frac{N_{\mathrm{u}}}{N_{\mathrm{user}}} \sum_{o=1}^{O} \mathrm{BER}\{o\},$$

Algorithm for Time Domain Resource Allocation

- Two-dimensional optimization problem
- Hungarian Algorithm (HA) finds solution fast
- HA needs polynomial time vs. double factorial for full search
- Symmetric cost matrix with entries for each pair

FD Resource Allocation – Problem Formulation

- Combined optimization of resource allocation and user pairing
  $\Rightarrow$ three-dimensional optimization problem
- A three-dimensional assignment problem is in general $\mathcal{NP}$-hard
- Task: find that mapping function $u(o, p)$, that either maximizes the sum capacity or minimizes the average BER for all users
- Each value of $o \in \{1,\ 2,\ \ldots,\ O\}$ stands for the number of the considered frequency chunk

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FD Resource Allocation – Full Search

- Compute sum capacity (C_{sum}) or BER for all possible groups ($\mathrm{BER}_{\mathrm{total}}$) and group/chunk combinations $u(o,p)$
- Choose combination with the maximum sum capacity/minimum average BER
- If the frequency bandwidth is divided into $O = N_{\mathrm{user}}/N_{\mathrm{u}}$ disjoint chunks the total number of combinations is

$$\mathcal{O}_{\mathrm{FS}} = \binom{N_{\mathrm{user}}}{N_{\mathrm{u}}} \cdot \binom{N_{\mathrm{user}} - N_{\mathrm{u}}}{N_{\mathrm{u}}} \cdots \binom{2 \cdot N_{\mathrm{u}}}{N_{\mathrm{u}}} \cdot \binom{N_{\mathrm{u}}}{N_{\mathrm{u}}}$$
$$= \frac{N_{\mathrm{user}}!}{(N_{\mathrm{u}}!)^{N_{\mathrm{user}}/N_{\mathrm{u}}}}$$

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FD Resource Allocation – Hungarian Algorithm

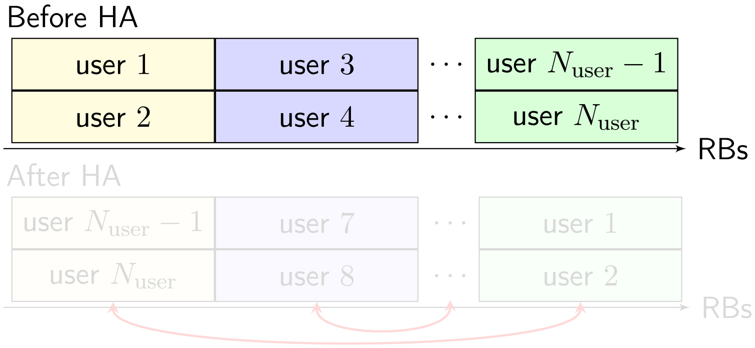

Figure: Example for result of HA for the optimization of the resource allocation of pairs.

HA finds the best assignment of O given user groups to O chunks.

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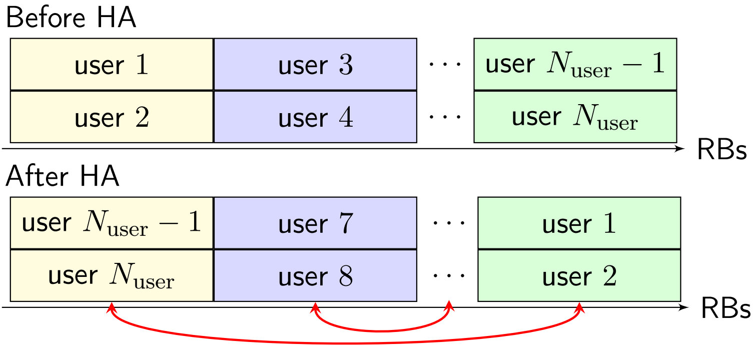

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FD Resource Allocation – Binary Switching Algorithm I

- An exchange of users of different groups can be realized by the binary switching algorithm (BSA)
- Based on
  - Simply try to exchange users between different groups
  - Compute the resulting cost/weight function $\mathrm{BER}_{\mathrm{total}}/C_{\mathrm{sum}}$
  - Perform the exchange which yields the maximum decrease/increase of the cost/weight function of all considered trials
- To limit complexity of the BSA $\Rightarrow$ confine the number of users that can be switched simultaneously to N_{sw}
- Number of total switches for $N_{\mathrm{sw}} = 1$

$$\mathcal{O}_{\mathrm{BSA}} = N_{\mathrm{u}} \cdot (N_{\mathrm{user}} - N_{\mathrm{u}}) + N_{\mathrm{u}} \cdot (N_{\mathrm{user}} - 2N_{\mathrm{u}}) + \ldots + N_{\mathrm{u}} \cdot N_{\mathrm{u}}$$
$$= \frac{1}{2}(N_{\mathrm{user}}^2 - N_{\mathrm{u}} \cdot N_{\mathrm{user}}).$$

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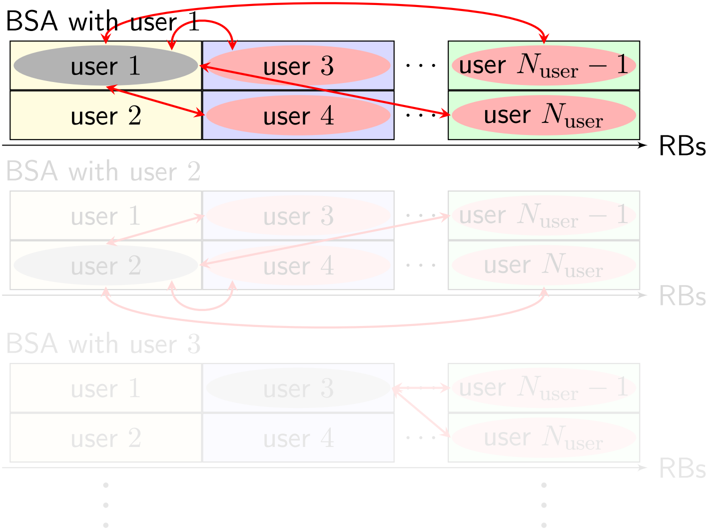
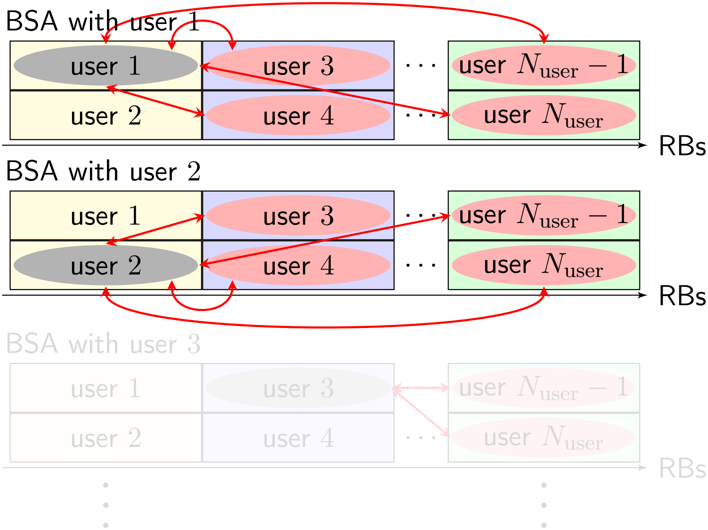
FD Resource Allocation – Binary Switching Algorithm II



FD Resource Allocation – Binary Switching Algorithm II



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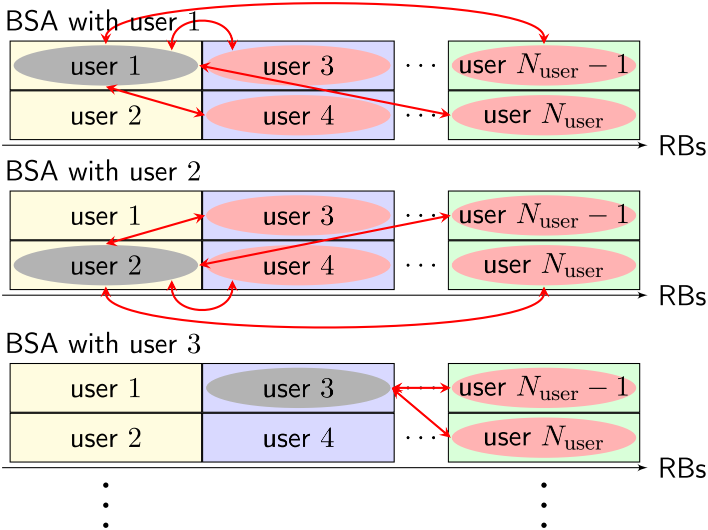

FD Resource Allocation – Combined HA+BSA I

- Initialization: random groups assigned to random chunks
- First step: use HA to find optimum allocation of given groups to chunks
  BUT HA does not exchange users between chunks
- Second step: BSA exchanges $N_{\mathrm{sw}} = 1$ users between chunks
- Iterations: repeat first and second step for N_{it} iterations
- Convergence of the algorithm is guaranteed in principle
  BUT resulting solution might not be the optimum one

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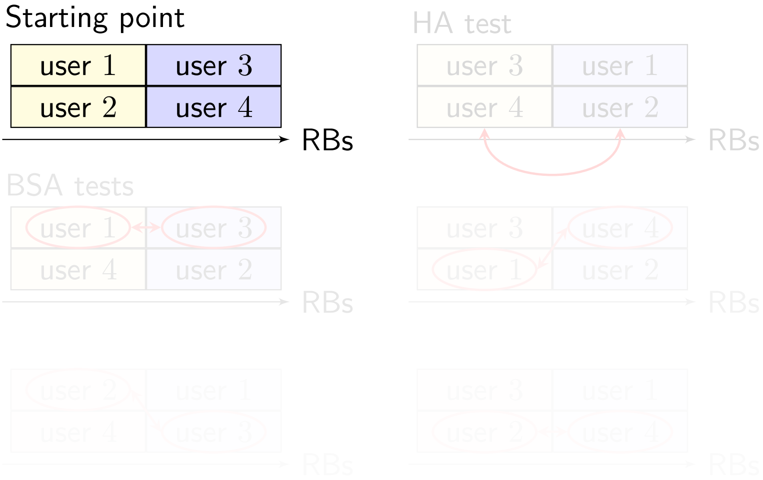
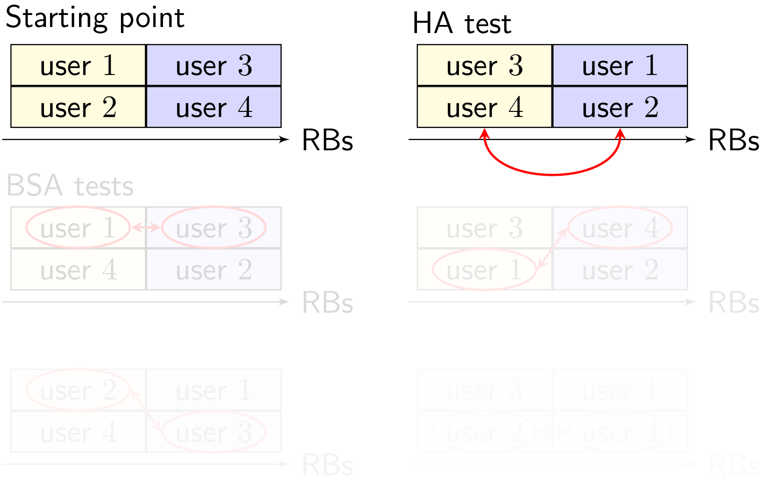


Example for HA+BSA for the pairing and resource allocation of $N_{\text{user}} = 4$ users and $N_{\text{u}} = 2$ users per group.

FD Resource Allocation – Combined HA+BSA II



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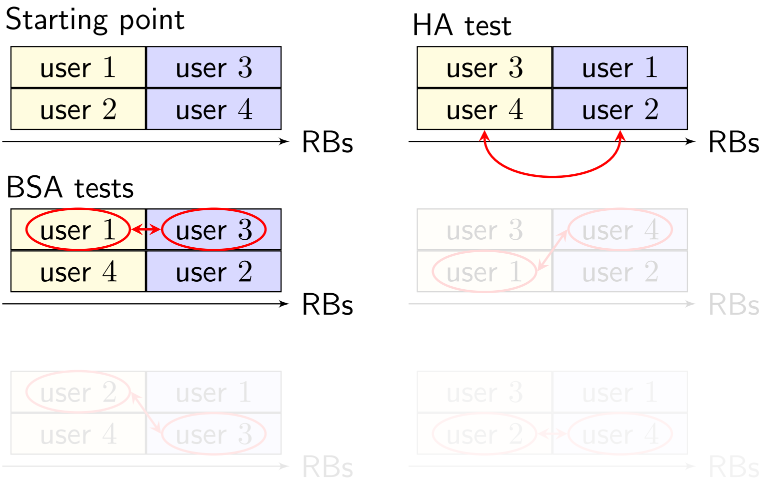

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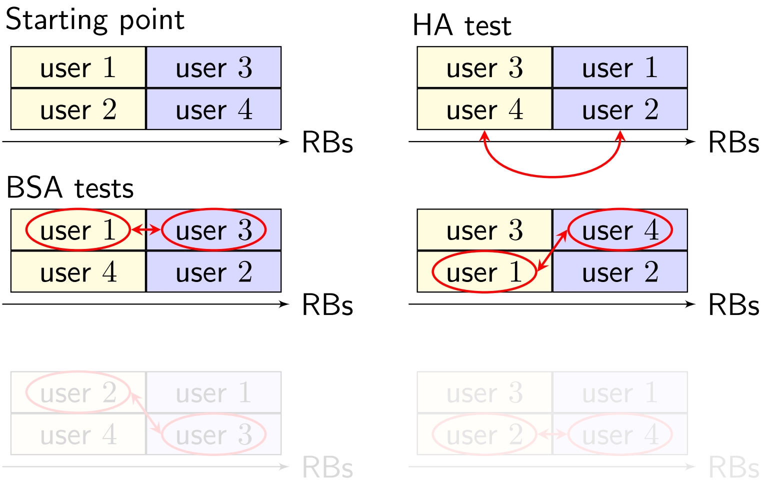

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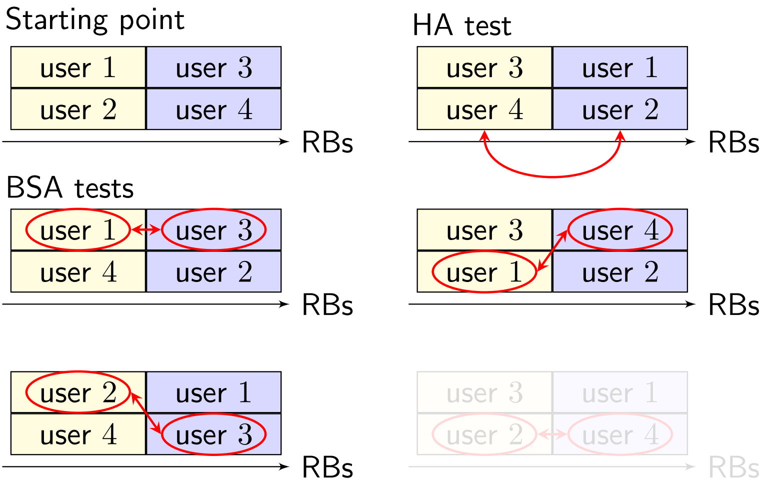

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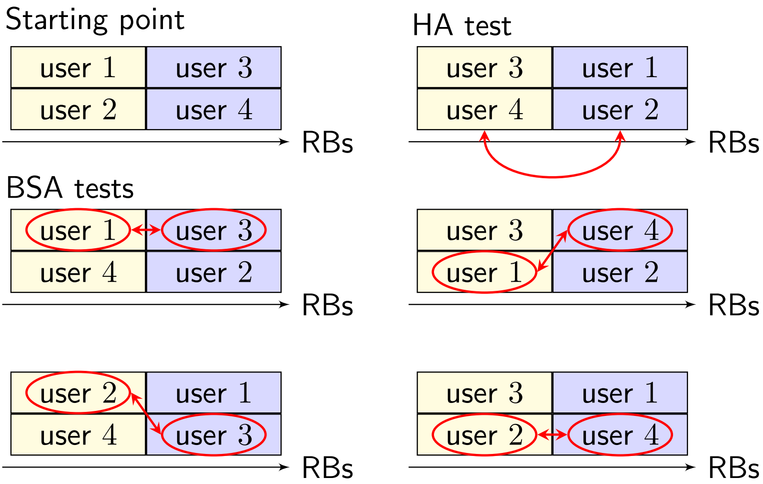


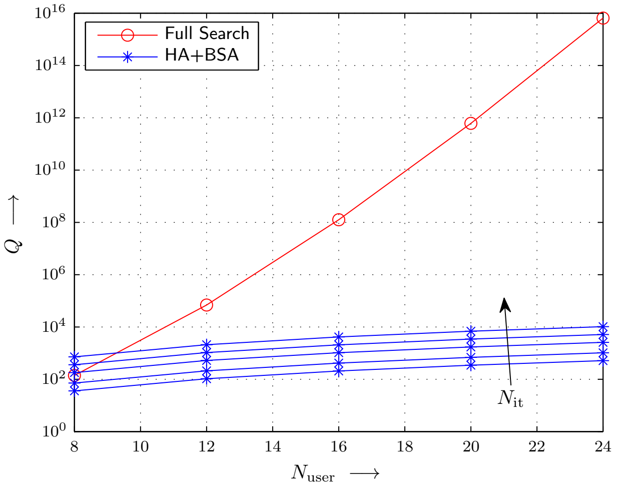
Example for HA+BSA for the pairing and resource allocation of $N_{\text{user}} = 4$ users and $N_{\text{u}} = 2$ users per group.

FD Resource Allocation – Complexity Analysis

Number of computations of the capacity/BER for the HA+BSA with N_{it} iterations

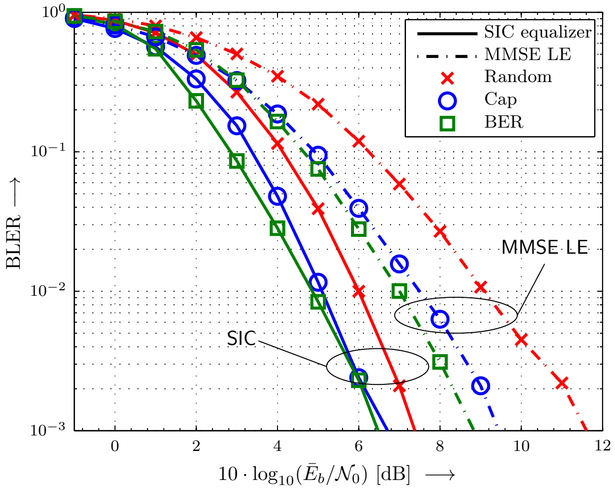
$$\begin{aligned} Q_{\mathrm{HA+BSA}} =& N_{\mathrm{it}} \cdot \left(\frac{1}{{N_{\mathrm{u}}}^2} N_{\mathrm{user}}^2 + (N_{\mathrm{user}}^2 - N_{\mathrm{u}} \cdot N_{\mathrm{user}}) \right) \\ =& N_{\mathrm{it}} \cdot \left(\left(\frac{1}{{N_{\mathrm{u}}}^2} + 1 \right) N_{\mathrm{user}}^2 - N_{\mathrm{u}} \cdot N_{\mathrm{user}} \right). \end{aligned}$$

FD Resource Allocation – Complexity Analysis



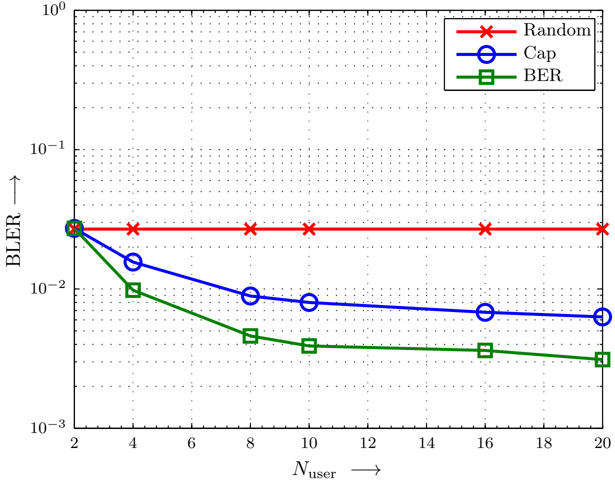
Complexity Q of the full search and of the HA+BSA for $N_{\text{u}} = 4$ and $N_{\text{it}} \in \{1,\ 2,\ 5,\ 10,\ 20\}$, respectively.

Numerical Results I



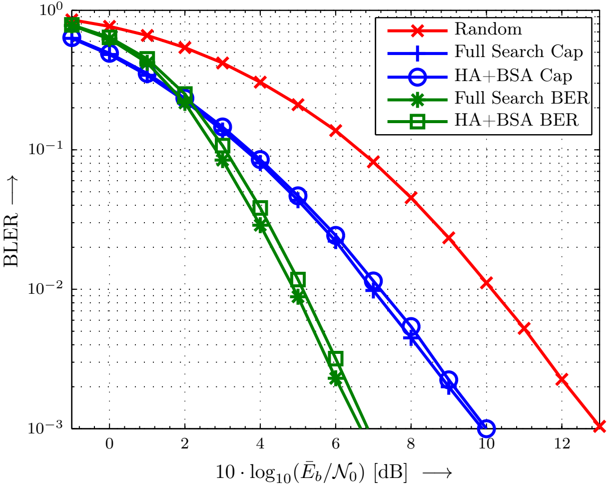
BLER versus $\bar{E}_{\mathrm{b}}/\mathcal{N}_0$ for TD resource allocation, $N_{\mathrm{user}} = 20$, $N_{\mathrm{u}} = 2$, different pairing criteria, 10 MHz bandwidth.

Numerical Results II



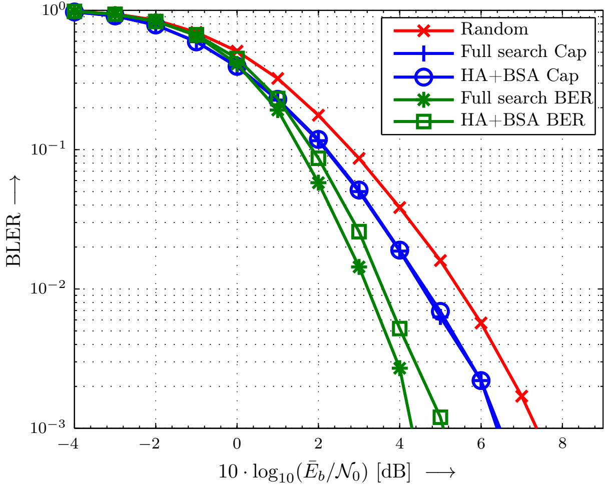
BLER versus N_{user} for TD resource allocation, $\bar{E}_{\text{b}}/\mathcal{N}_0 = 8$ dB, $N_{\text{u}} = 2$, different pairing criteria, MMSE LE, 10 MHz bandwidth.

Numerical Results III



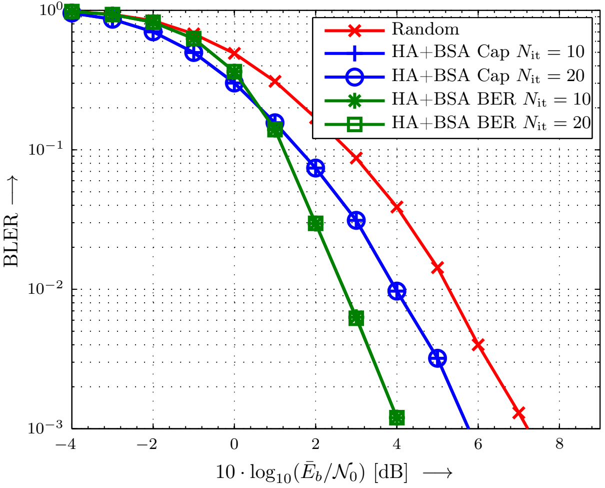
BLER versus $\bar{E}_\text{b}/\mathcal{N}_0$ for FD res. alloc., $N_\text{user} = 10$, $N_\text{u} = 2$, $N_\text{it} = 20$, different pairing criteria, MMSE LE, 10 MHz bandwidth.

Numerical Results IV



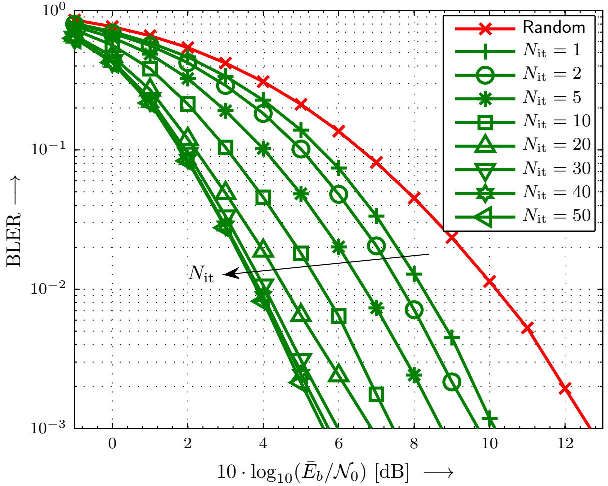
BLER versus $\bar{E}_\mathrm{b}/\mathcal{N}_0$ for FD resource allocation, $N_\mathrm{user} = 8$, $N_\mathrm{u} = 4$, $N_\mathrm{it} = 10$, MMSE LE, 20 MHz bandwidth.

Numerical Results V



BLER versus $\bar{E}_{\rm b}/\mathcal{N}_0$ for FD resource allocation, $N_{\rm user} = 16$, $N_{\rm u} = 4$, MMSE LE, 20 MHz bandwidth.

Numerical Results VI



BLER versus $\bar{E}_{\mathrm{b}}/\mathcal{N}_0$ for FD resource allocation, $N_{\mathrm{user}} = 20$, $N_{\mathrm{u}} = 2$, BER grouping, MMSE LE, 20 MHz bandwidth.

Summary

- Random user grouping and resource allocations wastes huge potential
- Hungarian Algorithm finds optimum solution for two dimensional time domain resource allocation and user pairing problem
- Full search finds optimum solution for three dimensional frequency domain resource allocation and user grouping problem with extremely high complexity
- Proposed iterative HA+BSA finds close to optimum solution for frequency domain resource allocation and user grouping problem at much lower complexity

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Future Work

- Lower bound for capacity pairing (orthogonality in random Gaussian channels, famous balls-and-bins problem)
- Power allocation not yet implemented
- Rate adaptation not considered
- 4×4 V-MIMO with two transmit antennas per mobile
- $\Rightarrow$ Use of network simulator necessary

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