

Channel Polarization and Polar Coding

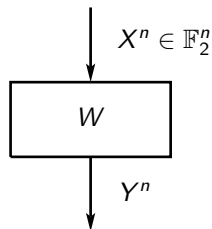
Erdal Arıkan

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Bilkent University
Ankara, Turkey

EURECOM
Sophia Antipolis
2 July 2010

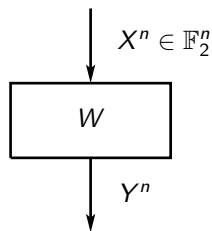
Channel Polarization

- Given a binary input DMC W ,
- i.i.d. uniformly distributed inputs $(X_1, \dots, X_n) \in \{0, 1\}^n$,
- in one-to-one correspondence with binary 'data' $(U_1, \dots, U_n) \in \{0, 1\}^n$.
- Observe that U_i are i.i.d., uniform on $\{0, 1\}$.



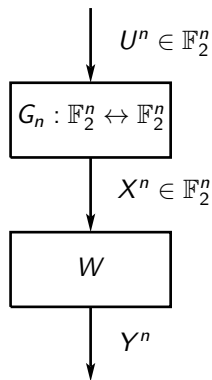
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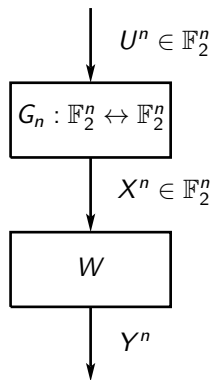
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Channel Polarization

$$\begin{aligned} I(U^n; Y^n) &= I(X^n; Y^n) \\ &= \sum_i I(X_i; Y_i) \\ &= \sum_i I(W) \end{aligned}$$

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Notation: $I(P)$ denotes the mutual information between the input and output of a channel P when input is uniformly distributed.

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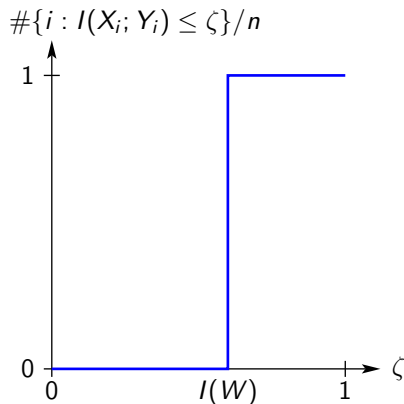
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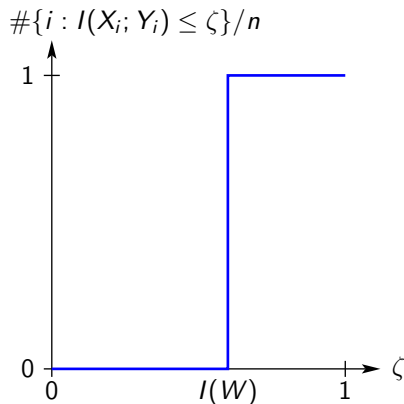


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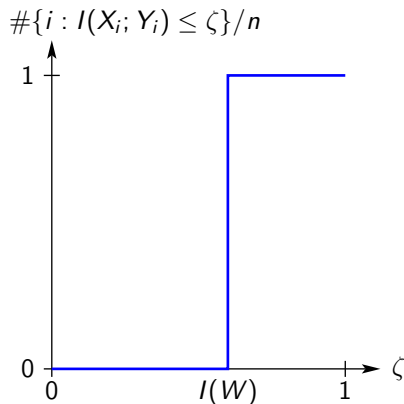


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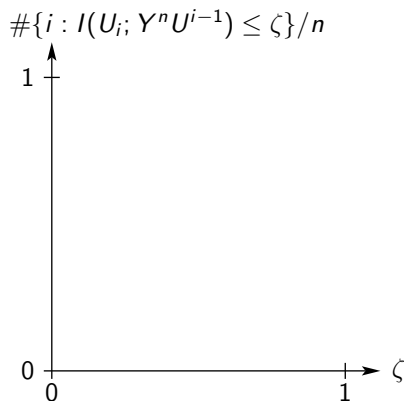


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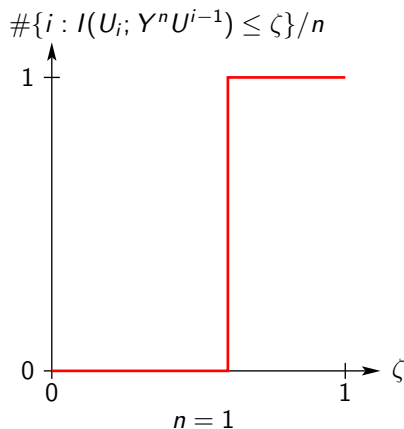


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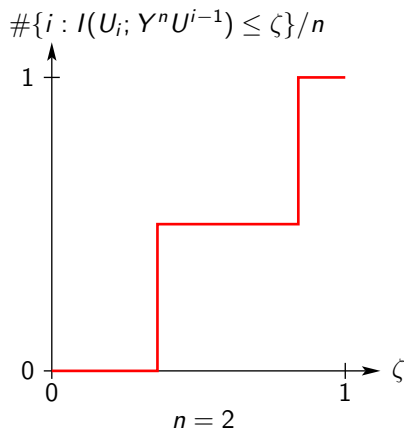


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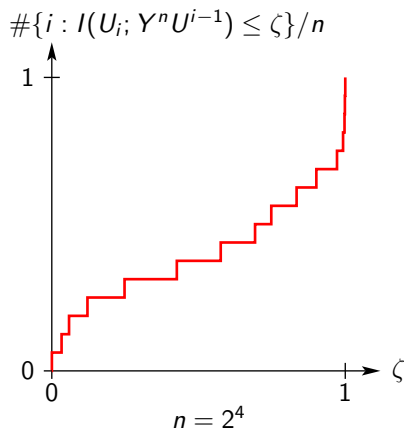


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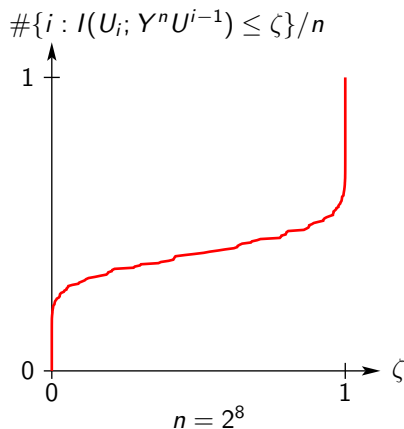


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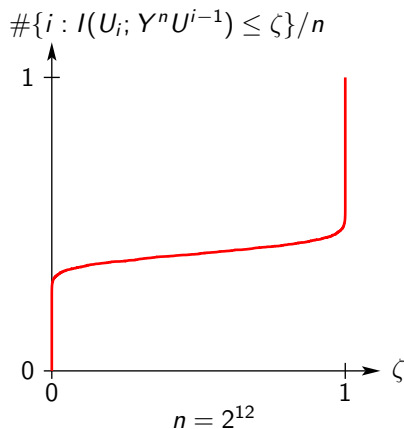


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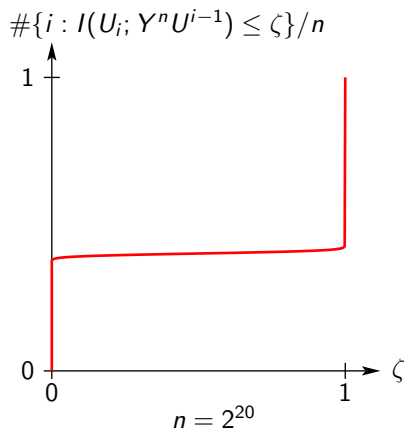


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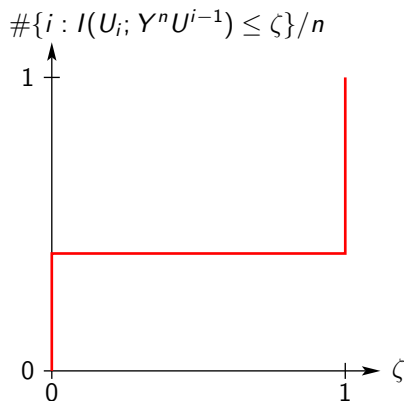


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We say **channel polarization** takes place if it is the case that almost all of the numbers $I(U_i; Y^n U^{i-1})$ are near the extremal values, i.e., if

$$\frac{1}{n} \#\{i : I(U_i; Y^n U^{i-1}) \approx 1\} \rightarrow I(W)$$

and

$$\frac{1}{n} \#\{i : I(U_i; Y^n U^{i-1}) \approx 0\} \rightarrow 1 - I(W).$$

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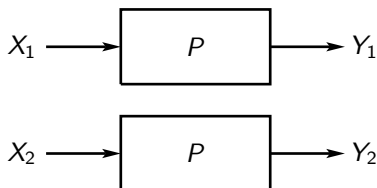
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Given two copies of a binary input channel $P : \mathbb{F}_2 \rightarrow \mathcal{Y}$



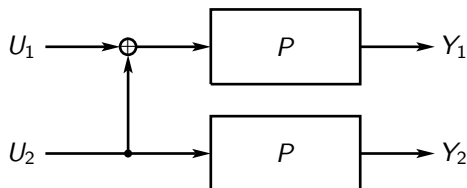
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$$P^-(y_1 y_2 | u_1) = \sum_{u_2} \frac{1}{2} P(y_1 | u_1 + u_2) P(y_2 | u_2)$$

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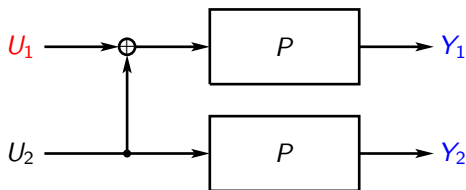
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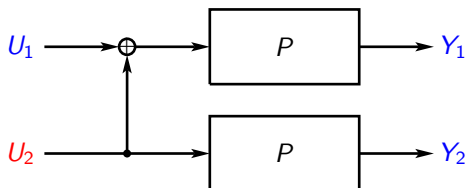
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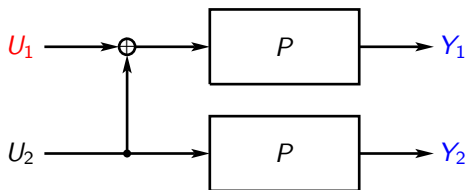
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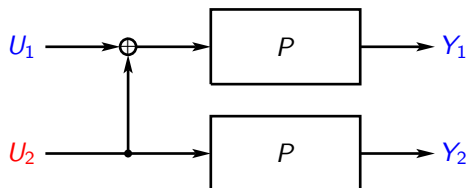
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- Observe that

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}.$$

- With independent, uniform U_1, U_2 ,

$$\begin{aligned} I(P^-) &= I(U_1; Y_1 Y_2), \\ I(P^+) &= I(U_2; Y_1 Y_2 U_1). \end{aligned}$$

- Thus,

$$\begin{aligned} I(P^-) + I(P^+) &= I(U_1 U_2; Y_1 Y_2) \\ &= I(X_1; Y_1) + I(X_2; Y_2) \\ &= 2I(P), \end{aligned}$$

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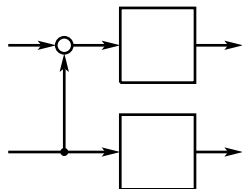
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- Duplicate W and obtain W^- and W^+ .
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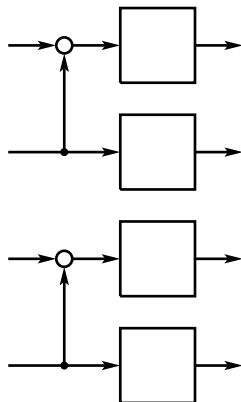
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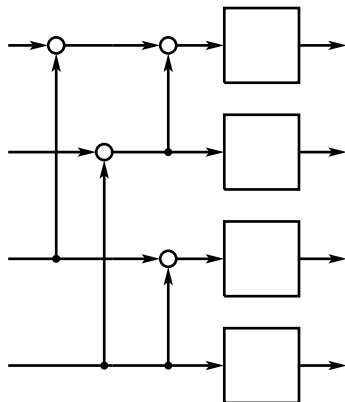
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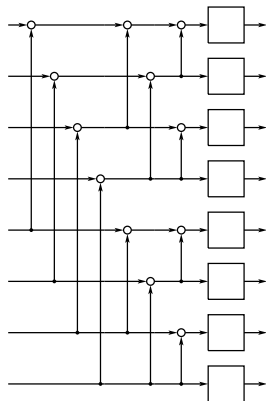


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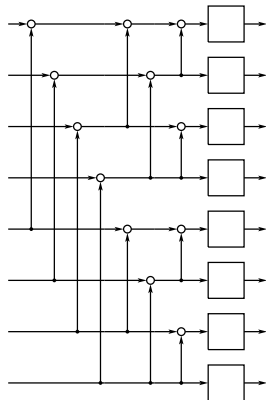
• ...



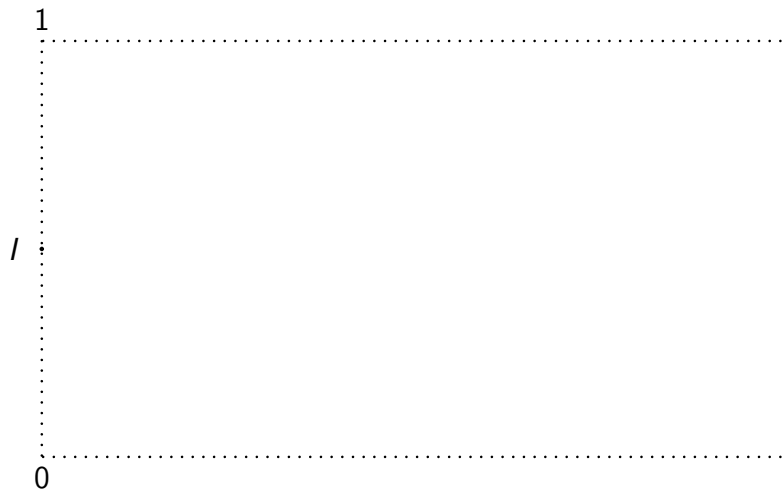
Polarization – HowTo

What we can do once, we can do many times: Given W ,

- Duplicate W and obtain W^- and W^+ .
- Duplicate W^- (W^+),
- and obtain W^{--} and W^{-+} (W^{+-} and W^{++}).
- Duplicate W^{--} (W^{-+} , W^{+-} , W^{++}) and obtain W^{---} and W^{--+ (W^{-+-} , W^{-++} , W^{+--} , W^{+-+} , W^{++-} , W^{+++}).
- ...



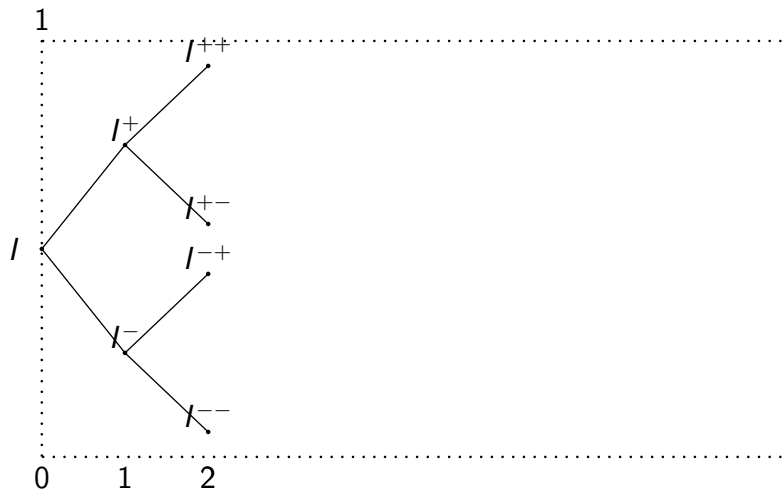
Polarization Process



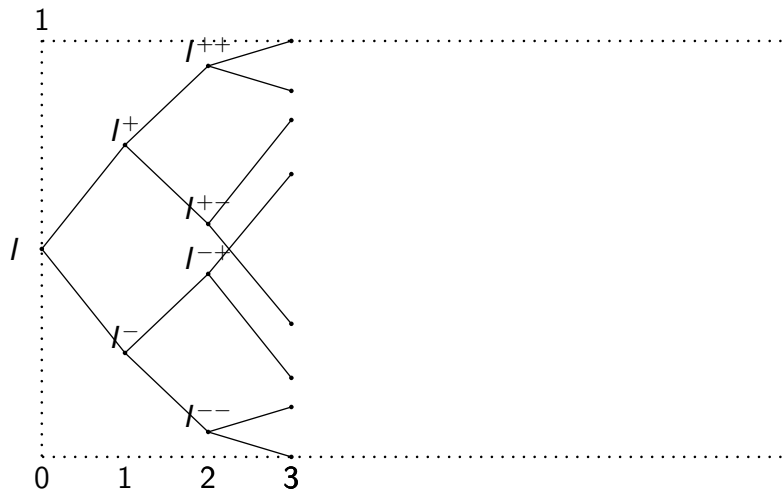
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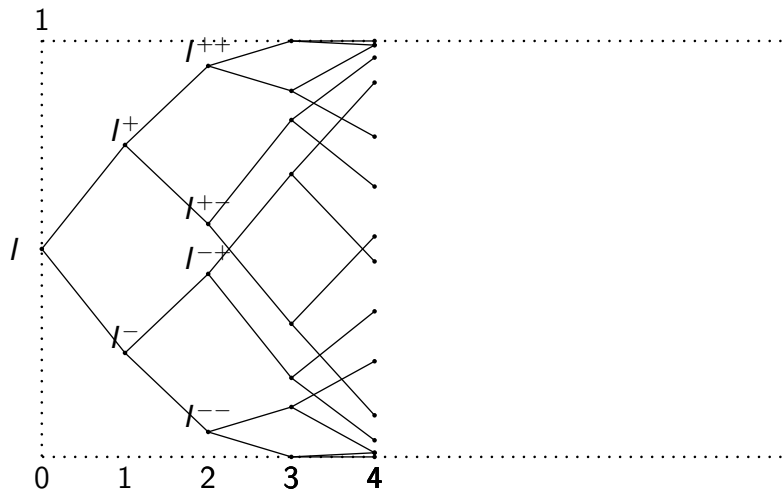
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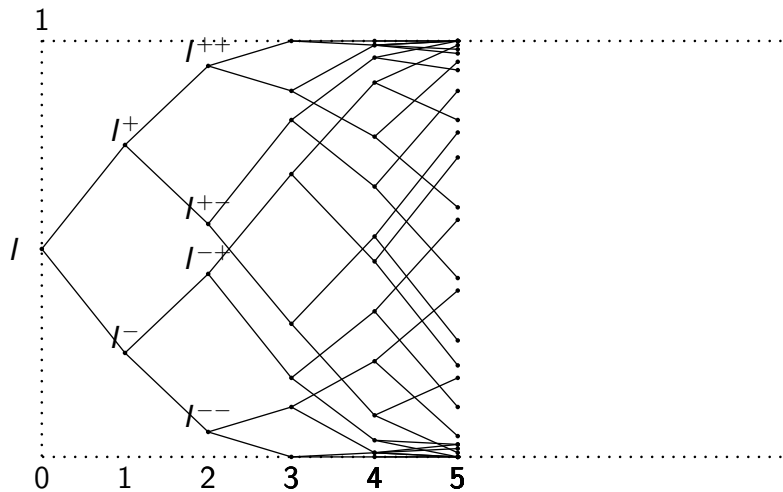
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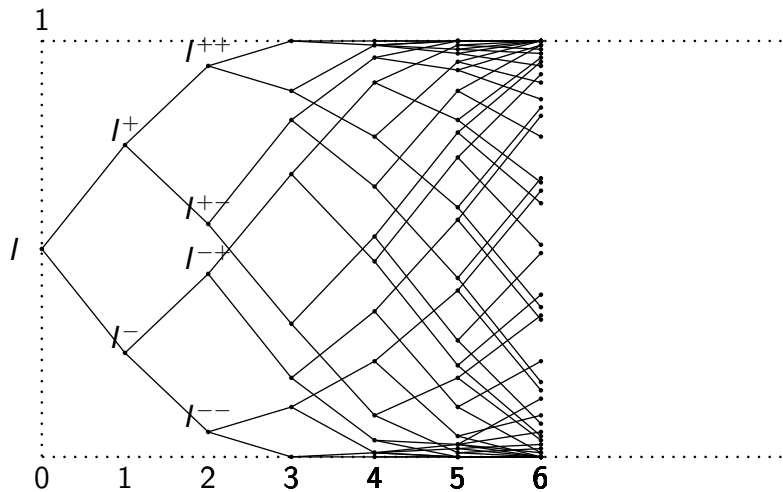
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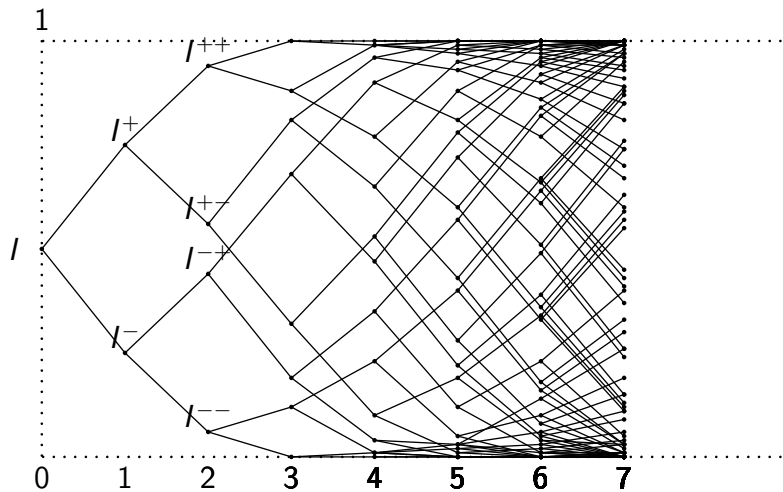
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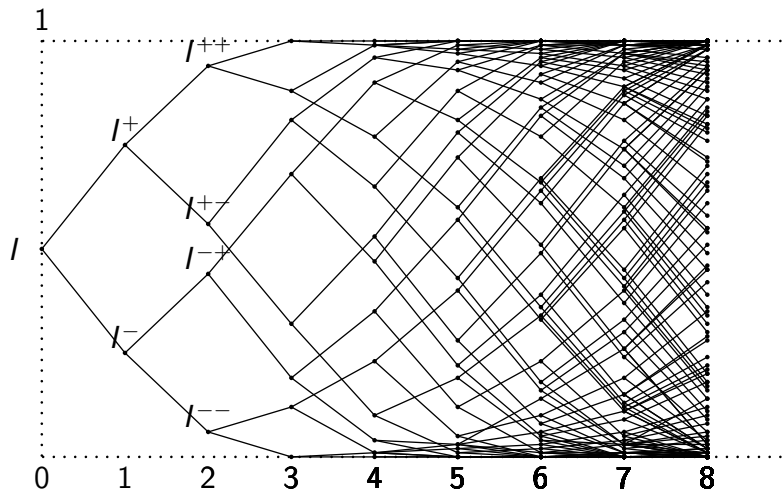
Polarization process



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Polarization process



Polar coding

If polarization takes place and we wish to communicate at rate R :

- Pick n , and $k = nR$ **good indices** i such that $I(U_i; Y^n U^{i-1})$ is high,
- let the transmitter set U_i to be **uncoded** binary data for good indices, and set U_i to random but publicly known values for the rest,
- let the receiver decode the U_i successively: U_1 from Y^n ; U_i from $Y^n \hat{U}^{i-1}$.
- One would expect this scheme to do well as long as $R < I(W)$.

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Polar coding complexity and performance

With the particular one-to-one mapping described here and with the **successive cancellation decoding**

- polarization codes are ' $I(W)$ achieving',
- encoding complexity is $n \log n$,
- decoding complexity is $n \log n$,
- probability of error decays like $2^{-\sqrt{n}}$ (proved with Telatar, 2008).
- However, for small ℓ their performance is surpassed by the usual suspects.

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Polar coding in other scenarios

Polar coding has been shown (mostly by EPFL researchers) to achieve the information-theoretic limits in the following scenarios

- Noiseless source coding
- Rate-distortion problem
- Slepian-Wolf problem
- Wyner-Ziv problem
- Gelfand-Pinsker problem
- Multi-access channels
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- Wyner wiretap channel
- Above results generalized to q 'ary channels and sources

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