Large Systems Analysis of Cellular Network MIMO

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Outline

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Why Network MIMO?

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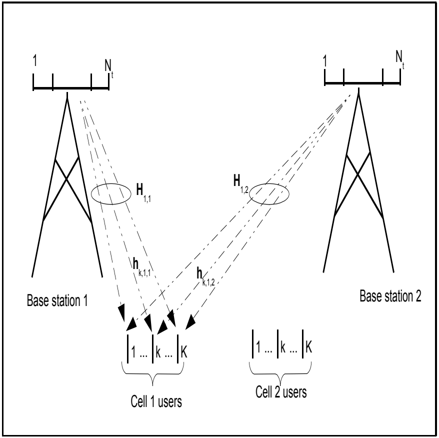
- Hence, interest in BS cooperation schemes:
  1. Network MIMO
  2. Interference Avoidance

Three architectures



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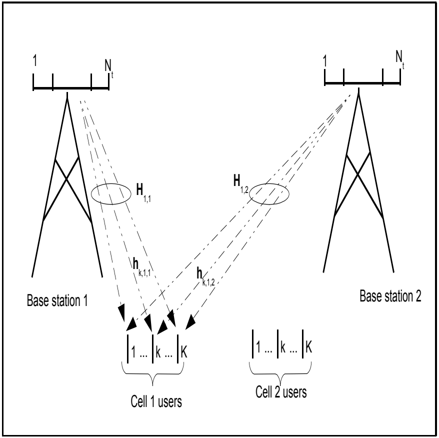

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   - BS's only aware of own-cell data and linearly precode to own-cell mobiles

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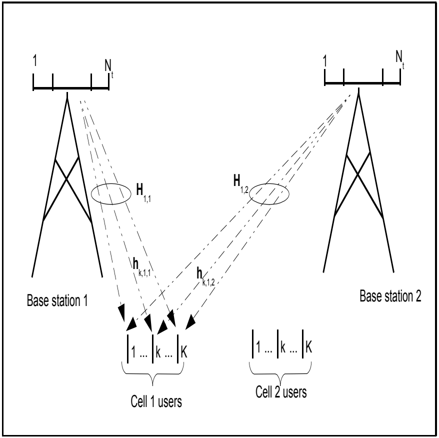

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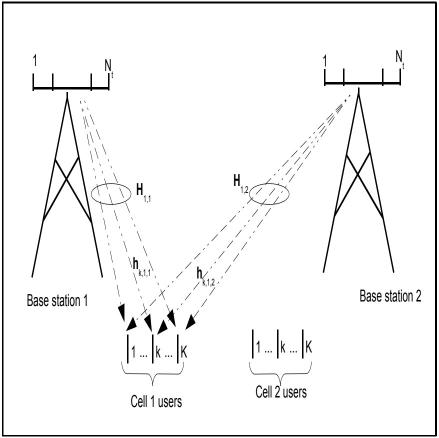

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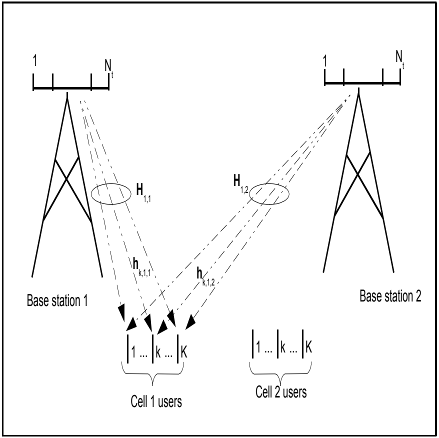

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   - BS's precode as if they were single isolated cells, but with more noise at mobile receivers

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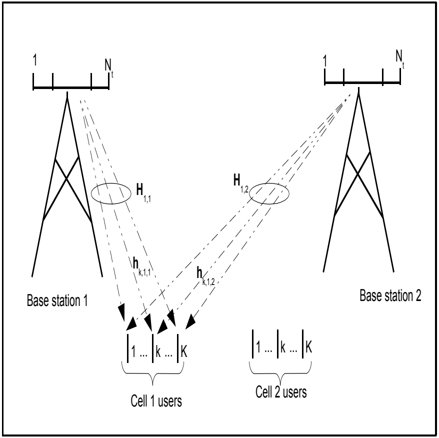

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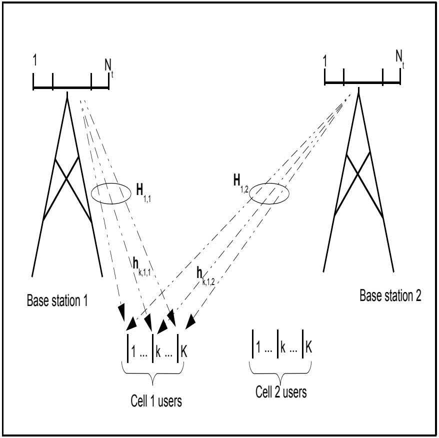

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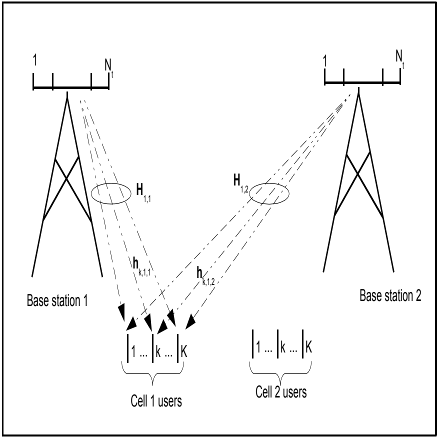

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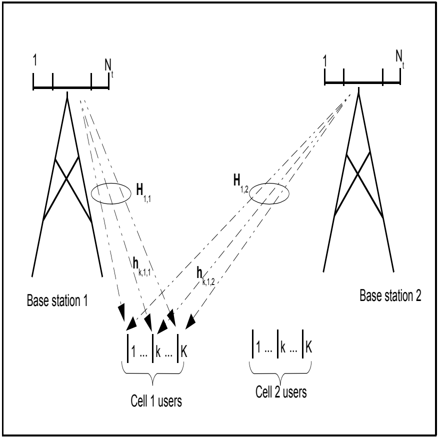

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- Precoding becomes a joint, two-cell optimization

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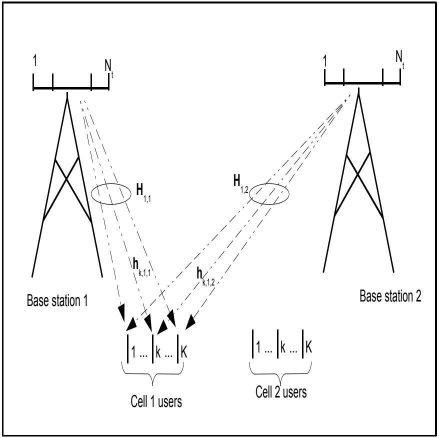

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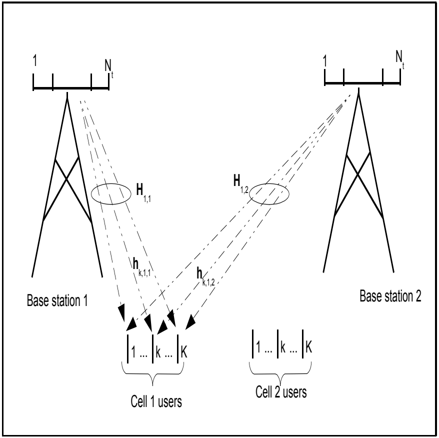

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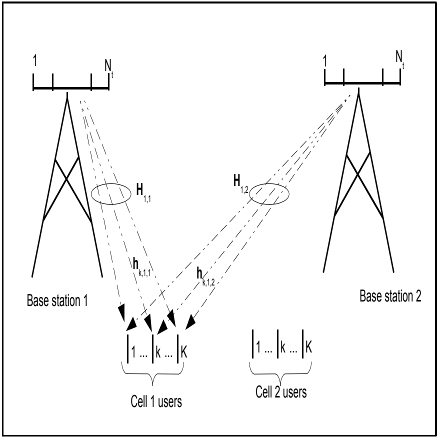

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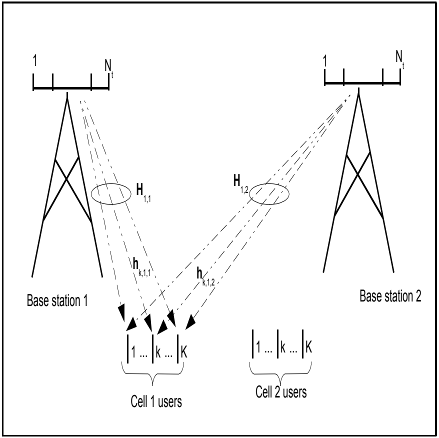

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How do these three approaches compare?

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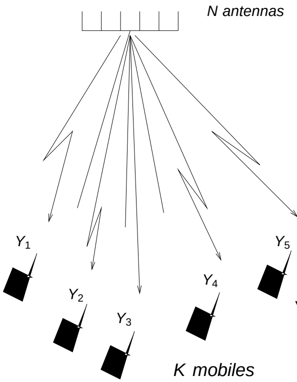
Linear precoding

In SCP, we have coupled (interfering) MIMO broadcast channels (MIMO-BC).



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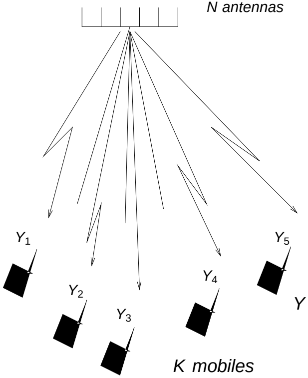

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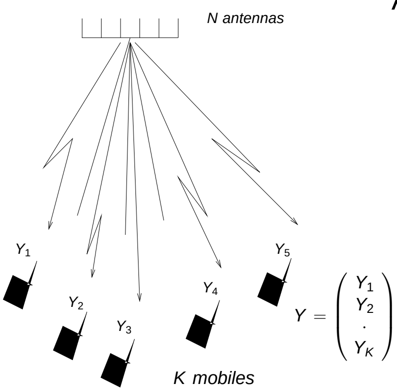

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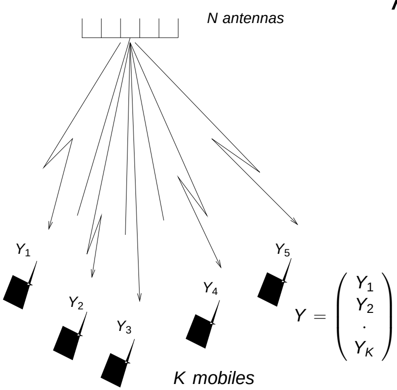


A MIMO BC is as follows:

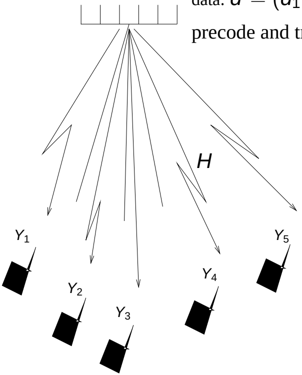
- There are K single-antenna receivers (mobiles)
- N transmit antennas at the BS
- Denote the total received signal (at all mobiles) by the $K \times 1$ received vector $\mathbf{Y}$

Linear precoding

Linear precoding in the MIMO-BC:

data: $d = (d_1, d_2, ..., d_K)^t$

precode and transmit: Wd



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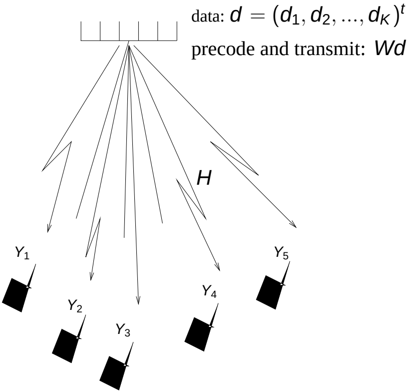

$$\mathbf{Y} = \mathbf{HWd} + \mathbf{Z}$$

where

- $\mathbf{H}$ is the $K \times N$ MIMO channel matrix
- $\mathbf{W}$ is the $N \times K$ precoding matrix
- $\mathbf{d}$ is the vector of data symbols, and $\mathbf{z}$ is the noise vector

Zero forcing schemes

The following pre-coding matrices are well known:

- zero-forcing (ZF): precode so as to null the interference at all mobiles

$$\mathbf{W}^{(ZF)} = c_1 \mathbf{H}^H \left[\mathbf{H}\mathbf{H}^H \right]^{-1}$$

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$$\mathbf{W}^{(ZF)} = c_1 \mathbf{H}^H \left[\mathbf{H}\mathbf{H}^H \right]^{-1}$$

- regularized zero-forcing (RZF): similar to zero-forcing, but with an additional regularization term added

$$\mathbf{W}^{(RZF)} = c_2 \mathbf{H}^H \left[\mathbf{H}\mathbf{H}^H + \alpha \mathbf{I}_N \right]^{-1}$$

where $\mathbf{I}_N$ is the $N \times N$ identity matrix, and α is a regularization parameter.

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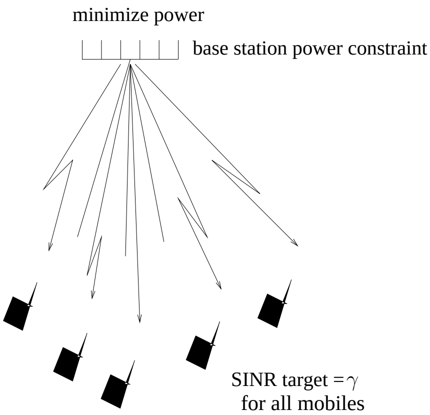

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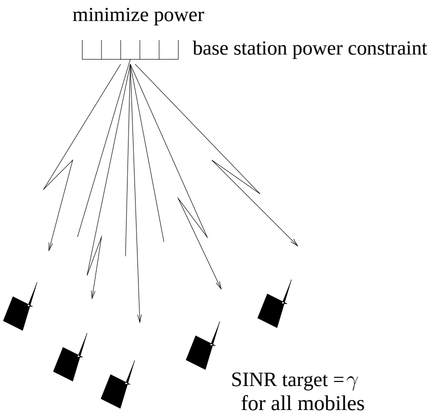

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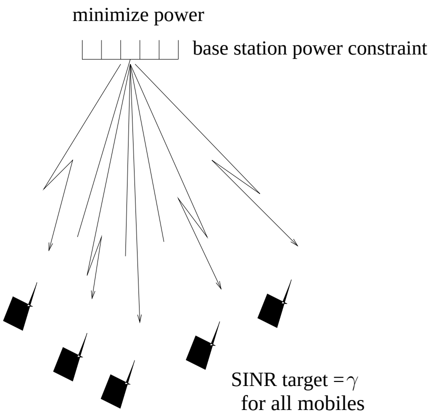

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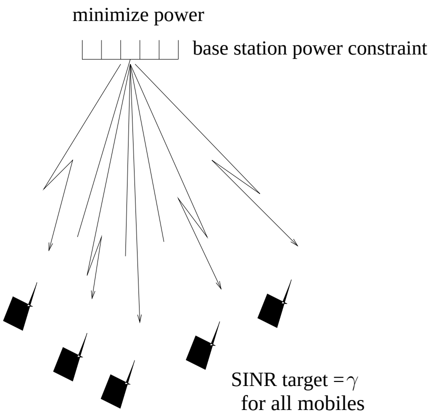

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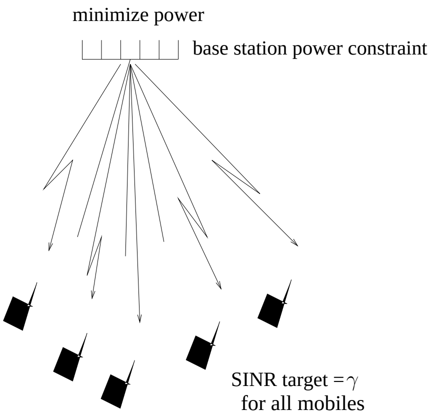

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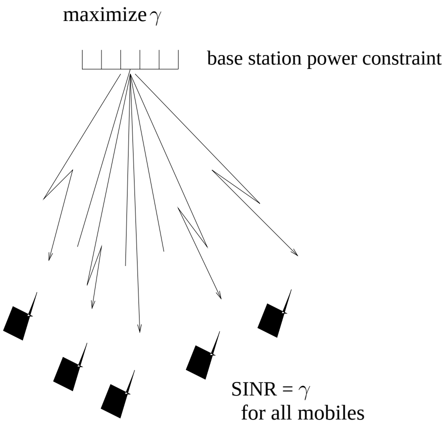

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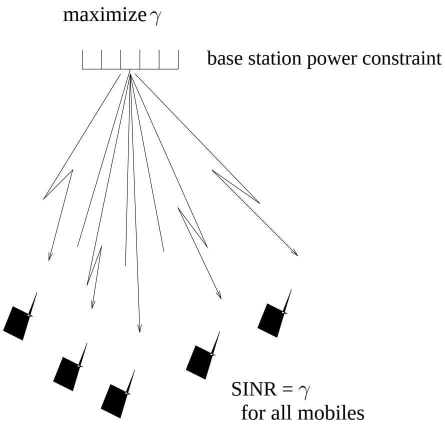

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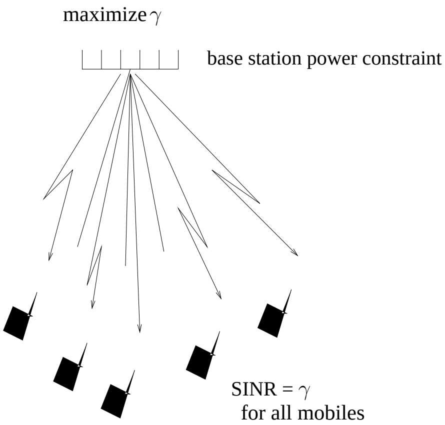

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We will also extend the theory to the case of MCP

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We show that the CBf strategy in the CBf paper "converges" to GRZF beamforming in our model.

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- Simplify beamforming design for the finite system case

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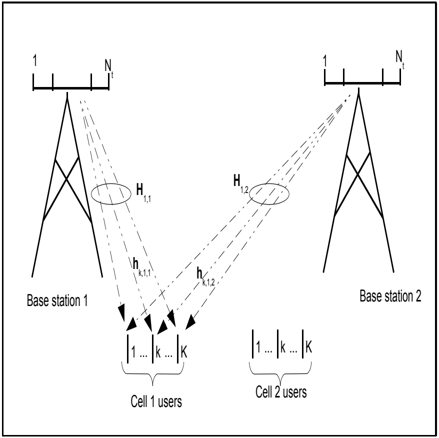
The model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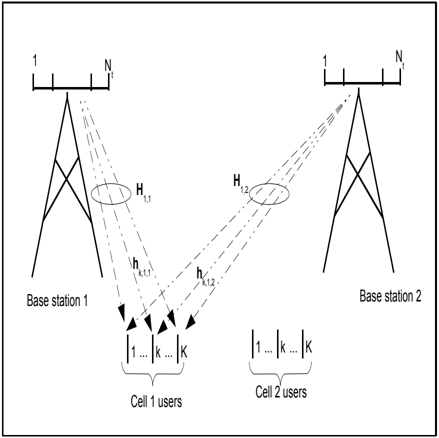
N_t
1
N_t
$\mathbf{H}_{1,1}$
$\mathbf{H}_{1,2}$
$\mathbf{h}_{k,1,1}$
$\mathbf{h}_{k,1,2}$
Base station 1
Base station 2
1 ... k ... K
1 ... k ... K
Cell 1 users
Cell 2 users

The model



- Focus on two cell setup

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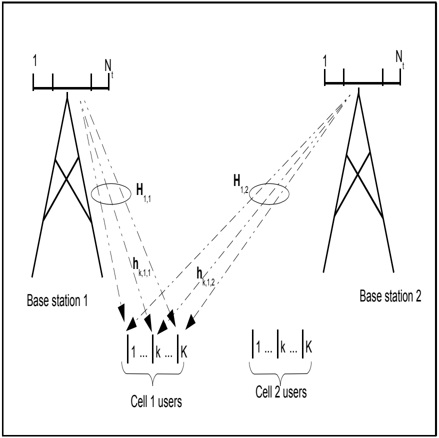

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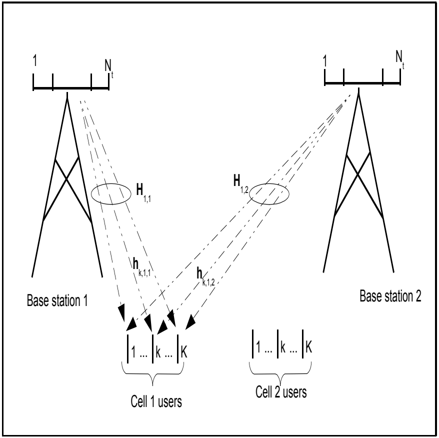

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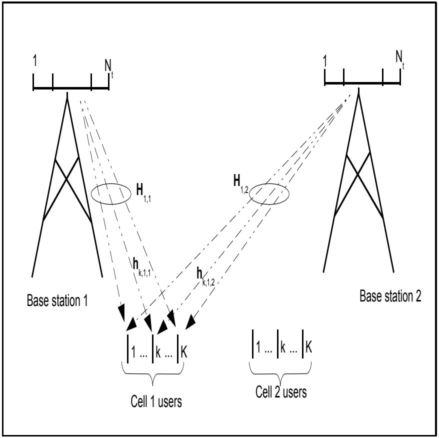

- Focus on two cell setup
- MS's in cell j have
  - i.i.d. $\mathcal{CN}(0,1)$ channels to their 'serving' base station, and
  - $\mathcal{CN}(0,\epsilon)$ channels to the other base station

Power minimization problem for SCP

Theorem

Assume $N, K \to \infty$ such that $\frac{K}{N} \to \beta < \infty$. Then the target SINR of γ is achievable if and only if $\beta \left(\frac{\gamma}{1+\gamma} + \epsilon\gamma \right) < 1$.

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- *Up to a constant, the optimal DL beamformer for user k in cell j is*

$$\mathbf{w}_{kj}^{SCP} = \left(\mathbf{I}_N + \frac{\bar{\lambda}}{N} \sum_{\bar{k} \neq k} \mathbf{h}_{\bar{k},j,j}^H \mathbf{h}_{\bar{k},j,j} \right)^{-1} \mathbf{h}_{k,j,j}^H. \qquad (1)$$

where $\bar{\lambda} = \dfrac{\gamma}{1 - \beta\frac{\gamma}{1+\gamma} - \beta\epsilon\gamma}$

Power minimization problem for CBf

Theorem

Assume $N, K \to \infty$ *such that* $\frac{K}{N} \to \beta < \infty$*. Then the target SINR of* γ *is achievable if and only if* $\beta\left(\frac{\gamma}{1+\gamma} + \frac{\epsilon\gamma}{1+\epsilon\gamma}\right) < 1$*.*

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$$\mathbf{w}_{kj}^{Coord} = \left(\mathbf{I}_N + \frac{\bar{\lambda}}{N} \sum_{(\bar{k},\bar{j}) \neq (k,j)} \mathbf{h}_{\bar{k},\bar{j},j}^H \mathbf{h}_{\bar{k},\bar{j},j}\right)^{-1} \mathbf{h}_{k,j,j}^H. \qquad (2)$$

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Subject to per base station power constraint P, as $N, K \to \infty$ such that $\frac{K}{N} \to \beta < \infty$, the maximum asymptotic network-wide achievable SINR is the unique solution to the following fixed point equation:

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- *MCP:* $\gamma^*_{MCP} = \frac{1}{\beta}\frac{1}{\frac{\sigma^2}{(1+\epsilon)P}+\frac{1}{1+\gamma^*_{MCP}}}$.

Outline

Numerical Results

Applicability to finite systems

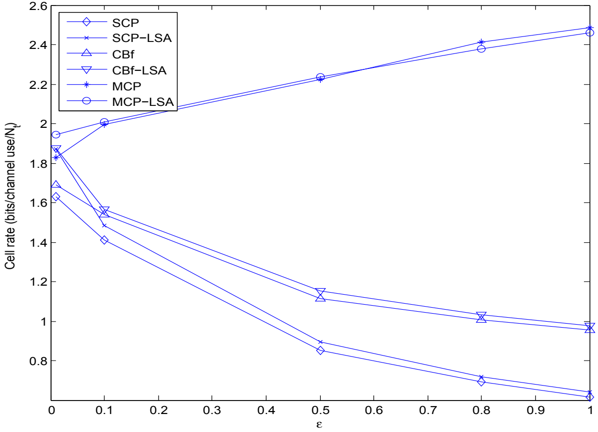

Figure: Large system analysis results vs. finite system optimization for $K = 3, N_t = 4$ and SNR = 10 dB.

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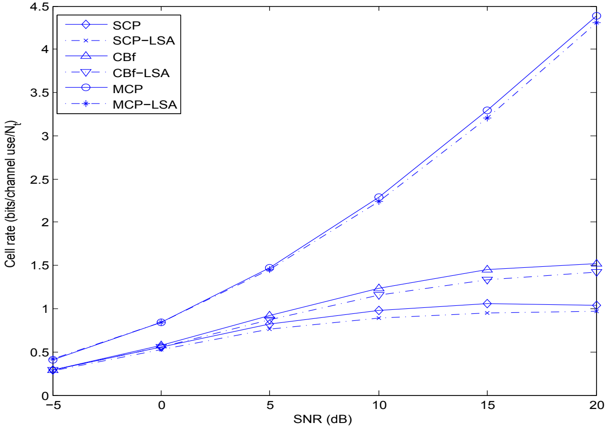

Figure: Large system analysis results vs. application of asymptotically optimal beamforming in the finite case for $K = 3, N_t = 4$ and $\epsilon = 0.5$.

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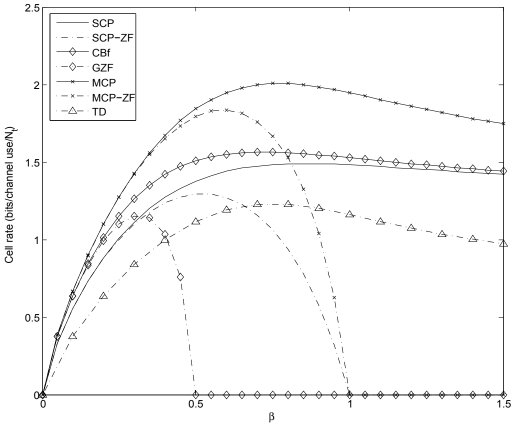

Figure: Effect of cell loading β on rate achieved for SNR = 10dB, $\epsilon = .1$

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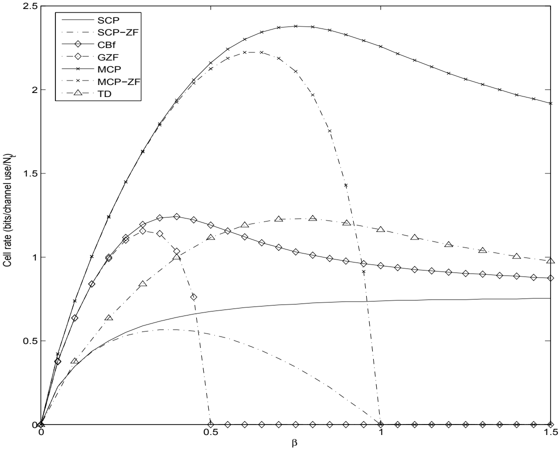

Figure: Effect of cell loading β on rate achieved for SNR = 10dB, $\epsilon = .8$

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