Large Systems Analysis of Cellular Network
MIMO

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Outline

Motivation

Linear Precoding

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Numerical results

Conclusions
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Motivation

Why Network MIMO?
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- In cellular systems, *reuse 1* considered for increased spectral efficiency.
Why Network MIMO?

• In cellular systems, *reuse 1* considered for increased spectral efficiency.
• But cells are not isolated.

⇒ INTERFERENCE!
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⇒ INTERFERENCE!

- Hence, interest in BS cooperation schemes:
  1. Network MIMO
  2. Interference Avoidance
Motivation

Three architectures
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1. In single cell processing (SCP),
   - BS’s only aware of own-cell data and linearly precode to own-cell mobiles
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   - BS’s only aware of own-cell data and linearly precode to own-cell mobiles
   - BS’s oblivious to interference created in other cell
   - Single-user detection at mobiles
   - BS’s precode as if they were single isolated cells, but with more noise at mobile receivers
Three architectures

2. In coordinated beamforming (CBf),
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As with SCP:

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But:
- Both base stations aware of system-wide channel gains
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- BS’s only aware of own-cell data and linearly precode to own-cell mobiles
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But:

- Both base stations aware of system-wide channel gains
- Precoding becomes a joint, two-cell optimization
Three architectures

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   - BS’s aware of all mobiles’ data (system-wide data knowledge)
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How do these three approaches compare?
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Linear precoding

In SCP, we have coupled (interfering) MIMO broadcast channels (MIMO-BC).

\[
Y = \begin{pmatrix}
Y_1 \\
Y_2 \\
\vdots \\
Y_K
\end{pmatrix}
\]
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- $N$ transmit antennas at the BS

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In SCP, we have coupled (interfering) MIMO broadcast channels (MIMO-BC).

A MIMO BC is as follows:

- There are $K$ single-antenna receivers (mobiles)
- $N$ transmit antennas at the BS
- Denote the total received signal (at all mobiles) by the $K \times 1$ received vector $\mathbf{Y}$
Linear precoding in the MIMO-BC:

Data: \( d = (d_1, d_2, \ldots, d_K)^t \)

Precode and transmit: \( Wd \)
Linear Precoding

Linear precoding in the MIMO-BC:

Data: $d = (d_1, d_2, ..., d_K)^t$

Precode and transmit: $Wd$

$$Y = HWd + Z$$

Where:

- $H$ is the $K \times N$ MIMO channel matrix
- $W$ is the $N \times K$ precoding matrix
- $d$ is the vector of data symbols, and $z$ is the noise vector
Zero forcing schemes

The following pre-coding matrices are well known:

- zero-forcing (ZF): precode so as to null the interference at all mobiles

\[ W^{(ZF)} = c_1 H^H \left[ H H^H \right]^{-1} \]
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  \[
  W^{(ZF)} = c_1 H^H \left[ H H^H \right]^{-1}
  \]

- regularized zero-forcing (RZF): similar to zero-forcing, but with an additional regularization term added
  \[
  W^{(RZF)} = c_2 H^H \left[ H H^H + \alpha I_N \right]^{-1}
  \]

where $I_N$ is the $N \times N$ identity matrix, and $\alpha$ is a regularization parameter.
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One may also seek optimal precoding matrices.
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minimize power

base station power constraint

SINR target = $\gamma$
for all mobiles
Power minimization problem

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We will consider power minimization subject to rate targets:

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- The objective is to minimize total power subject to the SINR target and per base station power constraints
Rate maximization problem

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- Each base station has the same average power constraint
- The objective is to maximize the SINR target subject to the per base station power constraints
Optimization in cellular networks

Such problems can be considered for SCP, CBf, and MCP, respectively.
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Our large systems analysis will shed light on this question.
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We will also extend the theory to the case of MCP
Connections with generalized ZF

For SCP

- It was recognized in SCP paper that with a lot of symmetry in channel, optimal BF turns out to be RZF
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- The large systems analysis of CBF, will give rise to a novel beamformer: GRZF (generalized regularized zero forcer)
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Both RZF (for SCP) and GRZF (for CBf) are much easier to implement than the optimal solution
We show that the CBf strategy in the CBf paper “converges” to GRZF beamforming in our model.
Summary of base station cooperation

- **Local CSIT** at each base station and **no data sharing**
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- Local CSIT at each base station and no data sharing → Single cell processing (SCP)
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- Efficiently compare these architectures
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Using large system analysis, we can

- Efficiently compare these architectures
- Simplify beamforming design for the finite system case
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The model

Base station 1

Cell 1 users

Base station 2

Cell 2 users

$H_{1,1}$

$h_{k,1}$

$h_{k,2}$

$H_{1,2}$
The model

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- Focus on two cell setup
- MS’s in cell $j$ have
  - i.i.d. $\mathcal{CN}(0, 1)$ channels to their 'serving' base station, and
  - $\mathcal{CN}(0, \epsilon)$ channels to the other base station
Power minimization problem for SCP

Theorem
Assume $N, K \to \infty$ such that $\frac{K}{N} \to \beta < \infty$. Then the target SINR of $\gamma$ is achievable if and only if $\beta \left( \frac{\gamma}{1+\gamma} + \epsilon \gamma \right) < 1$. 
Power minimization problem for SCP

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- The per BS power converges to $P = \frac{\beta \sigma^2 \gamma}{\left( 1 - \beta \frac{\gamma}{1+\gamma} - \beta \epsilon \gamma \right)}$. 
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- The per BS power converges to $P = \frac{\beta \sigma^2 \gamma}{\left( 1 - \beta \frac{\gamma}{1+\gamma} - \beta \epsilon \gamma \right)}$.
- Up to a constant, the optimal DL beamformer for user $k$ in cell $j$ is

$$w_{kj}^{SCP} = \left( I_N + \frac{\lambda}{N} \sum_{\bar{k} \neq k} h_{k,j,j}^H h_{\bar{k},j,j} \right)^{-1} h_{k,j,j}^H. \quad (1)$$

where $\lambda = \frac{\gamma}{1 - \beta \frac{\gamma}{1+\gamma} - \beta \epsilon \gamma}$.
Power minimization problem for CBf

Theorem
Assume $N, K \to \infty$ such that $\frac{K}{N} \to \beta < \infty$. Then the target SINR of $\gamma$ is achievable if and only if
\[ \beta \left( \frac{\gamma}{1+\gamma} + \frac{\epsilon \gamma}{1+\epsilon \gamma} \right) < 1. \]
Power minimization problem for CBf

**Theorem**

Assume $N, K \to \infty$ such that $\frac{K}{N} \to \beta < \infty$. Then the target SINR of $\gamma$ is achievable if and only if $
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- The per BS power converges to $P = \frac{\beta \sigma^2 \gamma}{1 - \beta \left( \frac{\gamma}{1+\gamma} + \frac{\epsilon \gamma}{1+\epsilon \gamma} \right)}$.
- Up to a constant, the optimal DL beamformer for user $k$ in cell $j$ is

$$w_{kj}^{\text{Coord}} = \left( I_N + \frac{\bar{\lambda}}{N} \sum_{(\bar{k},\bar{j}) \neq (k,j)} h_{\bar{k},\bar{j}}^H h_{k,j,j}^{}\right)^{-1} h_{k,j,j}^{}.$$  \hspace{1cm} (2)

where $\bar{\lambda} = \frac{\gamma}{1 - \beta \left( \frac{\gamma}{1+\gamma} + \frac{\epsilon \gamma}{1+\epsilon \gamma} \right)}$.
Power minimization problem for MCP

**Theorem**

Assume $N, K \to \infty$ such that $\frac{K}{N} \to \beta < \infty$. Then the target SINR of $\gamma$ is achievable if and only if $\beta \frac{\gamma}{1+\gamma} < 1$. 
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Theorem
Assume $N, K \to \infty$ such that $\frac{K}{N} \to \beta < \infty$. Then the target SINR of $\gamma$ is achievable if and only if $\beta \frac{\gamma}{1+\gamma} < 1$.

- The per BS power converges to $P = \frac{1}{1+\epsilon} \frac{\beta \sigma^2 \gamma}{(1 - \beta \frac{\gamma}{1+\gamma})}$. 
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Theorem
Assume $N, K \to \infty$ such that $\frac{K}{N} \to \beta < \infty$. Then the target SINR of $\gamma$ is achievable if and only if $\beta \frac{\gamma}{1+\gamma} < 1$.

- The per BS power converges to $P = \frac{1}{1 + \epsilon} \frac{\beta \sigma^2 \gamma}{(1 - \beta \frac{\gamma}{1+\gamma})}$.

- Up to a constant, the optimal DL beamformer for user $k$ in cell $j$ is

$$w^{MCP}_{kj} = \left( I_{2N} + \frac{\bar{\lambda}}{N} \sum_{(\bar{k}, \bar{j}) \neq (k,j)} h^H_{\bar{k}, \bar{j}} h_{\bar{k}, \bar{j}} \right)^{-1} h^H_{k,j}. \quad (3)$$

where $\bar{\lambda} = \frac{1}{1 + \epsilon} \frac{\gamma}{(1 - \beta \frac{\gamma}{1+\gamma})}$.
Theorem

In each scenario, the rates either increase indefinitely with $\beta$, or are maximized at a finite $\beta$. 
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The rates are increasing with $\beta$ when

- $SCP$: $\frac{\sigma^2}{P} + \epsilon > 1$.
- $CBf$: $\frac{\sigma^2}{P} + \epsilon - 2\epsilon^2 > 1$. 
Maximum rates

Theorem

In each scenario, the rates either increase indefinitely with $\beta$, or are maximized at a finite $\beta$.

The rates are increasing with $\beta$ when

- **SCP**: $\frac{\sigma^2}{P} + \epsilon > 1$.
- **CBf**: $\frac{\sigma^2}{P} + \epsilon - 2\epsilon^2 > 1$.
- **MCP**: $\frac{\sigma^2}{P} > 1 + \epsilon$. 
Maximum rates

Theorem

Subject to per base station power constraint $P$, as $N, K \to \infty$ such that $\frac{K}{N} \to \beta < \infty$, the maximum asymptotic network-wide achievable SINR is the unique solution to the following fixed point equation:
Maximum rates

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- SCP: $\gamma_{SCP}^* = \frac{1}{\beta} \frac{1}{\frac{\sigma^2}{P} + \epsilon + \frac{1}{1 + \gamma_{SCP}^*}}$. 
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- **SCP**: $\gamma^{*}_{SCP} = \frac{1}{\beta} \frac{1}{\frac{\sigma^2}{P} + \epsilon + \frac{1}{1+\gamma^{*}_{SCP}}}.$

- **CBf**: $\gamma^{*}_{Coord} = \frac{1}{\beta} \frac{1}{\frac{\sigma^2}{P} \frac{1}{1+\gamma^{*}_{Coord}} + \frac{1}{1+\epsilon \gamma^{*}_{Coord}}}.$
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- **SCP:**
  \[ \gamma^*_{SCP} = \frac{1}{\beta} \frac{\sigma^2}{P} + \epsilon + \frac{1}{1 + \gamma^*_{SCP}}. \]

- **CBf:**
  \[ \gamma^*_{Coord} = \frac{1}{\beta} \frac{\sigma^2}{P} + \frac{1}{1 + \gamma^*_{Coord}} + \frac{\epsilon}{1 + \epsilon \gamma^*_{Coord}}. \]

- **MCP:**
  \[ \gamma^*_{MCP} = \frac{1}{\beta} \frac{\sigma^2}{(1 + \epsilon)P} + \frac{1}{1 + \gamma^*_{MCP}}. \]
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Figure: Large system analysis results vs. finite system optimization for $K = 3$, $N_t = 4$ and SNR = 10 dB.
**Numerical Results**

Applicability to finite systems

![Graph showing cell rate vs. SNR for different beamforming techniques](image)

**Figure:** Large system analysis results vs. application of asymptotically optimal beamforming in the finite case for $K = 3$, $N_t = 4$ and $\epsilon = 0.5$. 

**Graph Details:**
- **SNR (dB)** range from -5 to 20.
- **Cell rate (bits/channel use/N_t)** range from 0.5 to 4.5.
- Different lines represent:
  - SCP
  - SCP-LSA
  - CBf
  - CBf-LSA
  - MCP
  - MCP-LSA
The above derivations allow us to compare the different setups for different schemes without Monte Carlo simulations.
Numerical Results

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Figure: Effect of cell loading $\beta$ on rate achieved for SNR = 10dB, $\epsilon = .1$
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The above derivations allow us to compare the different setups for different schemes without Monte Carlo simulations.

Figure: Effect of cell loading $\beta$ on rate achieved for SNR = 10dB, $\epsilon = .8$
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- We compare SCP, CBF, MCP, with a time division SCP scheme, and with some ZF schemes