Optimal Cooperation in Large Wireless Networks

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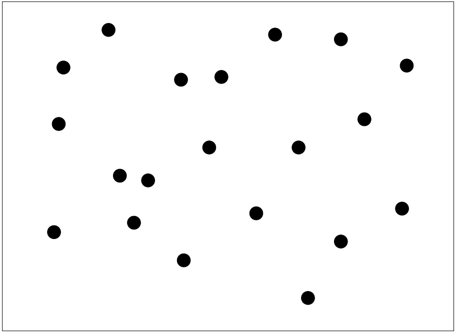
Simple network model



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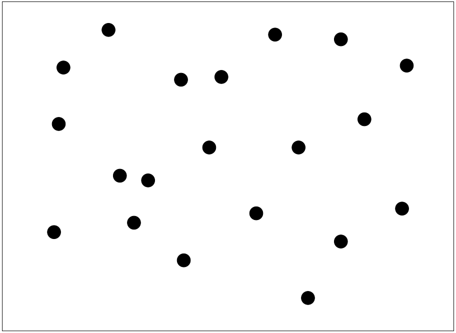

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  n nodes independently and uniformly distributed in a square area A

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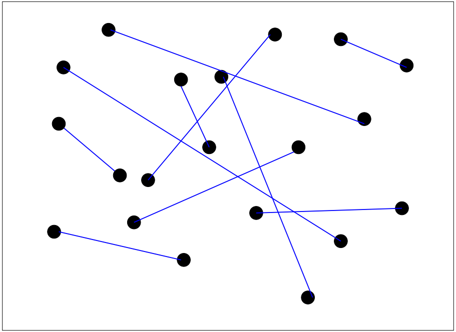
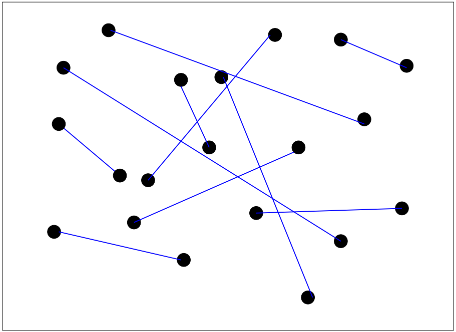


- uniform topology:
  n nodes independently and uniformly distributed in a square area A
- uniform traffic:
  order n source-destination pairs chosen at random in the network

Question



What is the maximum aggregate throughput scaling in such a network?

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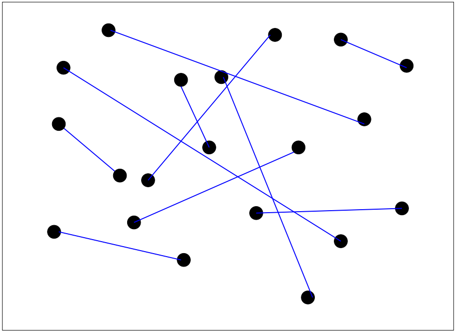

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aggregate throughput: $T(n) = n\,R(n)$,
where $R(n)$ = maximum achievable data rate per S-D pair

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- Özgür, Lévêque and Tse '07: An order n aggregate throughput scaling is achievable via an intelligent cooperation architecture and distributed MIMO transmissions.

- Franceschetti, Migliore and Minero '09: Independently of the technology, the aggregate throughput is limited by the spatial degrees of freedom in the network and scales at most as $\sqrt{n}$; multi-hopping is therefore the optimal strategy.

Two different communication models

$$y_i = \sum_{k \in \tau} h_{ik}\, x_k + z_i \qquad h_{ik} = \sqrt{G}\, \frac{\exp(j\theta_{ik})}{r_{ik}}$$

$\{x_k, k \in \tau\}$ = transmitted signals - power P each

z_i = background noise at node i - unit power

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2. Franceschetti et al. '09:

   $\theta_{ik} = 2\pi r_{ik}/\lambda$, where λ is the carrier wavelength

   line-of-sight model derived from Maxwell's equations

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- increasing number of nodes n

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Literature on wireless networks:

- increasing number of nodes n
- fixed power budget P per node
- network area A either fixed (dense network) or increasing linearly with n (extended network)

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Our plan

Characterize the aggregate throughput scaling of the network when

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3. n grows large, but there is space limitation
($A =$ limiting factor (line-of-sight model), P as large as we want)

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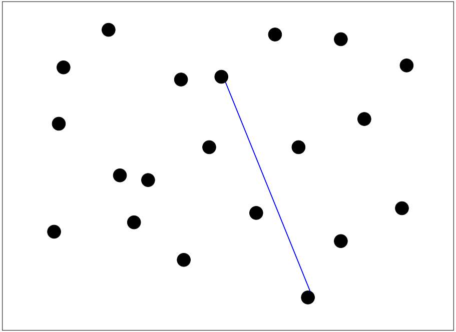
Question:

is the number of nodes itself a factor limiting the aggregate throughput?

Time-division



one communication at a time in the network

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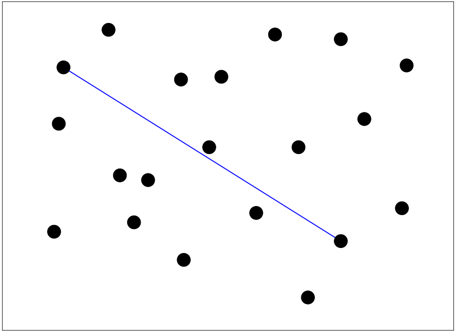

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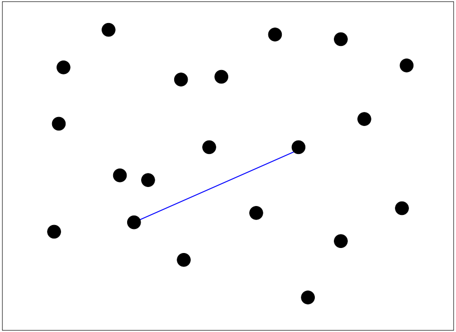

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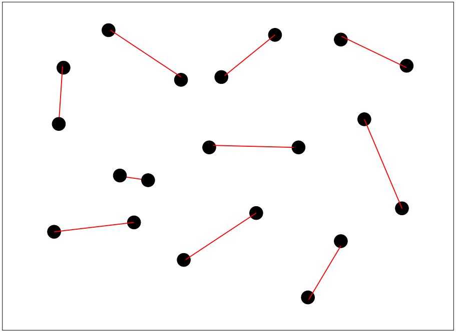
aggregate throughput $T(n) = \Theta(1)$

Multi-hop (Gupta-Kumar '00)



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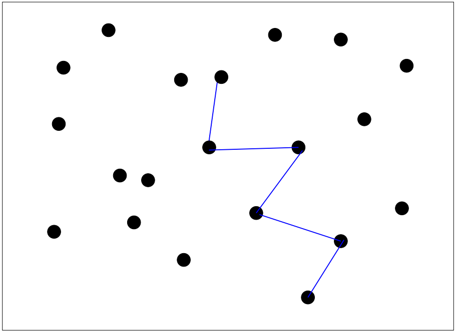

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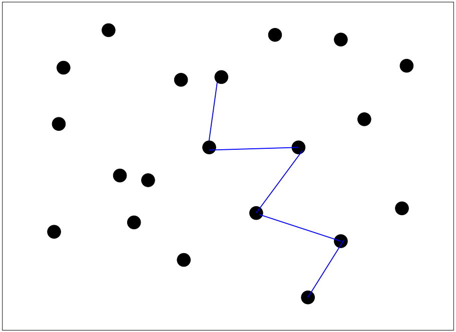

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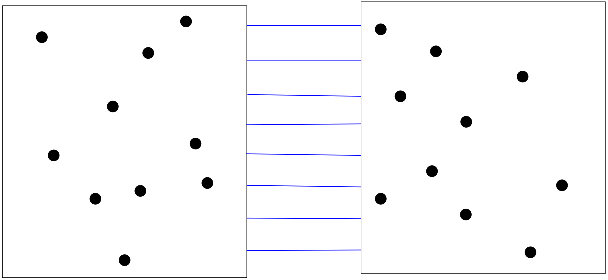
Parenthesis: distributed MIMO systems



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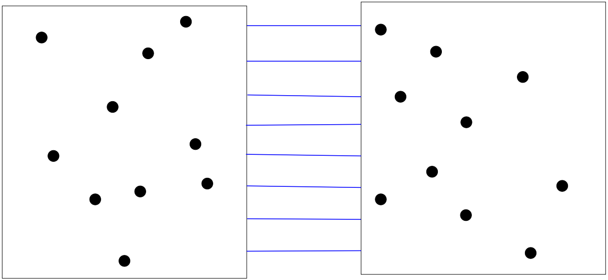

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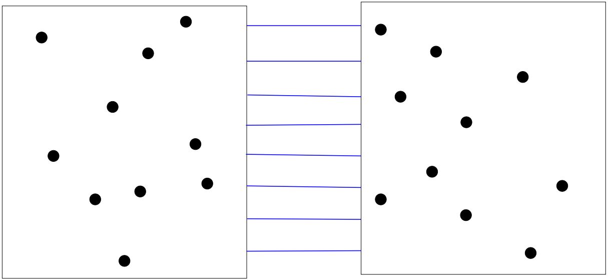

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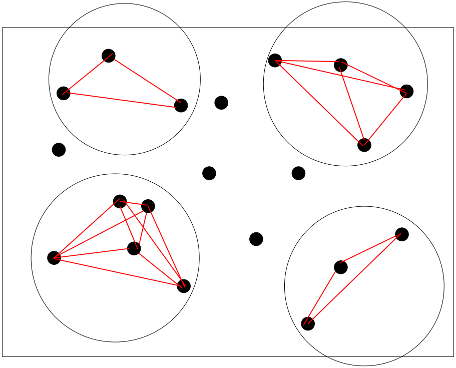
- ... as well as an aggreagtion phase at the receive cluster

Hierarchical cooperation scheme (Özgür-L-Tse '07)



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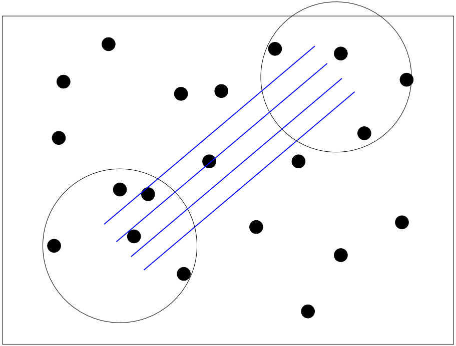

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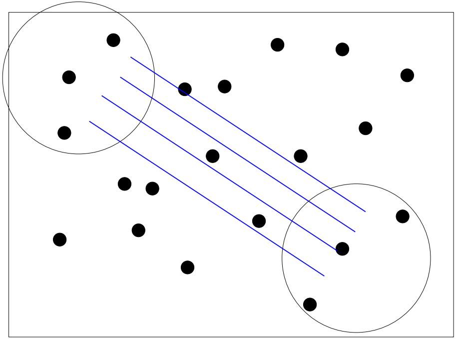

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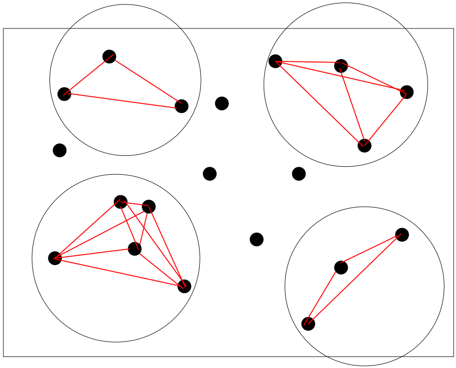

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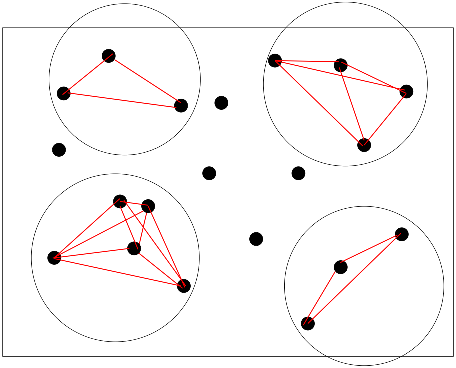

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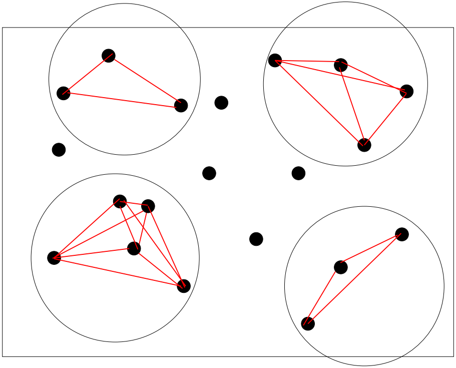

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- recursion: perform the same operation inside clusters now
- after h levels of recursion: aggregate throughput $T(n) = \Theta\left(n^{\frac{h}{h+1}}\right)$

Conclusion #1

in a network with a large number of nodes,
it is possible to sustain an aggregate throughput arbitrarily close to $\Theta(n)$
(provided that there is neither power nor space limitation)

2. Power limitation

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Question:

does hierarchical cooperation still outperform multi-hop in this case?

Power requirements

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- so hierarchical cooperation still outperforms multi-hop in this case, and an aggregae throughput $T(n)$ arbitrarily close to $\Theta(n)$ is achievable

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Remark:

The situation where the power path loss exponent $\alpha > 2$ is a different story! (see Özgür-Johari-Tse-L, Trans. on IT 2010).

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In a more general context, this result translates into:

$$T(n) = \begin{cases} O(\sqrt{n}) & \text{if } \sqrt{A}/\lambda \leq \sqrt{n} \\ O(\sqrt{A}/\lambda) & \text{if } \sqrt{n} \leq \sqrt{A}/\lambda \leq n \\ O(n) & \text{if } \sqrt{A}/\lambda \geq n \end{cases}$$

where λ is the carrier wavelength.

Temporary conclusion

- when area is a scarce resource, i.e. when $\sqrt{A}/\lambda \leq \sqrt{n}$, multi-hop achieves the optimal aggregate throughput scaling $T(n) = \Theta(\sqrt{n})$

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- can we do better than multi-hop in the intermediary regime?

Theorem

When $\sqrt{n} \leq \sqrt{A}/\lambda \leq n$ and $SNR = P/d^2 \geq 0$ dB, an aggregate throughput scaling arbitrarily close to

$$T(n) = \Theta(\sqrt{A}/\lambda)$$

is achievable via hierarchical cooperation (Özgür-L-Tse, ITA 2010).

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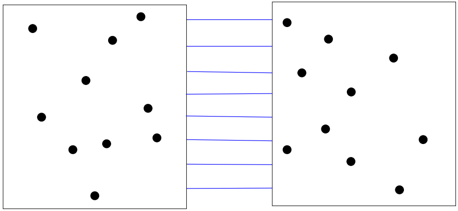
Remark:
A similar result has been obtained independently by Lee-Chung, ISIT 2010.

Proof idea: where does the spatial limitation kick in?



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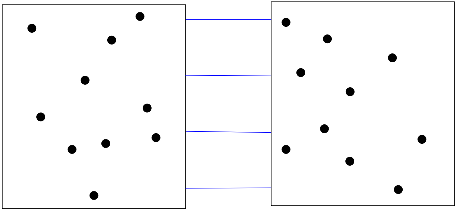

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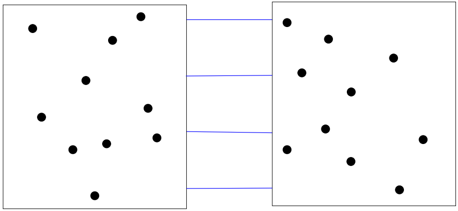

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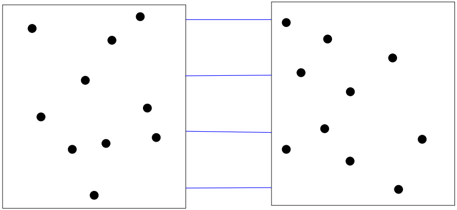

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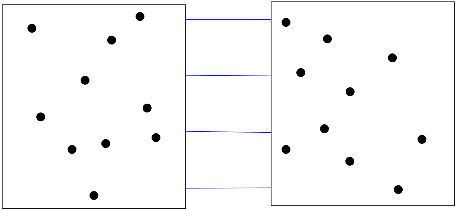

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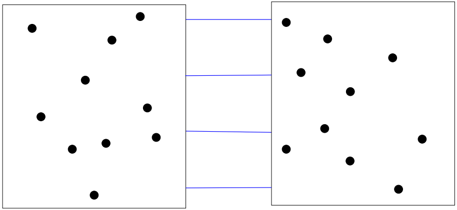
- at the highest level of the hierarchical scheme, it turns out that $D \sim \sqrt{A}$, so $\Omega(\sqrt{A}/\lambda)$ bits can be transmitted simultaneously

Proof idea (cont'd)



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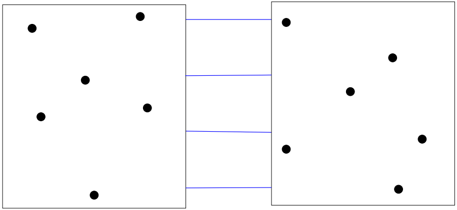

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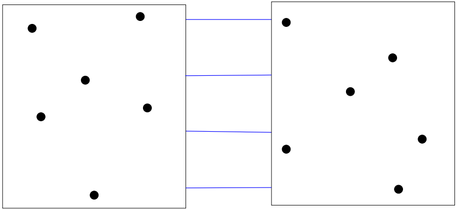

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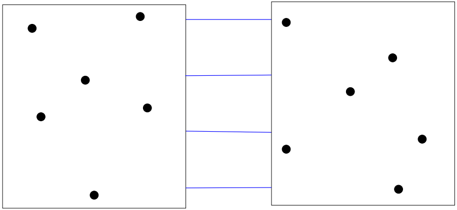

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- a simple solution again: reduce the number of nodes communicating simultaneously, so as to meet the spatial limitation
- !!! the area occupied by the nodes should be kept fixed !!!
- this way, a throughput of order arbitrarily close to $\Theta(\sqrt{A}/\lambda)$ is achievable via hierarchical cooperation

Conclusion #3

il $\sqrt{A}/\lambda \leq \sqrt{n}$, then multi-hop is optimal
if $\sqrt{A}/\lambda \geq \sqrt{n}$, then hierarchical cooperation is optimal
(provided that $SNR = P/d^2 \geq 0$ dB in both cases)

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- and no power limitation either ($d \sim 4$ m, $SNR \gg 0$ dB)

Open problem

What happens when both power and area are limiting factors?