

# ITERATIVE RECEIVER FOR SYNCHRONOUS CDMA USING HIDDEN MARKOV MODEL

Ejaz Khan and Dirk.T.M Sloek

Mobile Communications Department  
Institut Eurécom,  
B.P. 193, 06904 Sophia–Antipolis CEDEX, France  
Tel: +33 4 93 00 26 76, Fax: +33 4 93 00 26 27  
E-mail: `firstname.lastname@eurecom.fr`

**Abstract** - The expectation maximization (EM) algorithm is popular in estimating the parameters of the statistical models. In this paper, we consider application of the EM algorithm to Maximum Likelihood estimation. A Hidden Markov Model (HMM) formulation is used and EM algorithm is applied to estimate the parameters of the HMM which, in turn, are used to estimate received amplitudes of the users. The proposed method is compared with that of [8] and is found to be superior.

**Keywords** - Maximum Likelihood, expectation maximization, CDMA, hidden variable, HMM

## I. INTRODUCTION

Code Division Multiple Access (CDMA) is one of the most common multiple access techniques for wireless communication systems. In CDMA, all users in the channel transmit simultaneously, using quasi-orthogonal spreading codes to reduce inter-user interference. In recent years, many iterative algorithms have been proposed, Talwar, et al [6] proposed iterative least square with enumeration (ILSE) which solves the problem by estimating the channel by short training sequence or from previous estimates and find the data sequence over all possible data in the Finite Alphabet (FA). They also proposed iterative least square with projection (ILSP) which also initially estimates the channel with the same method as for ILSE and treats the problem as continuous optimization problem and projects the results onto closest discrete alphabet. In [7], a constrained Maximum Likelihood problem was considered with the data vector to lie with in hyper cube and called it as Box constrained ML. Similarly, they also proposed problem of maximizing likelihood function over sphere i.e. confine the solution vector to lie within the sphere and project the solution vector on the sphere. In this paper we consider the problem of estimating the received amplitudes of the users knowing only their spreading codes.

The HMM [1,5] is a stochastic model of the process that exhibits features that changes over time and is a finite set of states, each of which is associated with a (generally multi-dimensional) probability distribution. Transitions among the

Eurecom's research is partially supported by its industrial partners: Ascom, Cégétel, France Télécom, Hitachi, IBM France, Motorola, Swisscom, Texas Instruments, and Thomson CSF

states are governed by the set of probabilities called transition probabilities. In a particular state, an outcome or observation can be generated, according to the associated probability distribution. It is the outcome, not the state visible to the external observer and therefore states are "hidden" hence the name Hidden Markov Model. Direct maximization of the likelihood function for HMM is difficult task [5] and therefore we use expectation maximization algorithm to update the parameters of HMM iteratively. At each iteration the likelihood function is increased and converges to the stationary point, it is important to select initial values close to the global maximum. The rest of the paper is organized as follows: The signal model for the problem is described in section 2. Section 3 is devoted to describe HMM and the development of the EM based algorithm to estimate its parameters. In section 4 simulations are discussed and in section 5 conclusions are drawn.

## II. SIGNAL MODEL

We consider a  $K$  user synchronous CDMA system with processing gain  $P$ . After chip matched filtering and chip rate sampling, we can model the output of such system (in a single data symbol interval) as  $P$ -dimensional vector  $y$ , given by [4]

$$y = \bar{S}Cb + n \quad (1)$$

where  $\bar{S}$  is  $P \times K$  matrix whose columns are  $K$  users' normalized spreading sequences.

$$\bar{S} = [\bar{s}_1 | \bar{s}_2 | \dots | \bar{s}_K] \quad (2)$$

$C = \text{diag}(C_1, \dots, C_K)$ ,  $k$  users' received amplitudes,  $b = [b_1, \dots, b_K]^T$  contains the symbols transmitted by the users.  $n$  is  $P$ -dimensional white gaussian noise vector with variance  $\sigma^2$ . We assume that users' symbols are independent i.e.  $E[b_k b_l] = 1$  if  $k = l$  and 0 if  $k \neq l$ . We can rewrite equation (1) as

$$y = Hb + n \quad (3)$$

where  $H = \bar{S}C$  is  $P \times K$  dimensional matrix.

Given model of equation (3) our goal is to estimate  $C$  i.e. Amplitudes and  $b$  (users symbol) from multiple independent observation of  $y$ .

### III. DEVELOPMENT OF EM ALGORITHM FOR HMM

We formulate our problem as HMM in which the received signal is a function of Markov chain and then find ML solution to estimate its parameters. Direct ML estimation is computationally demanding. Fortunately, EM algorithm provides computationally efficient solution to the resulting optimization problem. We consider BPSK case, in which the transmitted data takes on two possible values  $\pm 1$ . The received data sequence in the presence of noise can be characterized as a stochastic process (function of Markov chain) that has an underlying Markovian finite-state structure. A Markov chain is capable of being in finite number of states which for our case is  $J = 2^K$ , where  $K$  is the number of users. The current state of the system is denoted by  $s_t$ . The probability of transition from a state at current (discrete) time  $t$  to any other state at time  $t + 1$  depends on the current state, and not on any prior states. The state transition probabilities  $a_{i,j}$  are given by

$$\begin{aligned} a_{i,j} &= P(s_{t+1} = j | s_t = i, s_{t-1} = i_1, \dots) \\ &= P(s_{t+1} = j | s_t = i) \end{aligned} \quad (4)$$

In our case, all the state transition have the same probability and is given by

$$a_{i,j} = P(s_{t+1} = j | s_t = i) = \frac{1}{J} \quad (5)$$

It is common to express state transition probabilities as matrix  $A$  with elements  $a_{i,j}$ ,  $1 \leq i, j \leq J$  where  $a_{i,j} \geq 0$  and  $\sum_{j=1}^J a_{i,j} = 1$  for each  $i$ . The initial state is chosen according to probability

$$\Pi = [P(s_0 = 1), \dots, P(s_0 = J)]^T = [\Pi_1, \dots, \Pi_J]^T, \quad (6)$$

where  $\Pi = \frac{1}{J}$ , for  $1 \leq i \leq J$  as all users symbols have the same probability.

In each state at time  $t$ ,  $s_t$ , an output is selected according to the density  $f(Y_t = y_t | s_t = i) = f_{s_t}(y)$  and is given by Gaussian distribution. The triple  $\lambda = (A, \Pi, f_s)$  defines HMM. Let the elements of HMM be parametrized by  $\theta$ . Given observation  $y = y_1, \dots, y_T$ , the ML estimate of  $\theta$  is given by

$$l_y(\theta) = \arg \max_{\theta} f(y_1, \dots, y_T | A(\theta), \Pi(\theta), f_s(\theta)) \quad (7)$$

This is complicated maximization problem. The EM algorithm [1,2,5], however, provides the power necessary to compute without difficulty. In the following lines we will describe EM algorithm. EM algorithm is an iterative approach to Maximum Likelihood Estimation (MLE), originally formalized in (Demster, Laird and Rubin). Each iteration is composed of two steps: an expectation (E) step and a maximization (M) step. The aim is to maximize the loglikelihood  $l(\theta; D) = \log L(\theta; D)$ , where  $\theta$  are parameters of the model and  $D$  are the data. Suppose that this optimization problem would be simplified by the knowledge of the additional variable  $\chi$ , known as missing or hidden data. The set

$D_c = D \cup \chi$  is referred to as the complete data set (in the same context  $D$  is referred to as incomplete data set). Correspondingly, the loglikelihood function  $l_c(\theta; D_c)$  is referred to as complete data likelihood.  $\chi$  is chosen such that the function  $l_c(\theta; D_c)$  would be easily maximized if  $\chi$  were known. However, since  $\chi$  is not observable,  $l_c$  is a random variable and cannot be maximized directly. Thus, the EM algorithm relies on integrating over the distribution of  $\chi$ , with the auxiliary function  $Q(\theta, \hat{\theta}) = E_{\chi}[l_c(\theta; D_c | D, \hat{\theta})]$ , which is the expected value of the complete data likelihood, given the observed data  $D$  and the parameter  $\hat{\theta}$  computed at the previous iteration. Intuitively, computing  $Q$  corresponds to filling the missing data using the knowledge of the observed data and previous parameters. The auxiliary function is deterministic and can be maximized. An EM algorithm iterates the following two steps, for  $k=1,2,\dots$ , until local or global maximum of the likelihood is found.

Expectation: Compute

$$Q(\theta; \theta^{(k)}) = E_{\chi}[l_c(\theta; D_c | D, \theta^{(k)})] \quad (8)$$

Maximization: Update the parameters as

$$\theta^{(k+1)} = \arg \max_{\theta} Q(\theta; \theta^{(k)}), \quad (9)$$

In some cases, it is difficult to analytically maximize  $Q(\theta; \theta^{(k)})$ , as required by the M-step of the above algorithm, and we are only able to compute a new value  $\theta^{(k+1)}$  that produces an increase of  $Q$  at each iteration. In this case we have so called generalized EM (GEM) algorithm.

We interpret data  $y$  as incomplete data which is part of complete data vector  $x = (y, s)$ . The pdf of the complete data can be written as

$$f(x|\theta) = f(y, s|\theta) = f(y|s, \theta)f(s|\theta) \quad (10)$$

where  $f(s|\theta)$  is given by

$$f(s|\theta) = \Pi_{s_0}(\theta) \prod_{t=1}^T a_{s_{t-1}, s_t}(\theta) \quad (11)$$

The pdf of the observation, conditioned upon unobserved states, factors in the following way [5]

$$f(y|s, \theta) = \prod_{t=1}^T f(y_t | s_t, \theta) \quad (12)$$

where  $f(y_t | s_t, \theta)$  is given by Gaussian distribution with mean  $\mu_{s_t} = H b_{s_t}$ . The expectation step of the EM algorithm is given by

$$Q(\theta | \theta^{(k)}) = E \left[ \log \left( f(y, s | \theta) \right) \middle| y, \theta^{(k)} \right] \quad (13)$$

Since the expectation is conditioned upon observation, the only random component comes from state variable. The E-step can thus be written, after some algebra and using Bayes'

rule, as

$$Q(\theta|\theta^{(k)}) = \frac{1}{f(y|\theta^{(k)})} \sum_{S \in S} f(y|s, \theta^{(k)}) f(s|\theta^{(k)}) \left[ \sum_{t=1}^T \left( -\log(2\Pi\sigma^2) - \frac{1}{2\sigma^2} (y_t - \mu_{s_t}(\theta))^T (y_t - \mu_{s_t}(\theta)) \right) + \log \pi_{s_0}(\theta) + \sum_{t=1}^T \log a_{s_{t-1}, s_t}(\theta) \right] \quad (14)$$

The maximization step which gives updated value of the parameters is given by:

For means:

$$\mu_s^{(k+1)} = \frac{\sum_{S \in S} f(y|s, \theta^{(k)}) f(s|\theta^{(k)}) \sum_{t:s_t=s} y_t}{\sum_{S \in S} f(y|s, \theta^{(k)}) f(s|\theta^{(k)}) \sum_{t:s_t=s} 1} \quad (15)$$

Similarly we can formulate update equations for initial state probability and transition probabilities by maximizing (14) with their proper constraints [5]. Fortunately, in our case initial state probability and transition probabilities are constant and are given by  $2^{-K}$  and  $2^{-K}$  respectively. The convergence of the EM algorithm to a solution and the number of iterations depends on the tolerance, the initial parameters, the data set, etc.

After convergence of the algorithm H is estimated, using (15), as

$$\hat{H} = \sum_S \mu_s b_s^T \left( \sum_S b_s b_s^T \right)^{-1} \quad (16)$$

The users' signal amplitudes is given by

$$\hat{C} = (\bar{S}^T \bar{S})^{-1} \bar{S}^T \hat{H} \quad (17)$$

#### IV. SIMULATIONS

In the following simulations we study the estimation error of the proposed algorithm for estimating received amplitudes of the users. The proposed algorithm was used for two, three and eight users. In all simulations the users signal amplitudes are set to be one. In figure 1, we compare the performance of the algorithm for two and three users. In figure 2, we compare the performance of the proposed algorithm with that of gaussian mixture method [8]. The method has been tested across a range of -5 to 30 dB signal to noise ratio (SNR) points. Due to very little variation in the result, 20 Monte Carlo trials are performed for each SNR point. Data block of 32 symbols are used in each simulation. The spreading gain is kept fixed, 32 in simulations. The method outperformed Gaussian mixture method [8] for all SNR points. Figure 3 shows the result for eight users with amplitudes set to one and we also show in the same figure the performance when one of the user (user of interest) having half the

received amplitude with respect to other 7 users. In all simulations the users' amplitudes are averaged for each SNR point. The users symbols can be detected by employing method of least squares or by choosing the maximum of the posteriori probabilities of the hidden states.

#### V. CONCLUSION

In this paper, we presented Hidden Markov Model (HMM) formulation to blindly demodulate the users data and estimate users' amplitudes. We propose the Expectation maximization (EM) based algorithm to estimate the parameters of HMM. Simulations results shows superiority of our method to the Gaussian mixture method and almost attains Cramer-Rao (CR) lower bound.

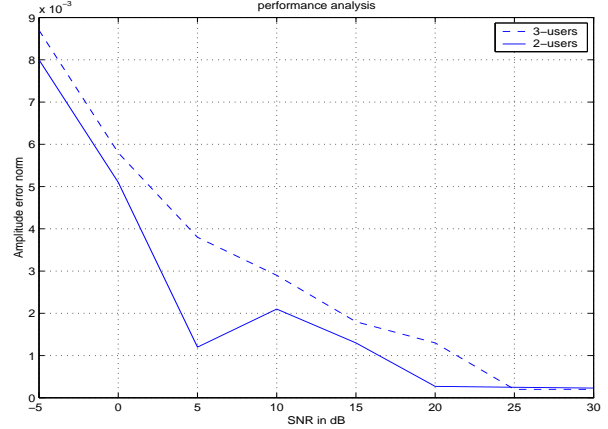


Fig. 1. Amplitude estimation error.

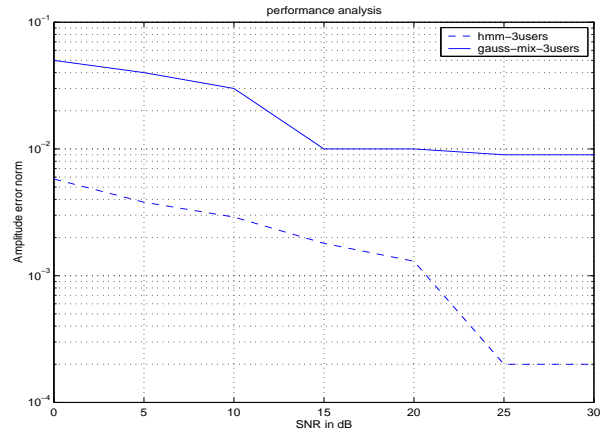


Fig. 2. Amplitude estimation error.

#### REFERENCES

- [1] J.A .Bilmes, "A gentle tutorial of EM algorithm and its application to parameter estimation for Gaussian mixture and hidden markov model," tech. rep., U.C.Berkely, April 1998.

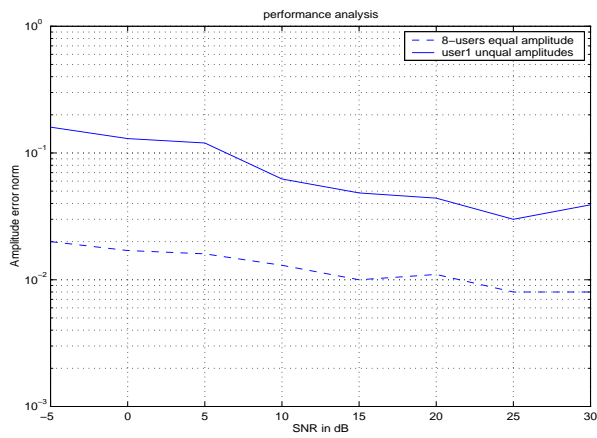


Fig. 3. Amplitude estimation error.

- [2] A .P .Dempster, N .M .Laird, and D .B .Rubin, "Maximum likelihood from incomplete data via EM algorithm," *J.Royal Statistical Society B*, no. 39, pp. 1–38, 1977.
- [3] G. McLachlan and T. Krishnan, *The EM algorithm and extension*. John Wiley and sons Inc, 1997.
- [4] Y .Yao and H .V .Poor, "EM based blind demodulation of synchronous cdma," *IEEE 6<sup>th</sup> Int. symp. on spread spectrum Tech. and Application*, 2000.
- [5] T . Moon, "The expectation maximization algorithm," *IEEE signal processing Magazine*, November 1996.
- [6] S .Talwar, M . Viberg, and A .Paulraj,"Blind separation of synchronous co-channel digital signals using an antenna array-Part-1: algorithms," *IEEE trans. signal Processing*, 44,pp. 1184-1197, 1996
- [7] P. H. Tan, L. K. Rasmussen and T. J. Lim, "Constrained maximum-likelihood detection in CDMA", *IEEE Trans. Commun.*,vol. 49, pp. 142-153, Jan. 2001.
- [8] E. Khan, D. T. M. Slock, "Iterative blind demodulation of synchronous CDMA", accepted for PIMRC, 2002.