MAXIMUM-LIKELIHOOD BLIND EQUALIZATION OF MULTIPLE FIR CHANNELS

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ABSTRACT

We pursue our Iterative Quadratic Maximum Likelihood (IQML) approach to blind estimation of multiple FIR channels. We use a parameterization of the noise subspace in terms of linear prediction quantities. This parameterization is robust w.r.t. a channel length mismatch. Specifically, when the channel length is overestimated, no problems occur. Underestimation leads to a reduced-order channel estimate. We introduce two Matched Filter Bounds (MFBs) to characterize the performance of receivers using reduced-order channel models. The first one (MFB1) uses the channel model to perform the spatio-temporal matched filtering that yields data reduction from multichannel to single-channel form. The rest of the processing remains optimal. MFB2 on the other hand bounds the performance of the Viterbi algorithm with the reduced channel model. It is shown that the reduced model provided by IQML is the one that maximizes MFB1. We also propose some low complexity techniques for obtaining consistent estimates with which to initialize IQML.

1. INTRODUCTION

Consider a sequence of symbols $a(k)$ received through m channels $y(k) = \sum_{i=0}^{N-1} h(i)a(k-i) + v(k) = H_N A_N(k) + v(k)$, $y(k) = [y_1^H(k) \cdots y_m^H(k)]^H$, $H_N = [h(0) \cdots h(N-1)]$, $A_N(k) = \left[a(k)^H \cdots a(k-N+1)^H\right]^H$, where superscript ^{H} denotes Hermitian transpose. Let $\mathbf{H}(z) = \sum_{i=0}^{N-1} h(i)z^{-i} = [\mathrm{H}_1^H(z) \cdots \mathrm{H}_m^H(z)]^H$ be the SIMO channel transfer function. Consider additive independent white Gaussian noise $v(k)$ with $r_{vv}(k-i) = \mathrm{E}\, v(k)v(i)^H = \sigma_v^2 I_m\, \delta_{ki}$, but deterministic transmitted symbols $a(k)$. Assume we receive M samples:

$$Y_M(k) \;=\; \mathcal{T}_M(H_N)\, A_{M+N-1}(k) + V_M(k) \tag{1}$$

where $Y_M(k) = [y^H(k) \cdots y^H(k-M+1)]^H$ and similarly for $V_M(k)$, and $\mathcal{T}_M(H_N)$ is a block Toepliz matrix with M block rows and $[H_N \;\; 0_{m\times(M-1)}]$ as first block row.

2. THE IQML ALGORITHM

A robust, though slightly approximate[1], approach to the ML estimation problem comes about as follows. In [1] we have shown that the multivariate prediction filter $\mathbf{P}(z)$ for the noise-free received signal satisfies $\mathbf{P}(z)\mathbf{H}(z) = h(0)$. The order of $\mathbf{P}(z)$ is $\underline{L} = \left\lceil \frac{N-1}{m-1} \right\rceil$. Now, if $h(0)^{\perp}$ is a $m \times (m-1)$ matrix such that $h(0)^{\perp\, H} h(0) = 0$, then $\overline{\mathbf{P}}(z) = h(0)^{\perp\, H}\mathbf{P}(z)$ is a $(m-1)\times m$ polynomial that satisfies $\overline{\mathbf{P}}(z)\mathbf{H}(z) = 0$. Asymptotically the ML criterion becomes the sum of the squares of $w(k)$ in

$$\begin{aligned} y(k) = \mathbf{H}(q)\, a(k) + v(k) \Rightarrow & \\ \overline{\mathbf{P}}(q)y(k) = \overline{\mathbf{P}}(q)v(k) = & \left(\overline{\mathbf{P}}(q)\overline{\mathbf{P}}^{\dagger}(q)\right)^{1/2} w(k) \end{aligned} \tag{2}$$

where $\overline{\mathbf{P}}^{\dagger}(z) = \overline{\mathbf{P}}^H(1/z^*)$ and $(.)^{1/2}$ is a minimum-phase factor of its argument. Note that $\mathrm{E}w(k)w(k)^H = \sigma_v^2 I_{m-1}$ and hence

$$(m-1)\sigma_v^2 = \mathrm{tr}\, r_{ww}(0) = \;\mathrm{tr}\; \oint \mathrm{P}_{\overline{\mathbf{P}}^{\dagger}(z)} \mathrm{S}_{yy}(z)\, \frac{dz}{z} \tag{3}$$

where $\mathrm{P}_{\mathbf{H}(z)} = \mathbf{H}(z)\left(\mathbf{H}^{\dagger}(z)\mathbf{H}(z)\right)^{-1}\mathbf{H}^{\dagger}(z)$ and tr denotes trace. This leads us to introduce an approximate ML problem as

$$\min_{\overline{\mathbf{P}}} \|\mathrm{P}_{\mathcal{T}^H_{M-L}(\overline{\mathbf{P}})} \mathbf{Y}_M\|_2^2 \tag{4}$$

for any $L \geq \underline{L}$. (4) can be solved in the IQML fashion.

A minimal parameterization for $\overline{\mathbf{P}}$ is

$$\overline{\mathbf{P}} = h(0)^{\perp H}\mathbf{P} = \left[\, h(0)^{\perp H} \quad Q \,\right] \;,\quad h(0)^{\perp H} = [I_{m-1}\; g]\mathcal{P} \tag{5}$$

where g ($(m-1)\times 1$) and Q ($(m-1)\times mL$) are the free parameters and $\mathcal{P}$ is a permutation matrix that permutes g into the column of $h(0)^{\perp H}$ that corresponds to the position of the largest element of $h(0)$. To determine this position requires an initial estimate of $h(0)$. One such estimate can be obtained by inspecting the estimate of $r_{yy}(N-1) = \sigma_a^2 h(N-1)h(0)^H$ (for white $a(k)$). Alternative estimates for $h(0)$ can be obtained from the initialization procedure for $\overline{\mathbf{P}}$ to be discussed. The optimization w.r.t. the unconstrained parameters Q and g in the IQML procedure is simple. With the parameters in the inverted matrix fixed at their values obtained from the previous iteration, the criterion is quadratic in g and Q and separable: first minimize w.r.t. Q and obtain Q as a function of g. Substitute this value of Q in the criterion and minimize the resulting quadratic criterion in g. A more robust parameterization (not dependent on the permutation matrix) of $h(0)^{\perp}$ is the following nonlinear parameterization in terms of $m-1$ complex numbers s_i, $|s_i| \leq 1$. For example for $m = 5$, we get

$$h(0)^{\perp H} = \begin{bmatrix} s_1 & c_1 & 0 & 0 & 0 \\ s_2 c_1 & -s_2 s_1 & c_2 & 0 & 0 \\ s_3 c_2 c_1 & -s_3 c_2 s_1 & -s_3 s_2 & c_3 & 0 \\ s_4 c_3 c_2 c_1 & -s_4 c_3 c_2 s_1 & -s_4 c_3 s_2 & -s_4 s_3 & c_4 \end{bmatrix}$$

[1] The dimension of the parameterized noise subspace will get slightly reduced, this has an asymptotically negligible effect.

where $c_i = \sqrt{1-|s_i|^2}$. Since in every iteration of the IQML procedure, the cost function is quadratic in s_i and c_i, minimization w.r.t. a particular pair s_i, c_i is straightforward. Minimization w.r.t. all s_i can be done by performing alternating minimizations w.r.t. $s_1, s_2, \ldots, s_{m-1}$ until convergence.

3. CONSISTENT INITIALIZATION

A good estimate of $\overline{\mathbf{P}}$ is required to initialize the IQML algorithm. In fact, with a consistent initialization, one iteration of the IQML algorithm yields an ABC (asymptotically best consistent) estimate. One starts by consistently estimating $\mathrm{R}_L^{y} = \mathrm{E}\, Y_L Y_L^H$ for $L \geq \underline{L}+1$. One approach then is to estimate σ_v^2 as $\lambda_{min}(\mathrm{R}_L^{y})$ and to perform linear prediction on $\mathrm{R}_L^{y} - \lambda_{min}(\mathrm{R}_L^{y})\, I$ as estimate of the noise-free covariance matrix. The prediction problem then gets truncated at a certain order where the $m \times m$ prediction error variance matrix is deemed to be of rank one. This gives some information on the channel length N. This information could be exploited to clean up the covariance matrix estimate: in principle $ryy(i) = 0,\ i \geq N$ and $\mathrm{rank}(ryy(N-i)) = i,\ i = 1, \ldots, m-1$. Then $\lambda_{min}(\mathrm{R}_L^{y})$ could be estimated again and the prediction problem could be solved again and this process could be reiterated.

3.1. Minimal Signal Subspace Fitting

If $\mathrm{R}_L^{y} = \sum_{i-1}^{mL} \lambda_i V_i V_i^H$ is the ordered eigendecomposition, then N can be inferred from the distribution of the eigenvalues. The channel can then be estimated by signal subspace fitting [2] $\min_{H_N} \left\| V_{L+N:mL}^H \mathcal{T}_L(H_N) \right\|_F^2$. A complete eigendecomposition is expensive though. However, it suffices to express orthogonality of the parameterized signal subspace to $m-1$ noise subspace vectors to be able to determine H_N [1] and hence to minimize w.r.t. H_N

$$\left\| V_{m(L-1)+2:mL}^H \mathcal{T}_L(H_N) \right\|_F^2 = \left\| H^t_N \mathcal{T}_N(V_{m(L-1)+2:mL}^{H\,t}) \right\|_2^2 \tag{6}$$

where superscript t denotes transposition of the blocks when considering the quantity as an appropriate block matrix. Indeed, since the noise subspace is (part of) the column space of $\mathcal{T}_{L-\underline{L}}^H\left(\overline{\mathbf{P}}_{\underline{L}}\right)$, there exists a matrix of combination coefficients W such that $V_{m(L-1)+2:mL} = \mathcal{T}_{L-\underline{L}}^H\left(\overline{\mathbf{P}}_{\underline{L}}\right) \mathrm{W}^H$. Letting $\mathbf{W}(z)$ be the z-transform of the $(m-1) \times (m-1)$ samples in the block row vector W, then the criterion in (6) can be rewritten as

$$\left\| \mathbf{W}(z)\, \overline{\mathbf{P}}(z)\, \mathbf{H}(z) \right\|^2 \ . \tag{7}$$

Since W is full rank, $\mathbf{W}(z)$ is generically full rank, and hence $\mathbf{W}^\dagger(z)\mathbf{W}(z)$ is generically a positive definite weighting function. This implies that if $\left\| \mathbf{W}(z)\, \overline{\mathbf{P}}(z)\, \mathbf{H}(z) \right\|^2 = 0$, then $\overline{\mathbf{P}}(z)\, \mathbf{H}(z) = 0$. This in turn implies that $\mathbf{H}(z)$ is the correct channel transfer function, Q.E.D.

Since in this case only the $m-1$ "smallest" eigenvectors are determined, the complete eigenvalue distribution is not available for the determination of N. N then needs to be estimated as indicated before. A misestimation of N is not unlikely. If N is estimated to be $N' > N$, then all solutions $H_{N'}$ that render the cost function in (6) zero satisfy $\overline{\mathbf{P}}_{\underline{L}}(z)\, \mathbf{H}_{N'}(z) = 0$ and hence are of the form $\mathbf{H}_{N'}(z) = \mathbf{H}_N(z)\mathrm{G}(z)$ where $\mathrm{G}(z)$ is a scalar polynomial of order $N'-N$. Hence this approach so far is not robust as a means to estimate the channel directly. However, $\mathbf{H}_{N'}(z) = \mathbf{H}_N(z)\mathrm{G}(z)$ can be interpreted as the overall transfer function fed by white symbols that first get colored by $\mathrm{G}(z)$ and then pass the correct channel $\mathbf{H}_N(z)$. The coloring of the channel input modifies the prediction filter $\mathbf{P}(z)$ obtained from $\mathbf{H}_{N'}(z)\mathbf{H}_{N'}^\dagger(z)$, but not the $\overline{\mathbf{P}}(z)$ part [2], which is the quantity of interest here. Below we show an example with $m = 2$, $N = 4$, $N' = 6$ and randomly generated real channel coefficients.

$$H_N = \begin{bmatrix} 0.9685 & 0.4201 & 1.6859 & -0.9020 \\ 0.6703 & -2.8728 & 0.0279 & -2.0533 \end{bmatrix}$$

$$\hat{H}_N = \begin{bmatrix} 0.2303 & 0.0999 & 0.4008 & -0.2145 \\ 0.1594 & -0.6830 & 0.0066 & -0.4882 \end{bmatrix}$$

$$\hat{H}_{N'} = \begin{bmatrix} -0.1175 & 0.0105 & -0.3129 & 0.1577 & -0.2924 & 0.1258 \\ -0.0813 & 0.3910 & -0.2791 & 0.6516 & -0.1341 & 0.2864 \end{bmatrix}$$

$$\bar{P}_N = \widehat{\bar{P}}_N = \widehat{\bar{P}}_{N'} = [\, 0.5691\ \ -0.8223\ \ -2.4391\ \ -0.3567\ \ 0.0237\ \ -1.4314\ \ -1.7433\ \ 0.7659\,]$$

4. MODEL REDUCTION

The IQML method proposed by Hua in [3] uses a different choice for the blocking equalizers:

$$\overline{\mathbf{P}}(z) = \begin{bmatrix} -\mathrm{H}_2(z) & \mathrm{H}_1(z) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\mathrm{H}_m(z) & 0 & \cdots & \mathrm{H}_1(z) \end{bmatrix} \tag{8}$$

which is also FIR. However, the noise subspace is in this case parameterized by the columnspace of a matrix that has many more columns than its column rank. Furthermore, if the channel order is overestimated as $N' > N$, then the channel estimate is of the form $\mathbf{H}_{N'}(z) = \mathbf{H}_N(z)\mathrm{G}(z)$ where the scalar polynomial $\mathrm{G}(z)$ of order $N'-N$ represents $N'-N$ zeros in common between the individual $H_{i,N'}(z)$. $\mathbf{H}_N(z)$ could again be found from $\mathbf{H}_{N'}(z)$ via the $\overline{\mathbf{P}}(z)$ of linear prediction as indicated before. In contrast to Hua's method, our parameterization of the noise subspace does not have the dimensionality problem. Furthermore, since in every iteration of the IQML problem, we solve a prediction problem, there is no problem with order overestimation because the superfluous prediction coefficients become automatically zero.

Furthermore, it is interesting to analyze what happens when the prediction order and hence the channel order gets underestimated. Let $\overline{\mathbf{P}}_{\underline{L}'}(z)$ be the blocking equalizer of reduced order. From $\overline{\mathbf{P}}_{\underline{L}'}(z)\mathbf{H}_{N'}(z) = 0$ we can determine the corresponding reduced-order channel estimate which will satisfy

$$\mathrm{P}_{\mathbf{H}_{N'}(z)} + \mathrm{P}_{\overline{\mathbf{P}}^\dagger_{\underline{L}'}(z)} = I_m \ . \tag{9}$$

For the investigation of the model reduction behavior of the ML criterion, we shall neglect the estimation variance. Hence, the ML criterion (4) divided by M leads asymptotically for large M (LLN) to the criterion

$$\begin{aligned} \mathrm{tr}\, rww(0) &= \mathrm{tr} \oint \mathrm{P}_{\overline{\mathbf{P}}^\dagger_{\underline{L}'}(z)}\, \mathrm{S}yy(z)\, \frac{dz}{z} \\ &= \mathrm{tr}\, ryy(0) - \mathrm{tr} \oint \mathrm{P}_{\mathbf{H}_{N'}(z)}\, \mathrm{S}yy(z)\, \frac{dz}{z} \\ &= \mathrm{tr}\, ryy(0) - \sigma_v^2 - \sigma_a^2 \oint \mathbf{H}_N^\dagger(z) \mathrm{P}_{\mathbf{H}_{N'}(z)} \mathbf{H}_N(z)\, \frac{dz}{z} \ . \end{aligned} \tag{10}$$

Before interpreting this last expression, we shall review the concept of the Matched Filter Bound (MFB).

For the multichannel $\mathbf{H}(z)$ shown in Fig. 1, the MFB is given by $\|\mathbf{H}\|^2 \sigma_a^2 / \sigma_v^2$ where we assume that the symbols

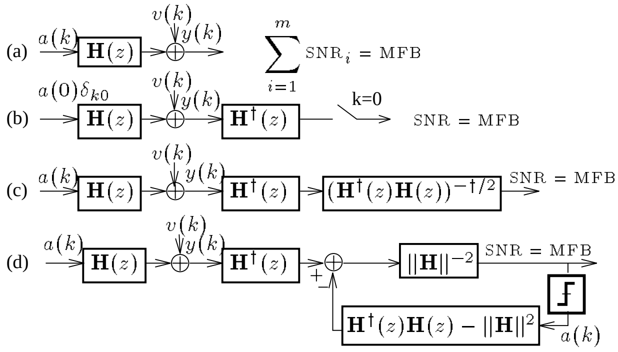

Figure 1. Four ways to get the MFB from SNRs.

$a(k)$ are white and the noise $v(k)$ is temporally and spatially white. Then the MFB can alternatively be calculated as the sum of the SNRs in the individual channels in (a), as the SNR of the appropriate output sample of the matched filter (MF) when transmitting only one sample in (b), as the SNR of the output of the whitened MF (WMF) in (c) of finally as the SNR at the decision element of the (unrealizable) non-causal DFE (NCDFE) in (d). The MFB that we indicated here is the one that corresponds to continuous transmission. The MFB becomes sample-dependent in the case of burst (packet) transmission. The MFB indicates the optimal symbol detection performance when the channel $\mathbf{H}(z)$ is completely known. We shall now discuss appropriate MFBs when a reduced-order channel model is used. Two levels of suboptimality ensue in that case. These correspond to the two ways of implementing ML sequence estimation (MLSE) in the multichannel case: either used a vectorial matched filter and work with a scalar signal, or work with the vector received signal directly.

4.1. Matched Filter Bound One (MFB1)

Assume we have a reduced-order model $z^{-d}\mathbf{H}_{N'}(z)$ of $\mathbf{H}_N(z)$ ($d \in \{0,1,\ldots,N-N'\}$). In a first step of suboptimality, we can consider that in the data reduction step from multichannel to single-channel, we use the MF matched to the reduced model $z^{-d}\mathbf{H}_{N'}(z)$. However, after this suboptimal data reduction, we shall allow optimal processing of the resulting single channel (this requires knowledge of $\mathbf{H}^{\dagger}_{N'}(z)\mathbf{H}_N(z)$ which represents less information than $\mathbf{H}_N(z)$ itself). In order to find the conventional MFB for the processing of the resulting scalar channel, it suffices to whiten the noise after the vector MF. The resulting scalar channel then indeed becomes one of additive white noise $n(k)$ as indicated in Fig. 2 so that the MFB can be calculated as considered before.

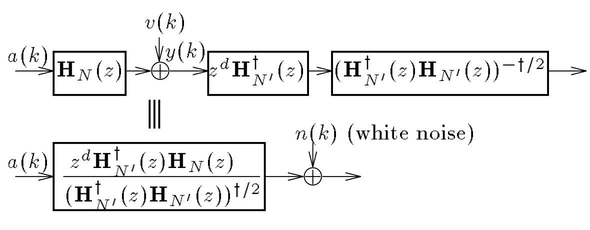

Figure 2. MFB1: reduced-order multichannel MF followed by a scalar MF.

We get for the continuous transmission MFB

$$\mathrm{MFB1} = \frac{\sigma_a^2}{\sigma_v^2} \oint \mathbf{H}^{\dagger}_N(z) \mathrm{P}_{\mathbf{H}_{N'}(z)} \mathbf{H}_N(z)\, \frac{dz}{z}\,. \qquad (11)$$

By comparing with (10), we find that asymptotically the reduced-order channel estimate obtained with our IQML method is the one that maximizes MFB1!

It is interesting to analyze the variation of MFB1(N') as a function of the reduced order N'. For $N = N'$ we get MFB1(N)= $\frac{\sigma_a^2}{\sigma_v^2}\|\mathbf{H}_N\|^2$. It is not difficult to show that in the limiting case $N'=1$ (purely spatial channel model), we get MFB1(N)= $\frac{\sigma_a^2}{\sigma_v^2}\lambda_{\max}\left(H_N H_N^H\right)$. We then can derive the following bounds

$$1 \leq \frac{\mathrm{MFB1}(N)}{\mathrm{MFB1}(1)} = \frac{\mathrm{tr}\left(H_N H_N^H\right)}{\lambda_{\max}\left(H_N H_N^H\right)} \leq \min(m,N) \qquad (12)$$

The lower bound is attained when $h(i) \sim h(0)$, $i = 1,\ldots,N-1$. The upper bound is attained when either $H_N H_N^H \sim I_m$ or $H_N^H H_N \sim I_N$, whichever is of full rank. In a statistical set-up, if the m channel impulse responses are i.i.d., then the upper bound is approached as the delay spread grows. Note that the case $N'=1$ corresponds to replacing a full spatio-temporal treatment by the cascade of a purely spatial combiner followed by a purely temporal treatment.

Since the IQML method will normally be applied to a burst of data $Y_M(k)$, it is interesting to pursue the burst mode equivalent of MFB1. Let $\mathcal{T}_N$ and $\mathcal{T}_{N'}$ denote $\mathcal{T}_M(H_N)$ and $\mathcal{T}_M(H_{N'})$ resp. and consider the Cholesky factorization $\mathcal{T}_{N'}^H\mathcal{T}_{N'} = LL^H$. Then the $M+N'-1$ reduced-order WMF outputs are

$$L^{-1}\mathcal{T}_{N'}^H Y = L^{-1}\mathcal{T}_{N'}^H\mathcal{T}_N A + L^{-1}\mathcal{T}_{N'}^H V \qquad (13)$$

The covariance matrix of the noise component is indeed $\sigma_v^2 I_{M+N'-1}$ while the covariance matrix of the signal part is $\sigma_a^2 L^{-1}\mathcal{T}_{N'}^H\mathcal{T}_N\mathcal{T}_N^H\mathcal{T}_{N'}L^{-H}$. The sum of the SNRs of all WMF outputs is then

$$\sum_{i=1}^{M+N'-1}\frac{\sigma_a^2}{\sigma_v^2}\left(L^{-1}\mathcal{T}_{N'}^H\mathcal{T}_N\mathcal{T}_N^H\mathcal{T}_{N'}L^{-H}\right)_{ii} = \frac{\sigma_a^2}{\sigma_v^2}\,\mathrm{tr}\left(\mathrm{P}_{\mathcal{T}_{N'}}\mathcal{T}_N\mathcal{T}_N^H\right) \qquad (14)$$

This point of view corresponds to (a) in Fig. 1. To find the equivalent of (d) in Fig. 1, consider passing the previous WMF output $L^{-1}\mathcal{T}_{N'}^H Y$ through the scalar MF $\mathcal{T}_N^H\mathcal{T}_{N'}L^{-H}$. This gives the $M+N-1$ outputs

$$\mathcal{T}_N^H\mathrm{P}_{\mathcal{T}_{N'}}Y = \mathcal{T}_N^H\mathrm{P}_{\mathcal{T}_{N'}}\mathcal{T}_N A + \mathcal{T}_N^H\mathrm{P}_{\mathcal{T}_{N'}}V \qquad (15)$$

The sum of the SNRs at the NCDFE detector for the $M+N-1$ elements of $A_{M+N-1}(k)$ is

$$\sum_{i=1}^{M+N-1}\frac{\sigma_a^2}{\sigma_v^2}\left(\mathcal{T}_N^H\mathrm{P}_{\mathcal{T}_{N'}}\mathcal{T}_N\right)_{ii} = \frac{\sigma_a^2}{\sigma_v^2}\,\mathrm{tr}\left(\mathrm{P}_{\mathcal{T}_{N'}}\mathcal{T}_N\mathcal{T}_N^H\right) \qquad (16)$$

which is the same as in (14).

In order to investigate the effect of the model reduction performed by the IQML method, some simulations were performed for $m = 2$ channels. In order to concentrate on the model reduction effects and not on the estimation errors, the averaged likelihood function was maximized. Since the noise is white, this boils down to maximizing $\mathrm{tr}\left(\mathrm{P}_{\mathcal{T}_{N'}}\mathcal{T}_N\mathcal{T}_N^H\right)$ for which we developed the details of the IQML method. This cost function is proportional to the average MFB1 for burst mode transmission. Fig. 3 shows the evolution of the average MFB1 as a function of N' for a case in which the two impulse responses are orthonormal and a case in which they are almost colinear.

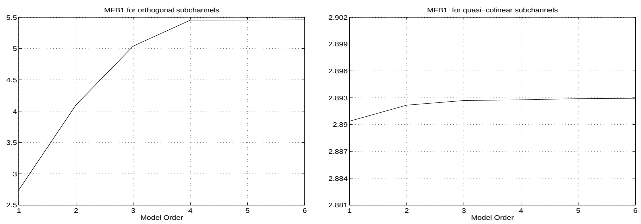

Figure 3. MFB1 as a function of $N' = 1, \ldots, N$ for $m = 2$, $M = 50$, $N = 6$ for orthonormal (left) and almost colinear (right) impulse responses.

4.2. Matched Filter Bound Two (MFB2)

We now go all the way in suboptimality. We will not only assume that the multichannel MF is based on the reduced channel model but in fact that the whole receiver is. To find the optimal performance in this case, consider MLSE. The received burst through the channel $\mathbf{H}_N(z)$ is $Y_M(k) = \mathcal{T}_M(\mathbf{H}_N)A_{M+N-1}(k)+V_M(k)$. Based on the reduced-order model $z^{-d}\mathbf{H}_{N'}(z)$, the MLSE problem is

$$\min_{\substack{a(i)\in\mathcal{A}\\ d\in\{0,1,\ldots,N-N'\}}} \|Y_M(k) - \mathcal{T}_M(H_{N'})A_{M+N'-1}(k-d)\|^2 \tag{17}$$

We obtain the MFB2 by considering the detection of a single symbol $a(i)$ assuming that the other symbols are known. It is easy to see that the continuous transmission version of this leads to the NCDFE depicted in Fig. 4.

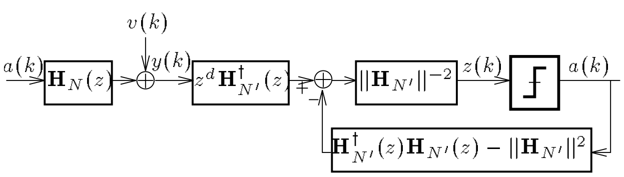

Figure 4. MFB2: MFB for MLSE with the reduced-order channel model.

The SNR at the NCDFE detector is

$$\text{MFB2} = \frac{\|\mathbf{H}_{N'}\|^2\sigma_a^2}{\sigma_v^2 + \sigma_a^2\|\mathbf{H}^\dagger_{N'}(z)(z^d\mathbf{H}_N(z) - \mathbf{H}_{N'}(z))\|^2/\|\mathbf{H}_{N'}\|^2}\,. \tag{18}$$

In contrast to MFB1, the delay d in the reduced-order channel model plays a role in MFB2. Note that the presence of an adjustable delay creates local minima for MLSE. Blind methods only allow the estimation of the channel up to a multiplicative constant. MFB2 on the other hand is quite sensitive to the choice of this scale factor. Within the spirit of blind methods, we have determined the magnitude of this scale factor on the basis of the variance of the received signal, which leads to $\|\mathbf{H}_{N'}\| = \|\mathbf{H}_N\|$. The determination of the phase of the scale factor is less obvious though. In simulations we have avoided this issue by restricting to real impulse responses. Some simulation results are shown in Fig. 5. There, the evolution of MFB1 and MFB2 as a function of N' is shown for the following two channels

$$\begin{aligned} H1 &= \begin{bmatrix} 1.0000 & 0.8000 & 0.5000 & 0.6000 & 0.1000 & 0.0050 \\ -1.5000 & 1.4000 & -0.9000 & 1.1000 & -0.0300 & 0.0050 \end{bmatrix} \\ H2 &= \begin{bmatrix} 1.0000 & 0.5000 & -0.1500 & 0.0550 & 0.0145 & -0.0014 \\ 1.5000 & -0.9500 & 0.3050 & 0.0695 & 0.0431 & -0.0043 \end{bmatrix} \end{aligned} \tag{19}$$

An example of the evolution of the averaged IQML cost function during the iterations of the IQML algorithm is shown in Fig. 6. We have found that the evolution is not always monotonic, contrary to what one might believe on

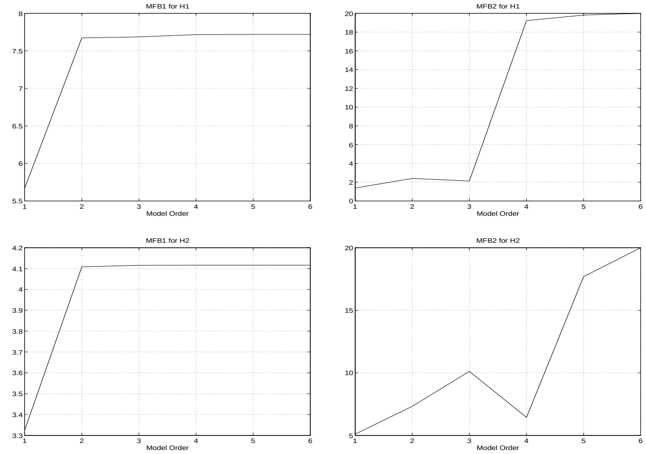

Figure 5. Comparison of MFB1 and MFB2 as a function of N' for channels *H1* and *H2*, $m = 2$, $N = 6$, $M = 50$.

the basis of Fig. 6. In fact, often a steady-state of small fluctuations is observed. However, we have also found that most of the convergence takes place in the first iteration for reasonably good initializations. Furthermore, the algorithm appears to be fairly insensitive to the initialization, various ways of obtaining a truncated $\overline{\mathbf{P}}$ almost always leading to the same result after convergence. The initialization used in Fig. 6 in fact is based on the prediction problem for the covariance matrix of a noisy signal with SNR=10dB (noise in the signal leads to a bias in $\overline{\mathbf{P}}$).

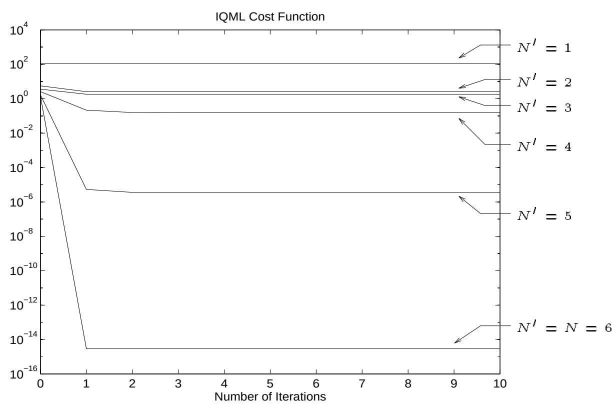

Figure 6. Evolution of the averaged IQML cost function during the iterations for *H1* and $N' = 1, \ldots, N$.

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