

# MULTISTREAM SPACE-TIME CODING BY SPATIAL SPREADING, SCRAMBLING AND DELAY DIVERSITY

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## ABSTRACT

The use of multiple transmitter and receiver antennas allows to transmit multiple signal streams in parallel and hence to increase communication capacity. To distribute the multiple signal streams over the MIMO channel, linear space-time codes have been shown to be a convenient way to reach high capacity gains with a reasonable complexity. The space-time codes that have been introduced so far are block codes, leading to the manipulation of possibly large matrices. To reduce complexity, we propose a flexible spatial spreading and scrambling framework which allows to transmit an arbitrary number of streams. The number of streams would in practice be adjusted to fit the channel rank. Special cases of partial scrambling, in the case in which  $N_{rx}$  is an integer fraction of  $N_{tx}$ , or no scrambling, when  $N_{rx} \geq N_{tx}$ , are also considered.

## 1. INTRODUCTION

Spatial multiplexing has been introduced independently in a 1994 Stanford University patent by A. Paulraj and by Foschini [1] at Bell Labs. Spatial multiplexing can be viewed as a limiting case of Spatial Division Multiple Access (SDMA) in which the various mobile users are collocated in one single user multi antenna mobile terminal. In that case, the various users are no longer distinguishable on the basis of their (main) direction (DOA) since all antennas are essentially collocated. Nevertheless, if the scattering environment is sufficiently rich, the antenna arrays at TX and RX can see the different DOAs of the multiple paths. One can then imagine transmitting multiple data streams, one stream per path. For this, the set of paths to be used should be resolvable in angle at both TX and RX. Without channel knowledge at the TX, the multiple streams to be transmitted just get mixed over the multiple paths in the matrix channel. They can generally be linearly recovered at the RX if the channel matrix rank equals or exceeds the number of streams. This rank equals the number of paths that are simultaneously resolvable at TX and RX. The assumptions we shall adopt for the proposed approach are no channel knowledge at TX, perfect channel knowledge at RX, frequency-flat channels for most of the paper.

## 2. LINEAR PREFILTERING APPROACH

We consider here the case of full rate transmission ( $N_s = N_{tx}$ ), when  $N_{rx} \geq N_{tx}$  such that the rank of the channel possibly equals the number of streams  $N_s$ . A general ST coding setup is sketched

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in Fig. 1. The incoming stream of bits gets transformed to  $N_s$  symbol streams through a combination of channel coding, interleaving, symbol mapping and demultiplexing. The result is a vector stream of symbols  $b_k$  containing  $N_s$  symbols per symbol period. The

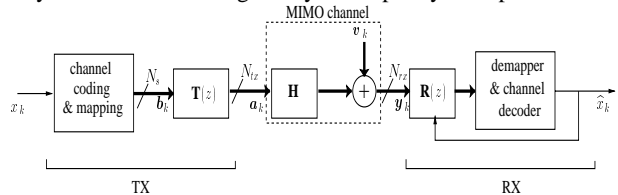
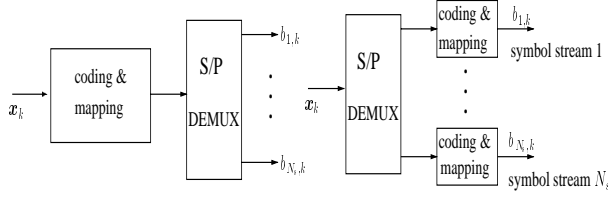


Fig. 1. General ST coding setup.

$N_s$  streams then get mapped linearly to the  $N_{tx}$  transmit antennas and this part of the transmission is called linear ST precoding. The output is a vector stream of symbols  $a_k$  containing  $N_{tx}$  symbols per symbol period. The linear precoding is spatiotemporal since an element of  $b_k$  may appear in multiple components (space) and multiple time instances (time) of  $a_k$ . The vector sequence  $a_k$  gets transmitted over a MIMO channel  $\mathbf{H}$  with  $N_{rx}$  receive antennas, leading to the symbol rate vector received signal  $y_k$  after sampling. The linear precoding can be considered to be an inner code, while the nonlinear channel coding etc. can be considered to be an outer code. As the number of streams is a factor in the overall bitrate, we shall call the case  $N_s = N_{tx}$  the full rate case, while  $N_s = 1$  corresponds to the single rate case. Instead of multiple antennas, more general multiple channels can be considered by oversampling, by using polarization diversity or other EM component variations, by working in beamspace, or by considering in phase and in quadrature (or equivalently complex and complex conjugate) components. In the case of oversampling, some excess bandwidth should be introduced at the transmitter, possibly involving spreading which would then be part of the linear precoding. As we shall see below, channel capacity can be attained by a full rate system without precoding ( $\mathbf{T}(z) = I$ ). In that case, the channel coding has to be fairly intense since it has to spread the information contained in each transmitted bit over space (across TX antennas) and time, see the left part in Fig. 2 and [2]. The goal of introducing the linear precoding is to simplify (possibly going as far as eliminating) the channel coding part [3]. In the case of linear dispersion codes [4],[5], transmission is not continuous but packet-wise (block-wise). In that case, a packet of  $T$  vector symbols  $a_k$  (hence a  $N_{tx} \times T$  matrix) gets constructed as a linear combination of fixed matrices in which the combination coefficients are symbols  $b_k$ . A particular case is the Alamouti code which is a full diversity single rate code corresponding to block length  $T = N_{tx} = 2$ ,  $N_s = 1$ . In the first part of this paper we shall focus on continuous transmission in which linear precoding corresponds to MIMO

prefiltering. This linear convolutive precoding can be considered as a special case of linear dispersion codes (making abstraction of the packet boundaries) in which the fixed matrices are time-shifted versions of the impulse responses of the columns of  $\mathbf{T}(z)$ , see Fig. 1. Whereas in the absence of linear precoding, the last



**Fig. 2.** Two channel coding, interleaving, symbol mapping and demultiplexing choices.

operation of the encoding part is spatial demultiplexing (serial-to-parallel (S/P) conversion) (see left part of Fig. 2), this S/P conversion is the first operation in the case of linear precoding, see the right part of Fig. 2. After the S/P conversion, we have a mixture of channel coding, interleaving and symbol mapping, separately per stream. The existing BLAST systems are special cases of this approach. VBLAST is a full rate system with  $\mathbf{T}(z) = \mathbf{I}_{N_{tx}}$  which leads to quite limited diversity. DBLAST is a single rate system with  $\mathbf{T}(z) = [1 \ z^{-1} \ \dots \ z^{-(N_{tx}-1)}]^T$  which leads to full diversity (delay diversity). We would like to introduce a prefiltering matrix  $\mathbf{T}(z)$  without taking a hit in capacity, while achieving full (spatial) diversity. The MIMO prefiltering will allow us to capture all diversity (spatial, and frequential for channels with delay spread) and will provide some coding gain. The optional channel coding per stream then serves to provide additional coding gain and possibly (with interleaving) to capture the temporal diversity (Doppler spread) if there is any. Finally, though time-invariant filtering may evoke continuous transmission, the prefiltering approach is also immediately applicable to block transmission by replacing convolution by circular convolution.

### 2.1. Capacity

Consider the MIMO AWGN channel

$$\mathbf{y}_k = \mathbf{H} \mathbf{a}_k + \mathbf{v}_k = \mathbf{H} \mathbf{T}(q) \mathbf{b}_k + \mathbf{v}_k \quad (1)$$

where the noise power spectral density matrix is  $S_{vv}(z) = \sigma_v^2 I$ ,  $q^{-1} \mathbf{b}_k = \mathbf{b}_{k-1}$ . The **ergodic capacity** when channel knowledge is absent at the TX and perfect at the RX is given by:

$$\begin{aligned} C(S_{aa}) &= E_H \frac{1}{2\pi j} \oint \frac{dz}{z} \ln \det(I + \frac{1}{\sigma_v^2} \mathbf{H} S_{aa}(z) \mathbf{H}^H) \\ &= E_H \frac{1}{2\pi j} \oint \frac{dz}{z} \ln \det(I + \frac{1}{\sigma_v^2} \mathbf{H} \mathbf{T}(z) S_{bb}(z) \mathbf{T}^\dagger(z) \mathbf{H}^H) \\ &= E_H \frac{1}{2\pi j} \oint \frac{dz}{z} \ln \det(I + \rho \mathbf{H} \mathbf{T}(z) \mathbf{T}^\dagger(z) \mathbf{H}^H) \end{aligned} \quad (2)$$

where we assume that the channel coding and interleaving per stream leads to spatially and temporally white symbols:  $S_{bb}(z) = \sigma_b^2 I$ , and  $\rho = \frac{\sigma_b^2}{\sigma_v^2}$ . The expectation  $E_H$  is here w.r.t. the distribution of the channel. As in [6], we assume the entries  $\mathbf{H}_{i,j}$  of the channel to be mutually independent zero mean complex Gaussian variables with unit variance (Rayleigh flat fading MIMO channel model). As stated in [7], to avoid capacity loss the prefilter  $\mathbf{T}(z)$  requires to be paraunitary ( $\mathbf{T}(z) \mathbf{T}^\dagger(z) = \mathbf{I}$ ). Motivated by the consideration of diversity also (see below), we propose to use the following paraunitary prefilter

$$\mathbf{T}(z) = \mathbf{D}(z) Q$$

$$\mathbf{D}(z) = \text{diag}\{1, z^{-1}, \dots, z^{-(N_{tx}-1)}\}, Q^H Q = I, |Q_{ij}| = \frac{1}{\sqrt{N_{tx}}} \quad (3)$$

where  $Q$  is a (constant) unitary matrix with equal magnitude elements. Note that for a channel with delay spread, the prefilter can be immediately adapted by replacing the elementary delay  $z^{-1}$  by  $z^{-L}$  for channel of length (delay spread)  $L$ . For the flat propagation channel  $\mathbf{H}$  combined with the prefilter  $\mathbf{T}(z)$  in (3), symbol stream  $n$  ( $b_{n,k}$ ) passes through the equivalent SIMO channel

$$\sum_{i=1}^{N_{tx}} z^{-(i-1)} \mathbf{H}_{\cdot,i} Q_{i,n} \quad (4)$$

which now has memory due to the delay diversity introduced by  $\mathbf{D}(z)$ . It is important that the different columns  $\mathbf{H}_{\cdot,i}$  of the channel matrix get spread out in time to get full diversity (otherwise the streams just pass through a linear combination of the columns, as in VBLAST, which offers limited diversity). The delay diversity only becomes effective by the introduction of the mixing/rotation matrix  $Q$ , which has equal magnitude elements for uniform diversity spreading.

### 2.2. Matched Filter Bound and Diversity

The Matched Filter Bound (MFB) is the maximum attainable SNR for symbol-wise detection, when the interference from all other symbols has been removed. Hence the multistream MFB equals the MFB for a given stream. For VBLAST ( $\mathbf{T}(z) = \mathbf{I}$ ), the MFB for stream  $n$  is

$$\text{MFB}_n = \rho \|\mathbf{H}_{\cdot,n}\|_2^2 \quad (5)$$

hence, diversity is limited to  $N_{rx}$ . For the proposed  $\mathbf{T}(z) = \mathbf{D}(z) Q$  on the other hand, stream  $n$  has MFB

$$\text{MFB}_n = \rho \frac{1}{N_{tx}} \|\mathbf{H}\|_F^2 \quad (6)$$

hence this  $\mathbf{T}(z)$  provides the same full diversity  $N_{tx} N_{rx}$  for all streams. Larger diversity order leads to larger outage capacity.

### 2.3. Pairwise Probability of Error $P_e$

The received signal is:

$$\mathbf{y}_k = \mathbf{H} \mathbf{T}(q) \mathbf{b}_k + \mathbf{v}_k = \mathbf{H} \mathbf{D}(q) Q \mathbf{b}_k + \mathbf{v}_k = \mathbf{H} \mathbf{D}(q) \mathbf{c}_k + \mathbf{v}_k \quad (7)$$

where  $\mathbf{c}_k = Q \mathbf{b}_k = [c_1(k) \ c_2(k) \ \dots \ c_{N_{tx}}(k)]^T$ . We consider now the transmission of the coded symbols over a duration of  $T$  symbol periods. The accumulated received signal is then:

$$\mathbf{Y} = \mathbf{H} \mathbf{C} + \mathbf{V} \quad (8)$$

where  $\mathbf{Y}$  and  $\mathbf{V}$  are  $N_{rx} \times T$  and  $\mathbf{C}$  is  $N_{tx} \times T$ . The structure of  $\mathbf{C}$  will become clear below. Over a Rayleigh flat fading i.i.d. MIMO channel, the probability of deciding erroneously  $\mathbf{C}'$  for transmitted  $\mathbf{C}$  is upper bounded by (see [3]):

$$\mathbf{P}(\mathbf{C} \rightarrow \mathbf{C}') \leq \left( \prod_{i=1}^r \lambda_i \right)^{-N_{rx}} \left( \frac{\rho}{4} \right)^{-N_{rx} r} \quad (9)$$

where  $r$  and  $\lambda_i$  are rank and eigenvalues of  $(\mathbf{C} - \mathbf{C}')^H (\mathbf{C} - \mathbf{C}')$ . Introduce  $\mathbf{e}_k = \frac{1}{\sigma_b} (\mathbf{c}_k - \mathbf{c}'_k)$ , then:

$$\mathbf{C} - \mathbf{C}' = \begin{bmatrix} e_1(0) & e_1(1) & \dots & \dots & \dots & \dots \\ 0 & \ddots & \ddots & \dots & \dots & \dots \\ \vdots & \ddots & \ddots & \ddots & \dots & \dots \\ 0 & \dots & 0 & e_{N_{tx}}(0) & e_{N_{tx}}(1) & \dots \end{bmatrix} \quad (10)$$

Let  $i$  be the time index of the first error:

$$\mathbf{C} - \mathbf{C}' = \begin{bmatrix} 0 & \dots & 0 & e_1(i) & \dots & \dots & \dots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \dots & \dots \\ 0 & \dots & \dots & \dots & 0 & e_{N_{tx}}(i) & \dots \end{bmatrix}. \quad (11)$$

$$\prod_{n=1}^{N_{tx}} e_n(i) \neq 0 \quad (12)$$

the upper bound on the pairwise error probability becomes (maximized for a single error event  $i$ ):

$$\mathbf{P}(\mathbf{C} \rightarrow \mathbf{C}') \leq \left( \prod_{n=1}^{N_{tx}} |e_n(i)|^2 \right)^{-N_{rx}} \cdot \left( \frac{\rho}{4} \right)^{-N_{rx} N_{tx}}. \quad (13)$$

Hence, full diversity  $N_{rx} N_{tx}$  is guaranteed, and the coding gain is:  $\min_{e_i \neq 0} \prod_{n=1}^{N_{tx}} |e_n(i)|^2$ . The condition (12) is well known in the design of lattice constellations (see [8], [9]), a field based on the theory of numbers. A solution that satisfies our criteria of unitary matrix and equal magnitude components of  $Q$ , is the Vandermonde matrix:

$$Q^s = \frac{1}{\sqrt{N_{tx}}} \begin{bmatrix} 1 & \theta_1 & \dots & \theta_1^{N_{tx}-1} \\ 1 & \theta_2 & \dots & \theta_2^{N_{tx}-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \theta_{N_{tx}} & \dots & \theta_{N_{tx}}^{N_{tx}-1} \end{bmatrix} \quad (14)$$

where the  $\theta_i$  are the roots of  $\theta^{N_{tx}} - j = 0$ ,  $j = \sqrt{-1}$ .

**2.4. Optimality for QAM constellations in the case  $N_{tx} = 2^{n_t}$**   
For  $N_{tx} = 2^{n_t}$  ( $n_t \in \mathbb{Z}$ ),  $Q^s$  also leads to satisfaction of (12) [8], and guarantees for any constellation such that  $b_n(i) - b'_n(i) = a + jb \in \mathbb{Z}[j]$  ( $\mathbb{Z}[j] = \{a + jb \mid a, b \in \mathbb{Z}\}$ ), with  $b_i - b'_i \in (\mathbb{Z}[j]^{N_{tx}}/0)$ , that  $(N_{tx}^{N_{tx}/2} \prod_{n=1}^{N_{tx}} e_n(i)) \in \mathbb{Z}[j]/0$ , and hence:

$$\min_{e_i \neq 0} \prod_{n=1}^{N_{tx}} |e_n(i)|^2 \geq \left( \frac{1}{N_{tx}} \right)^{N_{tx}}. \quad (15)$$

For finite QAM constellations with  $(2M)^2$  points, any symbol can be written as:  $b_n(i) = d\{(2l-1) + j(2p-1)\}$  where  $d \in \mathbb{R}^{+*}$ ,  $l, p \in \{-M+1, -M+2, \dots, M\}$ . Then  $\frac{1}{\sigma_b}(b_n(i) - b'_n(i)) = \frac{2d}{\sigma_b}(l' + jp')$ ,  $l', p' \in \{-2M+1, -2M+2, \dots, 2M-1\}$  and  $\sigma_b^2 = \frac{2(4M^2-1)d^2}{3}$ . The lower bound of (15), which is valid in fact for any Vandermonde matrix  $Q$  of the form in (14) built with roots of a polynomial of order  $N_{tx}$  with coefficients in  $\mathbb{Z}[j]$  and satisfying a certain number of conditions [8] (hence  $Q^s$  is a special case of this family), becomes

$$\min_{e_i \neq 0} \prod_{n=1}^{N_{tx}} |e_n(i)|^2 \geq \left( \frac{4d^2}{\sigma_b^2} \right)^{N_{tx}} \left( \frac{1}{N_{tx}} \right)^{N_{tx}} = \left( \frac{4d^2}{N_{tx}\sigma_b^2} \right)^{N_{tx}}. \quad (16)$$

In what follows, we consider an upper bound for the coding gain for any matrix  $Q$  with normalized columns. The minimal product of errors  $\prod_n |e_n(i)|^2$  is upper bounded by a particular error instance corresponding to a single error in the  $b$ 's, when  $\frac{1}{\sigma_b}(b_i - b'_i) = \frac{2d}{\sigma_b} w_{n_0}$ , where  $w_{n_0}$  is the vector with one in the  $n_0^{\text{th}}$  coefficient and zeros elsewhere, hence

$$\min_{e_i \neq 0} \prod_{n=1}^{N_{tx}} |e_n(i)|^2 \leq \left( \frac{4d^2}{\sigma_b^2} \right)^{N_{tx}} \prod_{n=1}^{N_{tx}} |Q_{n,n_0}|^2. \quad (17)$$

Now, given that  $\sum_{n=1}^{N_{tx}} |Q_{n,n_0}|^2 = 1$ , then by applying Jensen's inequality, we get

$$\prod_{n=1}^{N_{tx}} |Q_{n,n_0}|^2 \leq \left( \frac{1}{N_{tx}} \right)^{N_{tx}}. \quad (18)$$

Hence,

$$\min_{e_i \neq 0} \prod_{n=1}^{N_{tx}} |e_n(i)|^2 \leq \left( \frac{4d^2}{\sigma_b^2} \right)^{N_{tx}} \left( \frac{1}{N_{tx}} \right)^{N_{tx}} = \left( \frac{4d^2}{N_{tx}\sigma_b^2} \right)^{N_{tx}} \quad (19)$$

is an upper bound for the coding gain for any matrix  $Q$  with normalized columns. Now, the intersection of the sets of matrices that lead to the lower bound (16) and the upper bound (19) includes the unitary matrix  $Q^s$  given in (14), which hence achieves the upper bound on the coding gain:  $\min_{e_i \neq 0} \prod_{n=1}^{N_{tx}} |e_n(i)|^2 = \left( \frac{4d^2}{N_{tx}\sigma_b^2} \right)^{N_{tx}}$ .

**Remark 1:** The Jensen's inequality (18) becomes an equality if and only if all the coefficients  $Q_{n,n_0}$ ,  $n = 1, \dots, N_{tx}$  have the same module  $1/\sqrt{N_{tx}}$ . This holds for any  $n_0 = 1, \dots, N_{tx}$ . Hence we conclude that a necessary condition on any unitary matrix  $Q$  to maximize the coding gain is to have all equal magnitude coefficients. This is equivalent to our condition to achieve the same maximum MFB for all streams (full diversity).

**Remark 2:** In the case when  $N_{tx} \neq 2^{n_t}$  (and using  $Q^s$ ), the coding gain is closely related to the size of the used QAM constellation, and is in general lower than the upper bound given above.

In principle it is possible to maintain  $N_s = N_{tx}$  under all circumstances of channel rank: the capacity decomposition shows an additive decomposition into  $N_{tx}$  streams with varying SINRs in any case. However, when  $N_s$  exceeds the channel rank, some of these SINRs may be much lower than some of the others, which leads to big discrepancies in terms of channel coding and bit rate management for the streams at the transmitter.

### 3. SPATIALLY SCRAMBLED STC

The columns of the matrix  $Q$  above can be considered as spatial spreading codes. When we want to lower the number of transmitted streams  $N_s < N_{tx}$ , we propose to furthermore introduce spatial scrambling. The received signal becomes:

$$\mathbf{y}_k = \mathbf{H} \mathbf{a}_k + \mathbf{v}_k = \mathbf{H} \mathbf{D}(q) \mathbf{S}_k Q \mathbf{b}_k + \mathbf{v}_k \quad (20)$$

where  $\mathbf{D}(q)$  introduces the same temporal diversity as before and  $Q$  is  $N_{tx} \times N_s$  matrix with constant magnitude entries and normalized rows. The columns of  $Q$  perform again spatial spreading.  $\mathbf{S}_k = \text{diag}(s_1(k), \dots, s_{N_{tx}}(k))$  contains the (constant unit amplitude) scrambling sequence,  $s_1(k)$  can be fixed to 1. The scrambling is introduced to allow uniform distribution of the transmitted signal over the TX antenna space such that

$$S \mathbf{a} \mathbf{a}^H(z) = E_S(\sigma_b^2 \mathbf{D}(z) \mathbf{S}_k Q Q^H \mathbf{S}_k^H \mathbf{D}^\dagger(z)) = \sigma_b^2 I \quad (21)$$

where  $E_S$  stands for expectation over the scrambling. Two flavors of scrambling can be introduced.

#### 3.1. Full spreading

In this case all  $s_n(k)$ ,  $n = 1, \dots, N_{tx}$  are iid and zero mean. The necessary and sufficient condition on  $Q$  to satisfy (21) is to have all equal row norms.

The upper bound on the pairwise error probability given in (13), remains the same with  $e_k = \frac{1}{\sigma_b} \mathbf{S}_k Q (b_k - b'_k)$ . The coding gain is then

$$\begin{aligned} \min_{e_i \neq 0} \prod_{n=1}^{N_{tx}} |e_n(i)|^2 &= \min_{b_i - b'_i \neq 0} \left( \prod_{n=1}^{N_{tx}} |s_n(i)|^2 \right) \left( \prod_{n=1}^{N_{tx}} \left| \frac{1}{\sigma_b} [Q (b_i - b'_i)]_n \right|^2 \right) \\ &= \frac{1}{\sigma_b^{N_{tx}}} \min_{b_i - b'_i \neq 0} \left( \prod_{n=1}^{N_{tx}} |[Q (b_i - b'_i)]_n|^2 \right) \end{aligned} \quad (22)$$

Again, and for the same reasons cited in the case  $N_s = N_{tx}$ , any matrix  $Q$  composed of  $N_s$  columns of  $Q^s$  (given in(14)) satisfies the criterion of diversity, MFB and optimize the coding gain for  $N_{tx} = 2^{nt}$ .

Suppose that  $N_{tx} = p N_s$  ( $p \in \mathbb{N}$ ). The family of matrices  $Q = \frac{1}{\sqrt{N_s}} \mathbf{P} [Q_1^H P_1, \dots, Q_m^H P_m]^H$ , where  $\mathbf{P}$  (resp.  $P_i$ ) is a  $N_{tx} \times N_{tx}$  (resp.  $N_s \times N_s$ ) permutation matrix.  $Q_i$  is a  $(p_i N_s) \times N_s$  matrix ( $\sum_{i=1}^m p_i = p$ ), whose columns are  $N_s$  columns of the Vandermonde matrix  $VDM(\theta_1, \dots, \theta_{p_i N_s})$ , where  $\theta_k$  are the roots of  $\theta^{p_i N_s} - j = 0$ ,  $j = \sqrt{-1}$ , are general solutions for the problem. These solutions optimize the coding gain if the  $(p_i N_s) i = 1, \dots, m$  are powers of 2.

The full scrambling framework appears to suggest straightforward extensions to the multiuser case, by providing different users with different scrambling sequences.

### 3.2. Partial spreading

As an example, consider the case  $N_{tx} = p N_s$  ( $p \in \mathbb{N}$ ), and  $\mathbf{S}_k = \text{diag}(s_1(k) I_{N_s}, \dots, s_p(k) I_{N_s})$ , with  $s_n(k), n = 1, \dots, N_{tx}$  iid and zero mean. The necessary and sufficient condition on  $Q = [Q_1^H, \dots, Q_p^H]^H$  to satisfy (21) is that all the square matrices  $Q_i$  be unitary. The upper bound on the pairwise error probability given in (13), leads to the same condition as in the previous section. A solution that satisfies the diversity criterion, the MFB and optimizes the coding gain for  $N_s = 2^{n_s}$  is  $Q_i = \frac{1}{\sqrt{N_s}} Q^s P_i$ ,  $i = 1, \dots, p$ , where  $P_i$  is a  $N_s \times N_s$  permutation matrix, and  $Q^s$  is the Vandermonde matrix  $VDM(\theta_1, \dots, \theta_{N_s})$ , where  $\theta_k$  are the roots of  $\theta^{N_s} - j = 0$ ,  $j = \sqrt{-1}$ .

### 4. ML RECEPTION

In principle, we can perform Maximum Likelihood reception since the delay diversity transforms the flat channel into a channel with finite memory. However, the number of states would be the product of the constellation sizes of the  $N_s$  streams to the power  $N_{tx} - 1$ . Hence, if all the streams have the same constellation size  $|\mathcal{A}|$ , the number of states would be  $|\mathcal{A}|^{N_s(N_{tx}-1)}$ , which will be much too large in typical applications. Suboptimal ML reception can be performed in the form of sphere decoding [10]. The complexity of this can still be too large though and therefore suboptimal receiver structures will be considered in the next section.

### 5. MIMO DFE RECEPTION

We consider here the case of the linear prefiltering approach (section2). Let  $\mathbf{G}(z) = \mathbf{H}\mathbf{T}(z) = \mathbf{H}\mathbf{D}(z)Q$  be the cascade transfer function of channel and precoding. The matched filter RX is

$$\begin{aligned} \mathbf{x}_k = \mathbf{G}^\dagger(q) \mathbf{y}_k &= \mathbf{G}^\dagger(q) \mathbf{G}(q) \mathbf{b}_k + \mathbf{G}^\dagger(q) \mathbf{v}_k \\ &= \mathbf{R}(q) \mathbf{b}_k + \mathbf{G}^\dagger(q) \mathbf{v}_k \end{aligned} \quad (23)$$

where  $\mathbf{R}(z) = \mathbf{G}^\dagger(z) \mathbf{G}(z)$ , and the psdf of  $\mathbf{G}^\dagger(q) \mathbf{v}_k$  is  $\sigma_v^2 \mathbf{R}(z)$ . The DFE RX is then:

$$\widehat{\mathbf{b}}_k = - \underbrace{\overline{\mathbf{L}}(q)}_{\text{feedback}} \mathbf{b}_k + \underbrace{\mathbf{F}(q)}_{\text{feedforward}} \mathbf{x}_k \quad (24)$$

where feedback  $\overline{\mathbf{L}}(z)$  is strictly ‘‘causal’’. Two design criteria for feedforward and feedback filters are possible: MMSE ZF and MMSE, see [7], where we introduced triangular MIMO feedback structures, allowing to incorporate channel decoding in the feedback, and leading to the stripping approach of Verdu & Müller or Varanası & Guess..

### 6. POLYNOMIAL EXPANSION DETECTOR

We consider here the case with spatial scrambling (section3).

$\mathbf{G}_k(z) = \mathbf{H}\mathbf{D}(z) \mathbf{S}_k Q$  is the cascade of the precoder and the channel. The matched filter RX is now

$$\mathbf{x}(k) = \mathbf{G}_k^\dagger(q) \mathbf{y}_k = \mathbf{R}_k(q) \mathbf{b}_k + \mathbf{G}_k^\dagger(q) \mathbf{v}_k. \quad (25)$$

$\mathbf{R}_k(z) = \sum_i \mathbf{R}_k(i) z^{-i}$ , let  $\mathbf{M}_k(z) = (\text{diag}(\mathbf{R}_k(0)))^{-1} \mathbf{R}_k(z)$  and  $\mathbf{x}'_k(z) = (\text{diag}(\mathbf{R}_k(0)))^{-1} \mathbf{x}(k)$  then

$$\begin{aligned} \widehat{\mathbf{b}}_k &= \mathbf{M}_k(z)^{-1} \mathbf{x}'_k(z) \\ &= (\mathbf{I} - \overline{\mathbf{M}}_k(z))^{-1} \mathbf{x}'_k(z) \end{aligned} \quad (26)$$

The correlation matrix  $\mathbf{M}_k(z)$  has a coefficient  $M[0]$  with a dominant unit diagonal in the sense that all other elements of the  $M[i]$  are much smaller than one in magnitude. Hence, the polynomial expansion approach suggests to develop  $(\mathbf{I} - \overline{\mathbf{M}}_k(z))^{-1} = \sum_{i=0}^{\infty} (\overline{\mathbf{M}}_k(z))^i$  up to some finite order, which leads to the iterative PE solution:

$$\begin{aligned} \widehat{\mathbf{b}}_k^{(-1)} &= 0 \\ i \geq 0 \quad \widehat{\mathbf{b}}_k^{(i)}[n] &= \mathbf{x}'_k(k) + \overline{\mathbf{M}}_k(z) \widehat{\mathbf{b}}_k^{(i-1)}. \end{aligned} \quad (27)$$

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