Multistage DS-CDMA Receivers with Pathwise Interference Mitigation

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Abstract—**In this paper we investigate the application of mulitple stage filters in the context of pathwise processing. Pathwise processing proposes to overcome one of the major difficulties encountered with linear DS-CDMA receivers in time-varying multipath propagation, namely the estimation of a large number of parameters from scarce training data. Pathwise Interference estimation allows the separation of the parameters into fastly and slowly varying parameters, thereby allowing the scarce training data to be used in the estimation of the fastly varying parameters with a short time constant while the slowly varying parameters can be estimated over a much larger time interval. This paper focuses on the application of polynomial expansion (PE) filters to pathwise processing and proposes the use of a weighting factor per signal component. We show that these weighting coefficients not only achieve significant improvements in the presence of power imbalances between users and paths w.r.t. scalar weighting, but also achieve further improvement due to the better estimation of the fastly varying parameters.**

Keywords—**Pathwise, Interference Cancellation, Multiuser Detection, DS-CDMA, Multistage, Polynomial Expansion**

I. Introduction

ONE of the main problems in linear multiuser detection is the amount of parameters that have to be estimated from relatively few training data. In particular, the fastly varying parameters of the mobile channel in a multipath, fading environment can pose serious difficulties to interference cancellation and data detection. Pathwise Interference Cancellation (PWIC) is an approach that allows to separate the parameters into fastly varying and slowly varying parameters, thereby allowing the scarce training data to be used in the estimation of the fastly varying parameters while the whole of the received signal can be used to estimate the slowly varying parameters over a much larger time interval. Since the interference cancellation takes place between individual multipath components before spatial-temporal recombination, the signal thus obtained contains the desired parameters at an improved SINR compared to the received signal and hence allows improved channel estimation [1][2][3].

Polynomial expansion (PE) is an approximation technique for LMMSE receivers and is particularly well suited for CDMA due to the presence of a large number of small correlations. The fundamental principle of PE is to avoid the relatively costly correlation matrix inverse required by an LMMSE/Decorrelator receiver by considering the correlation matrix to be a small perturbation of an identity matrix and approximating the inverse of the correlation matrix by a polynomial expansion in the perturbation matrix or, equivalently, in the correlation matrix itself. However, for PE to work, adapted weighting factors have to be introduced. By appropriately choosing the weighting coefficients, every additional term in the PE can be guaranteed to improve performance and hence divergence concerns get eliminated.

PE has, in various forms, received a fair amount of attention recently in the literature [4] [5] [6] [7] etc. Some works on PE have analysed the choice of scalar weighting factors on the basis of asymptotic system analysis, leading to weight values that can be determined a priori. In this paper, we propose to introduce diagonal weighting matrices which corresponds to one weighting factor per signal component. We shall see that such multiple coefficients not only improve performance substantially in the presence of power imbalances between users and paths, but also further improvement due to the fast adaptation of these weights is possible since the instantaneous channel states will reflect the power imbalances very strongly.

Moshavi, who first introduced PE [8], applied the polynomial expansion to the joint set of RAKE outputs for the various users. In this way, the polynomial expansion receiver involves only (de)spreading and channel (matched) filtering operations and hence is mostly parameterized in terms of the channel parameters (as opposed to the general coefficients of a general linear receiver). Honig and coworkers apply the PE principle to the received signal directly and were able to show [9] that PE is equivalent to the *Multistage Wiener Filter* [10] in this case. We propose to introduce polynomial expansion at the level of the pathwise RAKE outputs. As compared to Moshavi's approach, the PE is situated before the maximum ratio recombination of the path contributions and leads to pathwise interference cancellation which will allow to estimate the path parameters (amplitudes, or even angles in the spatio-temporal case) with improved SINR and hence with reduced estimation error. The diagonal weighting factors we introduce will hence provide a weighting per path (or even possibly per antenna element per path in the spatio-temporal case). Maximum ratio combining after pathwise PE corresponds then to a version of the G-RAKE (the path amplitudes multiplied by arbitrary eighting factors become arbitrary recombination coefficients).

*Eurécom's research is partially supported by its industrial partners: Ascom, Swisscom, Thomson-CSF, IBM France, CEGETEL, Motorola, France Télécom, Hitachi Europe and Texas Instruments. The research of Christian Fischer is supported by the Swiss National Science Foundation

II. Data Model

For the received DS-CDMA signal model, we assume the K users to be transmitting linearly modulated signals over a linear, specular multipath channel with additive gaussian noise in an asynchronous fashion. Furthermore, we assume that the basestation receiver utilizes an antenna array with Q elements. The channel impulse response is characterised for users $k \in [1 \dots K]$ by

$$\mathbf{h}_k(t) = \sum_{m=1}^{M} A_{k,m}\mathbf{h}(\theta_{k,m})\delta(t - \tau_{k,m})$$

where $\mathbf{h}_k$ and $\mathbf{h}_{k,m} = \mathbf{h}(\theta_{k,m})$ are column vectors of dimension Q, the number of sensors employed at the receiver. $\mathbf{h}_{k,m}$ defines the response of the antenna array and is a function of the Direction of Arrival (DoA), $\theta_{k,m}$, of the signal. For identifiability reasons, we chose the anntenna response vector to have unity power, $\mathbf{h}_{k,m}^H\mathbf{h}_{k,m} = 1$. Further, the specular channel is characterised by $A_{k,m}$ and $\tau_{k,i}$, the complex amplitude and the path delays, respectively. M is the number of specular paths. The channel parameters can be divided into two classes: fastly and slowly varying parameters. The slowly varying parameters are the delays, $\tau_{k,m}$, the DoA, $\theta_{k,m}$, and the short-term path power, $E|A_{k,m}|^2$. Hence, the fast varying parameters are the complex phases and amplitudes, $A_{k,m}$. At the receiver front-end, the received signal before sampling is written as

$$\begin{aligned}\mathbf{y}(t) &= \sum_{k=1}^{K}\Big\{\sum_{n=-\infty}^{\infty}\sum_{m=1}^{M}(A_{k,m}a_k[n]) \qquad (1)\\ &\quad\times\sum_{l=0}^{L-1}s_k[l]\mathbf{h}_{k,m}p(t-\tau_{k,m}-lT_c-nT)+\mathbf{n}(t)\Big\}\end{aligned}$$

$\mathbf{y}(t)$ and the Additive White Gaussian Noise (AWGN), $\mathbf{n}(t)$, are vector signals due to the use of multiple sensors and are of dimensions $Q \times 1$. $a_k[n], p(t)$ are the transmitted symbols for user k and the pulse-shaping filter, respectively. At the receiver front-end, the received signal given in equation (1) is lowpass-filtered and sampled at $1/T_s$. The spreading codes, $s_k(.)$ are assumed to be periodic of length $LT_c = T$ here. We obtain therefore

$$\mathbf{y}[n] = \sum_{i=-\infty}^{\infty}\mathbf{P}[n-i]\mathbf{SHAa}[i] + \mathbf{v}[n] \qquad (2)$$

where $\mathbf{y}[n] = [\mathbf{y}[n + 0 \cdot T_c/J] \dots \mathbf{y}[n + (LJ-1)\cdot T_c/J]^T$, i.e. we stacked all samples of the received signal for the duration of a symbol period T into $\mathbf{y}[n]$. $\mathbf{v}[n]$ is the sampled and low-pass filtered contribution of the noise, $\mathbf{n}(t)$. $\mathbf{a}[n] = [a_1(n)a_2(n)\dots a_K(n)]^T$ contains the data symbols of all K users for a given n, ^{T} indicating the matrix transpose, $\mathbf{A} = diag(\mathbf{A}_1 \dots \mathbf{A}_K)$ is the block diagonal matrix containing the complex amplitude coefficients for each user such that $\mathbf{A}_k = [A_{k,1}^H \dots A_{k,M}^H]^H$, $\mathbf{H} = diag(\mathbf{H}_1 \dots \mathbf{H}_K)$ where $\mathbf{H}_k = diag(\mathbf{h}_{k,1}\dots\mathbf{h}_{k,M})$ where both $\mathbf{H}_k$ and $\mathbf{H}$ are block diagonal matrices and $\mathbf{h}_{k,m}$ is a column vector. $\mathbf{S} = diag(\mathbf{S}_1 \dots \mathbf{S}_K)$ where $\mathbf{S}_k = [\mathbf{I}_M \otimes (\mathbf{s}_k \otimes \mathbf{I}_Q)]$; $\mathbf{s}_k = [s_k[0]\dots s_k[L-1]]^T$ represents the spreading code vector, $\mathbf{I}_M$ and $\mathbf{I}_Q$ denote identity matrices of dimensions $M \times M$ and $Q \times Q$, respectively. $\otimes$ signifies the Kronecker product. $\mathbf{P} = [\mathbf{p}_{n,1}\dots\mathbf{p}_{n,K}]$; $\mathbf{p}_{n,k} = [\mathbf{p}_{n,k,1}\dots\mathbf{p}_{n,k,M}]$ and

$$\mathbf{p}_{n,k,m} = \begin{bmatrix} \mathbf{p}_{n,k,m,0,0} & \cdots & \mathbf{p}_{n,k,m,0,L-1} \\ \vdots & \ddots & \vdots \\ \mathbf{p}_{n,k,m,LJ-1,1} & \cdots & \mathbf{p}_{n,k,m,LJ-1,L-1}\end{bmatrix}$$

where $\mathbf{p}_{n,k,m,r,l} = [p\,(nT + (r/J - l)T_c - \tau_{k,m}) \otimes \mathbf{I}_Q]$. Let us define the received signal in the q-domain where q is the advance operator, i.e. $qy_n = y_{n+1}$ w.r.t the symbol period. To this end, let us reformulate the received signal as given in (2) in the q-domain.

$$\begin{aligned}\mathbf{y}[n] &= \mathbf{P}(q)\mathbf{SHAa}[n] + \mathbf{v}[n]\\ &= \mathbf{E}(q)\mathbf{a}[n] + \mathbf{v}[n]\\ &= \mathbf{E}_k(q)a_k[n] + \sum_{i=1;i\neq k}^{K}\mathbf{E}_i(q)a_i[n] + \mathbf{v}[n] \qquad (3)\end{aligned}$$

where $\mathbf{P}(q) = \sum_i \mathbf{P}[i]q^{-i}$ and we split up the signal into user k's contribution and interference terms.

$$\begin{aligned}\underline{\underline{\mathbf{E}}}_k(q)\mathbf{H}_k\mathbf{A}_k &= \sum_{m=1}^{M}\underline{\underline{\mathbf{E}}}_{k,m}(q)\mathbf{h}_{k,m}A_{k,m} \qquad (4)\\ &= \sum_{m=1}^{M}\mathbf{E}_{k,m}(q)A_{k,m} = \underline{\mathbf{E}}_k(q)\mathbf{A}_k = \mathbf{E}_k(q)\end{aligned}$$

Furthermore, we can define

$$\begin{aligned}\mathbf{x}[n] &= \mathbf{H}^H\underline{\underline{\mathbf{E}}}^\dagger(q)\mathbf{y}[n] \qquad (5)\\ &= \underbrace{\mathbf{H}^H\underline{\underline{\mathbf{E}}}^\dagger(q)\underline{\underline{\mathbf{E}}}(q)\mathbf{H}}_{\mathbf{R}(q)=\mathbf{I}+\overline{\mathbf{R}}(q)}\mathbf{Aa}[n] + \mathbf{H}^H\underline{\underline{\mathbf{E}}}^\dagger(q)\mathbf{v}[n]\\ &= \underline{\mathbf{E}}^\dagger(q)\underline{\mathbf{E}}(q)\mathbf{Aa}[n] + \underline{\mathbf{E}}(q)^\dagger\mathbf{v}[n]\end{aligned}$$

where $\underline{\underline{\mathbf{E}}}^\dagger(q) = \underline{\underline{\mathbf{E}}}^H(1/q^*)$ is the *paraconjugate* and

$$\sum_i \mathbf{E}_{k,m}^\dagger[i]\mathbf{E}_{k,m}[-i] = 1, \forall\, k \in \{1\dots K\}, m \in \{1\dots M\}$$

$\mathbf{x}[n] = [x_{1,1}[n]\dots x_{K,M}[n]]^T$ are the matched filter or RAKE outputs, spatially but not temporally recombined. Assuming normalised spreading codes, $\mathbf{R}(q) = \underline{\mathbf{E}}^\dagger(q)\underline{\mathbf{E}}(q) = \sum_i \mathbf{R}[i]q^{-1}$ and $diag(\mathbf{R}[0]) = \mathbf{I}$ due to the normalisation of $\mathbf{h}_{k,m} : ||\mathbf{h}_{k,m}|| = 1$. From (5) it can be seen that the pathwise zero-forcing receiver for estimating $\mathbf{Aa}[n]$ is given by $\mathbf{R}^{-1}(q)$.

III. Polynomial Expansion in Pathwise Interference Cancellation

The principle of the polynomial expansion approach is based on the polynomial expansion in the matrix $\mathbf{R}(q)$ as introduced in the last section. In both the decorrelation receiver and the LMMSE receiver, most of the complexity is associated with the required correlation matrix inverse. Since $diag(\mathbf{R}[0]) = \mathbf{I}$, we can write $\mathbf{R}^{-1}(q) =$

$\left(\mathbf{I}+\overline{\mathbf{R}}(q)\right)^{-1} = \sum_{b=0}^{\infty}\left(-\overline{\mathbf{R}}(q)\right)^{b}$, provided that there is a matrix norm $|||\overline{\mathbf{R}}||| < 1$ to ensure convergence. We can approximate the inverse of the correlation matrix and hence the pathwise zero-forcing receiver by

$$\mathbf{R}^{-1}(q) \approx \tilde{\mathbf{R}}^{-1}(q) = \sum_{b=0}^{B}(-\overline{\mathbf{R}}(q))^{b} \quad (6)$$

where we have truncated the infinite summation to $B+1$ terms. However, as would be expected, such a truncation is suboptimal and can only improve SINR over the RAKE when the off-diagonal elements in $\mathbf{R}(q)$ are few and small, i.e. for low system loading factors. In [11] and [12] it was shown that the potential gains of such a PE receiver are limited to approximately the inverse of the loading factor, i.e. $1/\alpha$, where $\alpha = K/L$.

The performance of PE can be much improved by introducing scalar polynomial coefficients d_b according to some design criterion in (6) as has been documented in various publications e.g. [8][4][13][5][9] and we will hence not treat this case here. Instead, we propose to increase the degrees of freedom available to us by introducing a scalar coefficient *per path*. Let us define $\mathbf{D}_b = diag[d_{b,1}\ldots d_{b,KM}]$ and write the approximated inverse of $\mathbf{R}(q)$ as a polynomial in $\overline{\mathbf{R}}(q)$ or equivalently in $\mathbf{R}(q)$ since there is a one-to-one relationship between the expansions in $\mathbf{R}(q)$ and $\overline{\mathbf{R}}(q)$. Hence,

$$\tilde{\mathbf{R}}^{-1}(q) = \sum_{b=0}^{B}\mathbf{D}_b\overline{\mathbf{R}}^{b}(q) \quad (7)$$

Typically, we would only be interested in $B \in \{1,2\}$ stages after the RAKE in order to keep complexity at a reasonable level. Note that the additional complexity associated with every PE stage is about twice the complexity of the RAKE.

A. Pathwise filter design

In a pilot-assisted estimation scenario, we can therefore express the PE filter estimating $\mathbf{Aa}[n]$ for an arbitrary number of stages with a matrix polynomial coefficient per stage by

$$\mathbf{D}_i^o = \arg\min_{\mathbf{D}_i:i\in 0\ldots B} E||\mathbf{Aa}[n] - \sum_{b=0}^{B}\mathbf{D}_b\mathbf{R}^b(q)\mathbf{x}[n]||^2 \quad (8)$$

which can be solved through a set of linear equations. Looking at any row j in equation (8), we can equivalently write

$$\begin{aligned}\mathbf{d}_j^o &= \arg\min_{\mathbf{d}_j} E|A_j a_l[n] - \mathbf{d}_j\zeta_j[n]|^2\\ &= A_j E(a_l[n]\zeta_j^H[n])(E\zeta_j[n]\zeta_j^H[n])^{-1} \quad (9)\end{aligned}$$

where $j \in \{1\ldots KM\}$ is the path index, $l = \lceil\frac{j}{M}\rceil$ the corresponding datasymbol, $\mathbf{d}_j = [d_{0,j}\ldots d_{B,j}]$ and $\zeta_j = [z_{0,j}\ldots z_{B,j}]^T$, $\mathbf{z}_b[n] = \mathbf{R}^b(q)\mathbf{x}[n] = [z_{b,1}[n]\ldots z_{b,KM}[n]]^T$ and hence the problem decouples nicely into a path-by-path solvable problem. It worth noting that this is not the case when the polynomial coefficient matrix D_b is replaced by a scalar as the solution for the coefficients involves the summation over the paths j and hence there is no decoupling between paths nor users, i.e.

$$\begin{aligned}d_i^o &= \arg\min_{d_i:i\in 0\ldots B} E||\mathbf{Aa}[n] - \sum_{b=0}^{B} d_b\mathbf{R}^b(q)\mathbf{x}[n]||^2\\ &= \sum_j E(A_j a_l[n]\zeta_j^H[n])(\sum_j E\zeta_j[n]\zeta_j^H[n])^{-1}\end{aligned}$$

A variant of the approach in (8) is the sequential computation of the stages where each stage works on the error signal from the last stage, i.e.

$$\begin{aligned}\mathbf{D}_b^o &= \arg\min_{\mathbf{D}_b} ||\mathbf{Aa}[n] - \left(\widehat{\mathbf{Aa}}_{b-1}[n] + \mathbf{D}_b\mathbf{R}^b(q)\mathbf{x}[n]\right)||^2\\ &= \arg\min_{\mathbf{D}_b} ||\mathbf{e}_{b-1} - \mathbf{D}_b\mathbf{R}^b(q)\mathbf{x}[n]||^2\\ &= diag\left(E\ \mathbf{e}_{b-1}[n]\mathbf{z}_b^H[n]\right)\left(diag(E\ \mathbf{z}_b[n]\mathbf{z}_b^H[n])\right)^{-1}\\ \mathbf{e}_b[n] &= \mathbf{Aa}[n] - \widehat{\mathbf{Aa}}_b[n] \quad (10)\end{aligned}$$

Numerical results are shown in section IV. Path recombining after pathwise PE interference cancellation will give the symbol estimates:

$$\hat{\mathbf{a}}[n] = \mathbf{K}^H\mathbf{F}(q)\left[\mathbf{R}(q)\mathbf{Aa}[n] + \underline{\mathbf{E}}^{\dagger}(q)\mathbf{v}[n]\right]$$

where $\mathbf{K}$ is a general recombination matrix of the same block diagonal structure as $\mathbf{A}$, namely $\mathbf{K} = diag(\mathbf{K}_1,\ldots,\mathbf{K}_K)$. Maximum ratio combining is $\mathbf{K} = \mathbf{A}$. $\mathbf{F}(q)$ defines the linear filter corresponding to the PE approach above in (8). For the symbol estimate of user one, we have

$$\hat{a}_1[n] = \mathbf{K}_1^H\left[\mathbf{Z}_1(q)\mathbf{A}_1 a_1[n] + \overline{\mathbf{Z}}_1(q)\overline{\mathbf{A}}_1\overline{\mathbf{a}}_1[n] + \mathbf{X}(q)\mathbf{v}[n]\right]$$

where

$$\begin{aligned}\mathbf{K} &= diag(\mathbf{K}_1,\overline{\mathbf{K}}_1)\\ [\mathbf{I}_M\mathbf{0}]\,\mathbf{F}(q)\mathbf{R}(q) &= [\mathbf{Z}_1(q)\overline{\mathbf{Z}}_1(q)]\\ \mathbf{X}(q) &= [\mathbf{I}_M\mathbf{0}]\,\mathbf{F}(q)\underline{\mathbf{E}}^{\dagger}(q)\\ \mathbf{a}[n] &= [a_1[n]\overline{\mathbf{a}}_1^T[n]]^T\\ \mathbf{A} &= diag(\mathbf{A}_1,\overline{\mathbf{A}}_1)\end{aligned}$$

and $(.)_1$ is a signal model component acting on the useful signal contribution of user one whereas $\overline{(.)}_1$ defines the interfering terms. Hence, the output SINR of user one can be written as

$$\begin{aligned}SINR &= \frac{\sigma_a^2|\mathbf{K}_1^H\mathbf{Z}_1[0]\mathbf{A}_1|^2}{\mathbf{K}_1^H\mathbf{R}_1\mathbf{K}_1}\\ \mathbf{R}_1 &= \sigma_a^2\sum_{i\neq 0}\mathbf{Z}_1[i]\mathbf{A}_1\mathbf{A}_1^H\mathbf{Z}_1^H[i]\\ &\quad+\sigma_a^2\sum_i\overline{\mathbf{Z}}_1[i]\overline{\mathbf{A}}_1\overline{\mathbf{A}}_1^H\overline{\mathbf{Z}}_1^H[i] + \sigma_v^2\sum_i\mathbf{X}[i]\mathbf{X}^H[i]\end{aligned}$$

Maximum ratio combining is, however, not optimal and performance can be further improved by maximising the output SINR for the symbol estimate with respect to the recombining vector, $\mathbf{K}_1$. It can be shown that

$$SINR_{max} = \sigma_a^2\mathbf{A}_1^H\mathbf{Z}_1^H[0]\mathbf{R}_1\mathbf{Z}_1[0]\mathbf{A}_1$$

when the optimised recombination is given by $\mathbf{K}_1^o = \mathbf{R}_1^{-1}\mathbf{Z}_1[0]\mathbf{A}_1$ Numerical results are shown in the section IV.

B. Joint filter and recombination design

Using the pathwise recombination matrix $\mathbf{A}$ defined in the last section, we can write the symbol estimate resulting from the filtering and combining as

$$\hat{\mathbf{a}}[n] = \mathbf{A}^H \sum_{b=0}^{B} \mathbf{D}_b \mathbf{R}^b(q)\mathbf{x}[n] = \sum_{b=0}^{B} \mathbf{W}_b \mathbf{R}^b(q)\mathbf{x}[n] \quad (11)$$

where $\mathbf{W}_b = \mathbf{A}^H \mathbf{D}_b = diag(\omega_{b,1} \ldots \omega_{b,k})$ is another block diagonal matrix of the same structure as $\mathbf{K}$, stage $b = 0$ hence corresponds to a G-RAKE. Note however, that the direct application of this approach above would no longer provide the pathwise, SINR enhanced, outputs but the number of coefficients at our disposal remains at one scalar per path as can be seen from solving (11) per user:

$$\begin{aligned} \omega_k^o &= \arg\min_{\omega_k} ||a_k[n] - \omega_k \gamma_k[n]||^2 \\ &= E(a_k[n]\gamma_k^H[n])(E\gamma_k[n]\gamma_k^H[n])^{-1} \end{aligned} \quad (12)$$

where $\omega_k = [\omega_{0,k} \ldots \omega_{B,k}]$, $\omega_{k,b}$ has dimensions $M \times 1$ and $\gamma_k[n] = [\mathbf{z}_{0,k}[n] \ldots \mathbf{z}_{B,k}[n]]^T$. $\mathbf{z}_{b,k}[n] = [\mathbf{0} \ldots \mathbf{0}\ \mathbf{I_M}\ \mathbf{0} \ldots \mathbf{0}]\mathbf{z_b}[\mathbf{n}]$ where $\mathbf{0}$ is $M \times M$ and $\mathbf{z}_{b,k}[n]$ is simply the contribution in $\mathbf{z}_b[n]$ for the paths of user k. Alternatively, (12) can be solved using the Linearly Constrained Minimum Variance (LCMV) approach, shown for user 1 to simplify notation and without loss in generality, as follows

$$\begin{aligned} \omega_1^o &= \arg\min_{\omega_1} \omega_1 \mathbf{R}_{\gamma_1\gamma_1} \omega_1^H \text{ subject to } \omega_1 \Gamma_1[0]\mathbf{A}_1 = 1 \\ \omega_1^o &= \mathbf{A}_1^H \Gamma_1^H[0] \mathbf{R}_{\gamma_1\gamma_1}^{-1} \end{aligned} \quad (13)$$

where

$$\begin{aligned} \Gamma_1[0] &= (\mathbf{I}_B \otimes [\mathbf{I}_M\ \mathbf{0}])[\mathbf{R}[0] \ldots \mathbf{R}^{B+1}[0]]^T[\mathbf{I}_M\ \mathbf{0}]^T \\ \mathbf{R}_{\gamma_1\gamma_1} &= E(\gamma_1[n]\gamma_1^H[n]) \end{aligned} \quad (14)$$

The constraint ensures that the contribution of the data $a_1[n]$ in $\gamma_1[n]$ remains constant under the application of the filter while minimising the estimate output variance.

While the two solutions (by LMMSE and LCMV) are equivalent when all the parameters are known, note that the computation of the LMMSE filter from (12) requires the desired data signal, $a_k[n]$ or an estimate thereof, whereas no information on the fastly varying amplitudes $\mathbf{A}_k$ is required. For the LCMV approach, the situation is the inverse and for both cases, the estimates of either $a_k[n]$ or $\mathbf{A}_k$ need to be provided through an approach such as (8). Alternative structures for the data/amplitude estimate can also be found in [12].

In an adaptive filtering setting, the LCMV approach would be expected to be more sensitive to estimation errors, partly because of the error introduced in the minimisation constraint, partly because the estimation of the data can be assumed to be more robust than the estimation of the amplitudes since the data originates from a strictly finite alphabet.

A natural extension to our proposal to introduce diagonal weighting coefficient matrices instead of scalars, as well as providing an interesting basis for comparison, is to introduce a symbolwise (joint) approach where we apply a single scalar coefficient per symbol per stage instead of per path, i.e.

$$\mathbf{D}_b^o = \arg\min_{\mathbf{D}_b} ||\mathbf{a}[n] - \sum_{b=0}^{B} \mathbf{D}_b(\mathbf{A}^H\mathbf{R}(q)\mathbf{A})^b\mathbf{A}^H\mathbf{x}[n]||^2 \quad (15)$$

where $\mathbf{D}_b = diag(d_{b,1} \ldots d_{b,K})$ and $d_{b,k}$ are scalars. This can be solved in a user-by-user fashion, analogue to the procedure used in the LMMSE approach above. Comparitive results will be shown in the Simulation section. Note that this approach is a simple extension of the work in [8] which used a scalar coefficient per stage instead of a scalar per user.

IV. Simulations

The simulations show the output SINR at the symbol estimate as a function of the input SNR and are obtained for user 1. The SNR is computed w.r.t. the power of user 1. The spreading codes are periodic, and made up of iid random variables $s_{k,l} \in \frac{1}{L}\{+1, -1\}$. Delay spread is half a symbol period and the user delays are uniformly distributed for asynchronous channels. Where path recombining is necessary, maximum SINR recombining is used unless otherwise stated. Users have equal power on average and the simulations are run over 100 realizations. PE-D, PE-D SER, PE-D JOINT and PE-D SYMB denote the filters obtained in (8),(10),(11) and (15), respectively. In figure 1 we can see that both PE-D and PE-D SER are clearly outperformed by the two jointly optimized approaches, PE-D SYMB and PE-D JOINT. Compared with 3 where the situation is inversed. Note that the approaches PE-D and PE-D SER while being inferior to PE-D SYMB and PE-D JOINT by construction, have in effect M degrees of freedom more than than the joint approaches for the same number of stages due to the optimisation at recombining. Due to the small number of stages and the relatively low number of interferers (users and paths) in these simulations, this difference is quite notable. Comparing figure 2 and figure 3, one notices that the total number of paths in the system is equal in both, while the degrees of freedom are not. Indeed, in figure 2 we see that the pathwise approaches clearly outperform the symbolwise approach due to the higher number of paths. With a reduction in the number of paths, however, we can see from figure 3 and figure 1 that the performance gap narrows between the pathwise approach and the symbolwise.

V. Conclusions

Polynomial expansion (PE) is an approximation technique for LMMSE receivers and is particularly well suited for CDMA, due to the presence of a large number of small correlations. However, for PE to work, adjustment factors have to be introduced. We have shown that giving each signal component a separate scaling factor allows for improved performance at a small cost. Also, we have introduced PE at the path level, which allows for interference cancellation and hence improved parameter estimation at the path level. Further, new approaches to PE at symbol level have been introduced, providing more degrees of freedom than previous methods.

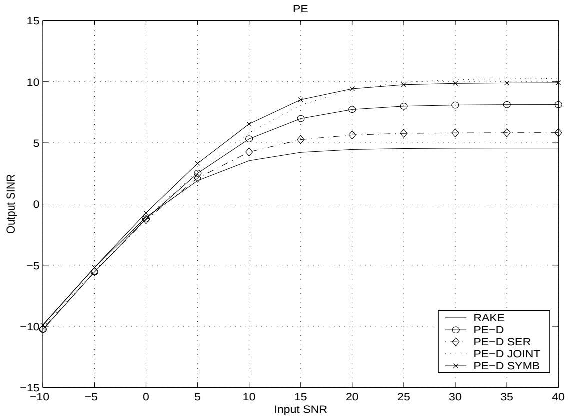

Fig. 1. L = 8,K = 4,M = 3, max. ratio combining

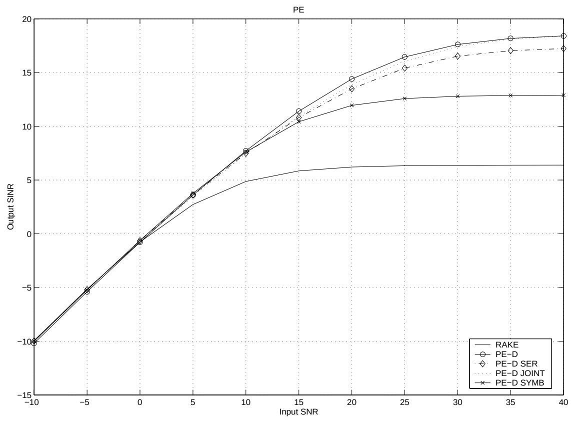

Fig. 2. L = 8,K = 3,M = 4, max. SINR combining

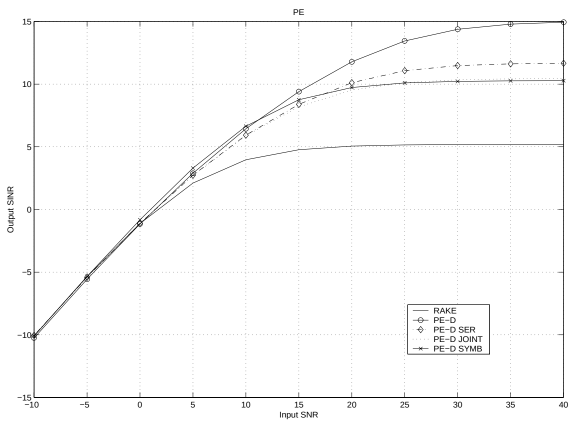

Fig. 3. L = 8,K = 4,M = 3, max. SINR combining

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