Multi-Cell MIMO Power Minimization via Rate Balancing with Partial CSIT

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***Abstract*—In this work, we consider the power allocation problem via rate balancing optimization with imperfect Channel State Information at the Transmitter (CSIT), namely: expected user rate balancing. In particular, we study two closely related optimization problems: maximizing the minimum ergodic user rate under per cell transmit power constraints, and minimizing overall transmit power while satisfying per user rate targets. The max-min rate approach combines an operation of balancing at the user level and sum rate maximization at the level of the user streams. For imperfect CSIT, we exploit an approximation of the expected rate as the Expected Signal and Interference Power (ESIP) rate, based on an original minorizer for every individual rate term. Then, the transmit power is minimized while fulfilling user rate requirements when the latter are feasible, and otherwise switches to weighted rate balancing under the power constraints. Also, we handle the power minimization problem with two variations: minimizing the total transmit power and minimizing the maximum cell transmit power. Simulation results show the effectiveness of the proposed solutions.**

***Index Terms*—Inter-cell interference coordination (ICIC), Coordinated Beamforming (CoBF), Multi-User MIMO, Rate Balancing, Power Minimization, Imperfect CSIT**

I. Introduction

Massive Multiple-Input Multiple-Output (MIMO) has become a key solution to increase the spectral efficiency of wireless cellular systems [1]. In fact, MIMO technology for wireless communications is now incorporated into wireless broadband standards since 3G. The basic idea behind MIMO technology is that the more antennas the transmitter and the receiver are equipped with, the more the available signal paths and signal streams, the better the performance in terms of data rate and energy efficiency [2]–[4].

In downlink communications, the base station (BS) equipped with multiple transmit antennas can serve multiple users within the same time and frequency resource block. Therefore, proper resource allocation is needed to fully harvest the gain in spectral and energy efficiency; for example: user scheduling, subcarrier allocation, *power allocation and precoder (receiver) design*. The latter represents the most important aspect to enhance the performance of the system at the physical layer, and can be combined with frequency subcarrier allocation and user scheduling to further boost the performance.

The power allocation optimization can be formulated as a maximization of some utility in terms of data rate. Depending on the chosen utility function, we can achieve different points on the Pareto optimal boundary. In other words, we cannot increase the rate of any of the active users without lowering the rate of another user [5]. The two most commonly used utility functions are $i)$ weighted sum rate [6]–[13] and $ii)$ weighted max-min fairness, also referred to as the balancing problem. The latter ensures fairness by providing the same quality-of-service for all users according to their priorities and makes this value as large as possible [14]. The weighted max-min fairness problem can be expressed for different objectives such as Signal-to-Noise-plus-Interference Ratio (SINR) [15]–[18], the Mean Squared Error (MSE) [19]–[21] and user rate [22]–[26]. Actually, in the single stream per user case (e.g. in MISO systems), balancing w.r.t. SINR, MSE or user rate is equivalent (in the unweighted case).

In our previous work [23]–[26], we have focused on (weighted) user rate balancing, in which we consider the weighting factors as user priorities since the considered optimization problem aims to maximize the minimum user rate in the system w.r.t. the users priorities. However, these weights can represent target rates, and every set of these targets also corresponds to a point on the boundary of the achievable rate region, which is defined as the set of all feasible rate points, when all users are active simultaneously under an overall power constraint. In this paper, we propose an original utility strategy that switches automatically between rate balancing and power minimization. Given a power constraint, we check if the rate targets are feasible. If they are not, we reduce the rates by performing weighted rate balancing with the target rates as weights. If on the other hand the target rates are feasible, we perform power minimization under the power constraints. The whole strategy is build on a rate balancing algorithm. Two power functions are considered for a multi-cell setting: total sum power over the cells or maximum cell power. And all algorithms are designed for imperfect Channel State Information at the Transmitter (CSIT) with a tighter expected rate approximation than the usual "use and forget" expected rate lower bound. This approximation is based on the Expected Signal and Interference Power (ESIP) rate, which allows straightforward minorization maximization.

II. System Model

We consider a MIMO system with C cells. Each cell c has one BS of M_c transmit antennas serving K_c users, with total number of users $\sum_c K_c = K$. We refer to the BS of user $k \in \{1, \ldots, K\}$ by b_k. Each user has N_k antennas. The channel between the kth user and the BS in cell c is denoted by $\boldsymbol{H}_{k,c} \in \mathbb{C}^{N_k \times M_c}$. We consider zero-mean white Gaussian noise $\boldsymbol{n}_k \in \mathbb{C}^{N_k \times 1}$ with distribution $\mathcal{CN}(0, \sigma_n^2 \boldsymbol{I})$ at the kth user.

We assume independent unity-power transmit symbols $\mathbf{s}_c = [\mathbf{s}_{K_{1:c-1}+1}^{\mathrm{T}} \dots \mathbf{s}_{K_{1:c}}^{\mathrm{T}}]^{\mathrm{T}}$, i.e., $\mathbb{E}[\mathbf{s}_c \mathbf{s}_c^{\mathrm{H}}] = \boldsymbol{I}$, where $\mathbf{s}_k \in \mathbb{C}^{d_k \times 1}$ is the data vector to be transmitted to the kth user, with d_k being the number of streams allowed by user k and $K_{1:c} = \sum_{i=1}^{c} K_i$. The latter is transmitted using the transmit filtering matrix $\boldsymbol{\mathcal{G}}_c = [\boldsymbol{\mathcal{G}}_{K_{1:c-1}+1} \dots \boldsymbol{\mathcal{G}}_{K_{1:c}}] \in \mathbb{C}^{M_c \times N_c}$, with $\boldsymbol{\mathcal{G}}_k = p_k^{1/2} \boldsymbol{G}_k$, $\boldsymbol{G}_k$ being the (unit Frobenius norm) beamforming matrix, p_k is non-negative downlink power allocation of user k and $N_c = \sum_{k:b_k=c} d_k$ is the total number of streams in cell c. Each cell is constrained with $P_{\max,c}$, i.e., the total transmit power in c is limited such that $\sum_{k:b_k=c} p_k \leq P_{\max,c}$. The received signal at user k in cell b_k is

$$\boldsymbol{y}_k = \underbrace{\boldsymbol{H}_{k,b_k} \boldsymbol{\mathcal{G}}_k \boldsymbol{s}_k}_{\text{signal}} + \underbrace{\sum_{\substack{i \neq k \\ b_i = b_k}} \boldsymbol{H}_{k,b_k} \boldsymbol{\mathcal{G}}_i \boldsymbol{s}_i}_{\text{intracell interf.}} + \underbrace{\sum_{j \neq b_k} \sum_{i:b_i=j} \boldsymbol{H}_{k,j} \boldsymbol{\mathcal{G}}_i \boldsymbol{s}_i}_{\text{intercell interf.}} + \boldsymbol{n}_k .$$

III. Joint Mean and Covariance Gaussian CSIT

In this section we drop the user index k and BS index c for simplicity. Assume that the channel has a (prior) Gaussian distribution with zero mean and separable correlation model

$$\boldsymbol{H} = \boldsymbol{C}_r^{1/2} \, \boldsymbol{H}' \, \boldsymbol{C}_t^{1/2} \tag{1}$$

where $\boldsymbol{C}_r^{1/2}$, $\boldsymbol{C}_t^{1/2}$ are Hermitian square-roots of the Rx and Tx side covariance matrices

$$\begin{array}{c} \mathrm{E}\, \boldsymbol{H}\, \boldsymbol{H}^H = \mathrm{tr}\{\boldsymbol{C}_t\}\, \boldsymbol{C}_r \\ \mathrm{E}\, \boldsymbol{H}^H \boldsymbol{H} = \mathrm{tr}\{\boldsymbol{C}_r\}\, \boldsymbol{C}_t \end{array} \tag{2}$$

Now, the Tx dispose of a (deterministic) channel estimate

$$\widehat{\boldsymbol{H}}_d = \boldsymbol{H} + \boldsymbol{C}_r^{1/2} \, \widetilde{\boldsymbol{H}}_d' \, \boldsymbol{C}_d^{1/2} \tag{3}$$

where again the elements of $\widetilde{\boldsymbol{H}}_d'$ are i.i.d. $\sim \mathcal{CN}(0,1)$, and typically $\boldsymbol{C}_d = \sigma_{\widetilde{\boldsymbol{H}}}^2 I_M$. The combination of the estimate with the prior information leads to the (posterior) LMMSE estimate

$$\widehat{\boldsymbol{H}} = \mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}_d} \boldsymbol{H} = \widehat{\boldsymbol{H}}_d \, (\boldsymbol{C}_t + \boldsymbol{C}_d)^{-1} \boldsymbol{C}_t, \; \boldsymbol{C}_p = \boldsymbol{C}_d \, (\boldsymbol{C}_t + \boldsymbol{C}_d)^{-1} \boldsymbol{C}_t \tag{4}$$

where the estimation error on $\widehat{\boldsymbol{H}}$ can be modeled as $\widehat{\boldsymbol{H}} - \boldsymbol{H} = \boldsymbol{C}_r^{1/2} \, \widetilde{\boldsymbol{H}}_p' \, \boldsymbol{C}_p^{1/2}$ with $\widehat{\boldsymbol{H}}$ and $\widetilde{\boldsymbol{H}}_p'$ being independent (or decorrelated if not Gaussian). Note that we get for the MMSE estimate of a quadratic quantity of the form

$$\mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}_d} \boldsymbol{H}^H \boldsymbol{H} = \widehat{\boldsymbol{H}}^H \widehat{\boldsymbol{H}} + \mathrm{tr}\{\boldsymbol{C}_r\} \boldsymbol{C}_p = \boldsymbol{R} \, . \tag{5}$$

Let us emphasize that this MMSE estimate implies $\boldsymbol{R} = \arg\min_{\boldsymbol{T}} \; \mathrm{E}_{\boldsymbol{H}|\hat{\boldsymbol{H}}_d} ||\boldsymbol{H}^H \boldsymbol{H} - \boldsymbol{T}||^2$. It averages out to

$$\mathrm{E}_{\widehat{\boldsymbol{H}}_d} \boldsymbol{R} = \; \mathrm{E}_{\boldsymbol{H},\widehat{\boldsymbol{H}}_d} \boldsymbol{H}^H \boldsymbol{H} = \; \mathrm{E}_{\boldsymbol{H}} \boldsymbol{H}^H \boldsymbol{H} = \mathrm{tr}\{\boldsymbol{C}_r\} \boldsymbol{C}_t \, . \tag{6}$$

Hence, if we want the best estimate for $\boldsymbol{H}^H \boldsymbol{H}$ (which appears in the signal or interference powers), it is not sufficient to replace $\boldsymbol{H}$ by $\widehat{\boldsymbol{H}}$ but also the (estimation error) covariance information should be exploited. Other useful expressions are

$$\mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}_d} \boldsymbol{H}^H \boldsymbol{Q} \boldsymbol{H} = \widehat{\boldsymbol{H}}^H \boldsymbol{Q} \widehat{\boldsymbol{H}} + \mathrm{tr}\{\boldsymbol{C}_r \boldsymbol{Q}\} \boldsymbol{C}_p \tag{7}$$

and $$\mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}_d} \boldsymbol{H} \boldsymbol{P} \boldsymbol{H}^H = \widehat{\boldsymbol{H}} \boldsymbol{P} \widehat{\boldsymbol{H}}^H + \mathrm{tr}\{\boldsymbol{C}_p \boldsymbol{P}\} \boldsymbol{C}_r \, . \tag{8}$$

Note that $\rho_P = \frac{\mathrm{tr}\{\widehat{\boldsymbol{H}}^H \widehat{\boldsymbol{H}}\}}{\mathrm{tr}\{\boldsymbol{C}_r\} \mathrm{tr}\{\boldsymbol{C}_p\}}$ is a form of Ricean factor that represents posterior channel estimation quality. It depends on the deterministic channel estimation quality $\rho_D = 1/\sigma_{\widetilde{\boldsymbol{H}}}^2$. Below we consider $\boldsymbol{C}_r = \boldsymbol{I}$, and the only covariance $\boldsymbol{C}$ we shall need is $\boldsymbol{C}_p$, hence we drop the subscript p. Perfect CSIT algorithms can be obtained by setting $\sigma_{\widetilde{\boldsymbol{H}}}^2 = 0$, leading to $\widehat{\boldsymbol{H}} = \boldsymbol{H}$ and $\boldsymbol{C}_p = 0$.

IV. Problem Formulation

In this work, we aim to optimize the power allocation in terms of the per user rates, for which we consider *i)* the rate balancing problem, referred to as Max-Min Rate (MMR), and *ii)* the total transmit PM problem, namely

MMR: $\max \min_{1 \leq k \leq K} r_k / r_k^o$
under total power constraints $P_{\max}$,

PM: minimize the total transmission power P
while fulfilling $\min_k \frac{r_k}{r_k^o} = \frac{r_k}{r_k^o} = 1 \;\; \forall k$.

with r_k being the kth user-rate

$$r_k = \ln\det\Big(\boldsymbol{I} + \boldsymbol{R}_{\overline{k}}^{-1} \boldsymbol{H}_{k,b_k} \boldsymbol{\mathcal{G}}_k \boldsymbol{\mathcal{G}}_k^{\mathrm{H}} \boldsymbol{H}_{k,b_k}^{\mathrm{H}}\Big) = \ln\det\Big(\boldsymbol{R}_{\overline{k}}^{-1} \boldsymbol{R}_k\Big) \, , \tag{9}$$

$$\boldsymbol{R}_{\overline{k}} = \sigma_n^2 \boldsymbol{I} + \sum_{l \neq k} \boldsymbol{H}_{k,b_l} \boldsymbol{\mathcal{G}}_l \boldsymbol{\mathcal{G}}_l^{\mathrm{H}} \boldsymbol{H}_{k,b_l}^{\mathrm{H}} \, , \tag{10}$$

$$\boldsymbol{R}_k = \boldsymbol{R}_{\overline{k}} + \boldsymbol{H}_{k,b_k} \boldsymbol{\mathcal{G}}_k \boldsymbol{\mathcal{G}}_k^{\mathrm{H}} \boldsymbol{H}_{k,b_k}^{\mathrm{H}} \, , \tag{11}$$

where $\boldsymbol{R}_{\overline{k}}$ and $\boldsymbol{R}_k$ are the interference plus noise and total received signal covariances, r_k° is the individual user rate target for user k, and $P = ||\boldsymbol{p}||_1$ is the total transmit power.

The PM optimization is interesting from a network operator's perspective. In fact, it minimizes intercell interference and improves the power efficiency of the system. Therein, the rate targets r_k^o are considered as feasible if and only if the optimum of MMR is greater than or equal to one, i.e.,

$$\frac{r_k}{r_k^o} \geq 1, \;\; \forall k.$$

While optimizing the PM problem, we have to take into account that the predefined target rates may be unsupported along with the power minimization. Therefore, the design of the algorithmic solution for PM should be in a two-stage approach: First test for feasibility, then minimize the transmission power. In case the rate targets are infeasible, the user rates are *fairly* balanced between users according to their targets, without reaching them. In other words, users achieve reduced rates. If this drop in rates is important, resource management is needed to properly relax the initial conditions (e.g., by reducing the number of users). Actually, in the presence of partial CSIT, we shall be interested in the expected (or ergodic) rates $\overline{r}_k = \; \mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}} r_k$. We shall need

$$\overline{\boldsymbol{S}}_{k,i} = \widehat{\boldsymbol{H}}_{k,b_i} \boldsymbol{G}_i \boldsymbol{G}_i^{\mathrm{H}} \widehat{\boldsymbol{H}}_{k,b_i}^{\mathrm{H}} + \mathrm{tr}\{\boldsymbol{G}_i^{\mathrm{H}} \boldsymbol{C}_{k,b_i} \boldsymbol{G}_i\} \boldsymbol{I}, \; \overline{\boldsymbol{S}}_k = \overline{\boldsymbol{S}}_{k,k} \tag{12}$$

$$\overline{\boldsymbol{R}}_{\overline{k}} = \mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}} \boldsymbol{R}_{\overline{k}} = \sigma_n^2 \boldsymbol{I} + \sum_{i \neq k} p_i \overline{\boldsymbol{S}}_{k,i} \, , \;\; \overline{\boldsymbol{R}}_k = \overline{\boldsymbol{R}}_{\overline{k}} + p_k \overline{\boldsymbol{S}}_k \tag{13}$$

We consider the following optimization problem

$$\begin{aligned} \min_{\boldsymbol{p},\boldsymbol{G}} \;\; & f(||\boldsymbol{p}||_1, t) \\ \text{s.t.} \;\; & r_k(\boldsymbol{p}, \boldsymbol{G}) / r_k^o \geq t, \;\; \forall k \\ & ||\boldsymbol{p}||_1 \leq P_{\max} \end{aligned} \tag{14}$$

$$\text{where } f(||\boldsymbol{p}||_1,t)=u(t-1)(||\boldsymbol{p}||_1+t)-t \tag{15}$$

$$\text{with } u(x)=\begin{cases}0, & x<0\\ 1, & x\geq 0\end{cases} \tag{16}$$

$$\text{and } t=\min_k \overline{r}_k/r_k^o \tag{17}$$

The problem in (14) describes MMR problem [26] when $t<1$. When $t\geq 1$ (14) becomes as follows

$$\begin{aligned} P^{\text{opt}} &= \min_{\boldsymbol{p},\boldsymbol{G}} \;\; ||\boldsymbol{p}||_1 \\ \text{s.t. } & \overline{r}_k(\boldsymbol{p},\boldsymbol{G})/r_k^o \geq 1, \;\; \forall k \\ & ||\boldsymbol{p}||_1 \leq P_{\max} \end{aligned} \tag{18}$$

V. Proposed Solution

Let us denote the function $\mathcal{R}(P_{\max},\boldsymbol{G})$ as follows

$$\begin{aligned} \mathcal{R}(P_{\max}) = \max_{\boldsymbol{G}} \mathcal{R}(P_{\max},\boldsymbol{G}) &= \max_{\boldsymbol{p},\boldsymbol{G}} \min_k \;\; \overline{r}_k(\boldsymbol{p},\boldsymbol{G})/r_k^o \\ & \text{s.t. } ||\boldsymbol{p}||_1 \leq P_{\max}, \;\; \forall k \end{aligned} \tag{19}$$

such that, at iteration (i), we have

$$\min_k \frac{\overline{r}_k(\boldsymbol{p}^{(i-1)},\boldsymbol{G}^{(i)})}{r_k^o} \leq \mathcal{R}(P_{\max},\boldsymbol{G}^{(i)}) \leq \max_k \frac{\overline{r}_k(\boldsymbol{p}^{(i-1)},\boldsymbol{G}^{(i)})}{r_k^o},$$

and at convergence

$$\min_k \frac{\overline{r}_k(\boldsymbol{p},\boldsymbol{G})}{r_k^o} = \mathcal{R}(P_{\max},\boldsymbol{G}) = \max_k \frac{\overline{r}_k(\boldsymbol{p},\boldsymbol{G})}{r_k^o}.$$

Let us now assume that $\mathcal{R}(P_{\max},\boldsymbol{G})>1$ holds, then $t>1$. In other words, the rate targets are feasible; thus, we have additional degrees of freedom that can be used to minimize the total transmission power. In fact, the PM problem is closely related to the MMR problem. Both of them become equivalent if we set $P_{\max}=P^{\text{opt}}$ in (19). Therefore, a modified version of the algorithms solving (19) from [23]–[26] can be used to solve PM in (18).

The designed algorithm is summarized within two steps:

- First, we have to make sure that the predefined targets $r_1^o,\ldots,r_K^o$ are feasible. In other words, there exists at least one iteration n which verifies $t^{(n)}\geq 1$. For that, the algorithm iterates the same steps as for the rate balancing problem until the condition is verified. In case the targets remain infeasible, i.e., $t^{(n)}<1$ for $n\rightarrow\infty$, we must relax the initial conditions.
- The second step is taken into consideration only if the targets are feasible. Thus, the condition $t^{(n)}\geq 1$ here is fulfilled, and the power minimum (18) can be found **by changing the power allocation policy** for the subsequent iterations. In fact, we proceed to minimizing the total transmit power while constraining $\overline{r}_k=r_k^o, \;\; \forall k$, instead of maximizing the achievable rate margin under total power constraint.

A. MaxMin Rate (MMR) Optimization

We start by solving the ergodic rate balancing problem in MMR expressed as follows

$$\begin{aligned} & \max_{\boldsymbol{G},p} \min_k \;\; \overline{r}_k/r_k^\circ \\ \text{s.t. } & \sum_{k:b_k=c} p_k \leq P_{\max,c}, \; c=1,\ldots,C \end{aligned} \tag{20}$$

with $P_{\max,c}$ being the power constraint for cell c.

The following approach will use a rate minorizer for every r_k, similar but not identical to what is used as in the DC programming approach which for the optimization of $\boldsymbol{G}_k$ keeps r_k and linearizes the $r_{\overline{k}}$. The (ergodic) rate balancing problem is approximated by the Expected Signal and Interference Power (ESIP) rate

$$\begin{aligned} \overline{r}_k = & \; \mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}} \ln\det\Big(\boldsymbol{I}+p_k\,\boldsymbol{G}_k^{\mathrm{H}}\,\boldsymbol{H}_{k,b_k}^{\mathrm{H}}\boldsymbol{R}_{\overline{k}}^{-1}\boldsymbol{H}_{k,b_k}\boldsymbol{G}_k\Big) \\ \approx & \; \ln\det\Big(\boldsymbol{I}+p_k\,\boldsymbol{G}_k^{\mathrm{H}}\;\mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}}\{\boldsymbol{H}_{k,b_k}^{\mathrm{H}}(\mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}}\boldsymbol{R}_{\overline{k}})^{-1}\boldsymbol{H}_{k,b_k}\}\boldsymbol{G}_k\Big) \\ = & \; \overline{r}_k^s = f_k^s(\frac{1}{p_k}\overline{\boldsymbol{R}}_{\overline{k}}) = \ln\det\Big(\boldsymbol{I}+\boldsymbol{G}_k^{\mathrm{H}}\,\overline{\boldsymbol{B}}_k(\frac{1}{p_k}\overline{\boldsymbol{R}}_{\overline{k}})\,\boldsymbol{G}_k\Big), \end{aligned} \tag{21}$$

$$\overline{\boldsymbol{B}}_k(\overline{\boldsymbol{T}}_k)=\widehat{\boldsymbol{H}}_{k,b_k}^{\mathrm{H}}\overline{\boldsymbol{T}}_k^{-1}\,\widehat{\boldsymbol{H}}_{k,b_k}+\mathrm{tr}\{\overline{\boldsymbol{T}}_k^{-1}\}\,\boldsymbol{C}_{k,b_k} \tag{22}$$

where the $\overline{r}_k$ approximation $\overline{r}_k^s$ in (21) in general is neither an upper nor lower bound but in the Massive MIMO limit becomes a tight upper bound.

Lemma 1. *The approximate $\overline{r}_k$, $\overline{r}_k^s$, can be obtained as $f_k^s(\frac{1}{p_k}\overline{\boldsymbol{R}}_{\overline{k}})=\min_{\overline{\boldsymbol{T}}_k}\underline{f}_k^s(\overline{\boldsymbol{T}}_k,\frac{1}{p_k}\overline{\boldsymbol{R}}_{\overline{k}})$, with $\underline{f}_k^s(\overline{\boldsymbol{T}}_k,\frac{1}{p_k}\overline{\boldsymbol{R}}_{\overline{k}})$:*

$$\underline{f}_k^s=\ln\det\Big(\boldsymbol{I}+\boldsymbol{G}_k^{\mathrm{H}}\overline{\boldsymbol{B}}_k(\overline{\boldsymbol{T}}_k)\,\boldsymbol{G}_k\Big)+\mathrm{tr}\{\breve{\boldsymbol{W}}_k(\overline{\boldsymbol{T}}_k-\frac{1}{p_k}\overline{\boldsymbol{R}}_{\overline{k}})\} \tag{23}$$

$$\textit{where}\;\breve{\boldsymbol{W}}_k=\overline{\boldsymbol{T}}_k^{-1}\big(\widehat{\boldsymbol{H}}_{k,b_k}\boldsymbol{X}_k\,\widehat{\boldsymbol{H}}_{k,b_k}^{\mathrm{H}}+\mathrm{tr}\{\boldsymbol{X}_k\,\boldsymbol{C}_{k,b_k}\}\boldsymbol{I}\big)\overline{\boldsymbol{T}}_k^{-1} \tag{24}$$

$$\textit{with}\;\boldsymbol{X}_k=\boldsymbol{G}_k\Big(\boldsymbol{I}+\boldsymbol{G}_k^{\mathrm{H}}\overline{\boldsymbol{B}}_k(\overline{\boldsymbol{T}}_k)\,\boldsymbol{G}_k\Big)^{-1}\boldsymbol{G}_k^{\mathrm{H}} \tag{25}$$

The optimizer is $\overline{\boldsymbol{T}}_k=\frac{1}{p_k}\overline{\boldsymbol{R}}_{\overline{k}}$. Also, $\underline{f}_k^s$ is a minorizer for $f_k^s(\frac{1}{p_k}\overline{\boldsymbol{R}}_{\overline{k}})$ as a function of $\frac{1}{p_k}\overline{\boldsymbol{R}}_{\overline{k}}$.

Indeed, since $f_k^s(.)$ is a convex function, it gets minorized by its tangent at any point:

$$f_k^s(\frac{1}{p_k}\overline{\boldsymbol{R}}_{\overline{k}})\geq\underline{f}_k^s=f_k^s(\overline{\boldsymbol{T}}_k)+\mathrm{tr}\{\frac{\partial f_k^s(\overline{\boldsymbol{T}}_k)}{\partial\overline{\boldsymbol{T}}_k}(\frac{1}{p_k}\overline{\boldsymbol{R}}_{\overline{k}}-\overline{\boldsymbol{T}}_k)\} \tag{26}$$

and $\breve{\boldsymbol{W}}_k=-\frac{\partial f_k^s(\overline{\boldsymbol{T}}_k)}{\partial\overline{\boldsymbol{T}}_k}$. Note that for the Perron-Frobenius theory, we need a function that is linear in $\frac{\boldsymbol{p}_{\overline{k}}}{p_k}$, hence we need to work with $\frac{1}{p_k}\overline{\boldsymbol{R}}_{\overline{k}}$ instead of $\overline{\boldsymbol{R}}_{\overline{k}}$.

The user ergodic rate balancing problem can be reformulated as

$$\begin{aligned} & \min_{t,\boldsymbol{G},\boldsymbol{p}} -t \\ & \text{s.t. } \; t\,r_k^\circ-\underline{f}_k^s\leq 0, \; \boldsymbol{c}_c^{\mathrm{T}}\boldsymbol{p}-P_{\max,c}\leq 0\,,\forall k,c. \end{aligned} \tag{27}$$

Introducing Lagrange multipliers to augment the cost function with the constraints leads to the Lagrangian

$$\max_{\lambda',\mu}\min_{t,\boldsymbol{G},\boldsymbol{p}}\mathcal{L}$$

$$\begin{aligned} \mathcal{L} &= -t+\sum_k\lambda_k'(t\,r_k^\circ-\underline{f}_k^s)+\sum_c\mu_c(\boldsymbol{c}_c^{\mathrm{T}}\boldsymbol{p}-P_{\max,c}) \\ &= -t-\sum_k\breve{\lambda}_k'\big(\ln\det\Big(\boldsymbol{I}+\boldsymbol{G}_k^{\mathrm{H}}\overline{\boldsymbol{B}}_k\,\boldsymbol{G}_k\Big)-\frac{1}{p_k}\mathrm{tr}\{\breve{\boldsymbol{W}}_k\overline{\boldsymbol{R}}_{\overline{k}}\} \\ &\quad +\mathrm{tr}\{\breve{\boldsymbol{W}}_k\overline{\boldsymbol{T}}_k\}-t\,r_k^o\big)+\sum_c\mu_c(\boldsymbol{c}_c^{\mathrm{T}}\boldsymbol{p}-P_{\max,c}) \end{aligned} \tag{28}$$

$$=-\,t+\sum_k\breve{\lambda}_k(\frac{1}{p_k\,\breve{\xi}_k}\mathrm{tr}\{\breve{\boldsymbol{W}}_k\overline{\boldsymbol{R}}_{\overline{k}}\}-1)+\sum_c\mu_c(\boldsymbol{c}_c^{\mathrm{T}}\boldsymbol{p}-P_{\max,c}) \tag{29}$$

$$\begin{aligned} \text{with } \breve{\xi}_k &= \mathrm{tr}\{\breve{\boldsymbol{W}}_k\overline{\boldsymbol{T}}_k\}+\ln\det\Big(\boldsymbol{I}+\boldsymbol{G}_k^{\mathrm{H}}\overline{\boldsymbol{B}}_k\,\boldsymbol{G}_k\Big)-t\,r_k^o, \\ \breve{\lambda}_k' &= \breve{\lambda}_k/\breve{\xi}_k, \overline{\boldsymbol{B}}_k=\overline{\boldsymbol{B}}_k(\overline{\boldsymbol{T}}_k), \end{aligned} \tag{30}$$

The balancing of the rates in (20) is equivalent to balancing the weighted interference plus noise powers in (29), i.e.,

$$\begin{aligned} & \max_{\breve{\lambda}}\min_{\boldsymbol{G},\boldsymbol{p}}\sum_k\frac{\breve{\lambda}_k}{\breve{\xi}_k}\frac{\mathrm{tr}(\breve{\boldsymbol{W}}_k\overline{\boldsymbol{R}}_{\overline{k}})}{p_k} \\ & \text{s.t. } \sum_{c=1}^C\theta_c\boldsymbol{c}_c^{\mathrm{T}}\boldsymbol{p}\leq\sum_{c=1}^C\theta_c P_{\max,c} \end{aligned} \tag{31}$$

where $\boldsymbol{c}_c$ is a column vector with $\boldsymbol{c}_c(j) = 1$ for $K_{1:c-1}+1 \leq j \leq K_{1:c}$, and 0 elsewhere. This problem formulation is a relaxation of (20), and $\boldsymbol{\theta} = [\theta_1 \cdots \theta_C]^{\mathrm{T}}$can be interpreted as the weights on the individual power constraints in the relaxed problem. The power constraint in (31) can be interpreted as a single weighted power constraint

$$(\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{C}_C^{\mathrm{T}})\,\boldsymbol{p} \leq \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{p}_{\max} \tag{32}$$

with $\boldsymbol{C}_C = [\boldsymbol{c}_1 \cdots \boldsymbol{c}_C] \in \mathbb{R}_+^{K_{1:C}\times C}$ and $\boldsymbol{p}_{\max} = [P_{\max,1} \cdots P_{\max,C}]^{\mathrm{T}}$, from which we get $\mu_c = \mu\theta_c$.

Now, define the following matrix (reparameterize $\boldsymbol{p} = \frac{\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{p}_{\max}}{\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{C}_C^{\mathrm{T}}\boldsymbol{p}'}\boldsymbol{p}'$ where now $\boldsymbol{p}'$ is unconstrained, and rewriting $\boldsymbol{p}'$ as $\boldsymbol{p}$)

$$\boldsymbol{\Lambda} = \breve{\boldsymbol{\xi}}^{-1}\breve{\boldsymbol{\Psi}} + \frac{1}{\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{p}_{\max}}\breve{\boldsymbol{\xi}}^{-1}\breve{\boldsymbol{\sigma}}\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{C}_C^{\mathrm{T}} \text{ with} \tag{33}$$

$$[\breve{\boldsymbol{\Psi}}]_{ij} = \begin{cases} \mathrm{tr}\{\breve{\boldsymbol{W}}_i(\widehat{\boldsymbol{H}}_{i,b_j}\boldsymbol{G}_j\boldsymbol{G}_j^{\mathrm{H}}\widehat{\boldsymbol{H}}_{i,b_j}^{\mathrm{H}} + \mathrm{tr}\{\boldsymbol{G}_j^{\mathrm{H}}\boldsymbol{C}_{i,b_j}\boldsymbol{G}_j\}\boldsymbol{I})\}, & i \neq j \\ 0, & i = j \end{cases} \tag{34}$$

$$\boldsymbol{\sigma}_i = \sigma_n^2\,\mathrm{tr}\{\breve{\boldsymbol{W}}_i\}, \breve{\boldsymbol{\xi}} = \mathrm{diag}(\breve{\xi}_1,\ldots,\breve{\xi}_K)\,, \tag{35}$$

we can reformulate (31) as

$$\Delta = \max_{\lambda:\sum_k \lambda_k = 1}\min_{\boldsymbol{p}}\sum_k \lambda_k\frac{[\boldsymbol{\Lambda}\,\boldsymbol{p}]_k}{p_k} \tag{36}$$

which is the Donsker–Varadhan–Friedland formula [27, Chapter 8] for the Perron Frobenius eigenvalue of Λ. A related formula is the Rayleigh quotient

$$\Delta = \max_{\boldsymbol{q}}\min_{\boldsymbol{p}}\frac{\boldsymbol{q}^T\boldsymbol{\Lambda}\,\boldsymbol{p}}{\boldsymbol{q}^T\boldsymbol{p}} \tag{37}$$

where $\boldsymbol{p}$, $\boldsymbol{q}$ are the right and left Perron Frobenius eigenvectors. Comparing (37) to (36), then apart from normalization factors, we get $\lambda_k/p_k = q_k$ or hence $\lambda_k = p_k q_k$.

The Tx BF and stream power optimization will be based on $\sum_i \frac{\breve{\lambda}_i}{\breve{\xi}_i}\underline{f}_i^s$, which from (28) becomes (apart from noise terms)

$$\sum_k \frac{\breve{\lambda}_k}{\breve{\xi}_k}\underline{f}_k^s = \sum_k \frac{\breve{\lambda}_k}{\breve{\xi}_k}\ln\det\Big(\boldsymbol{I} + \boldsymbol{G}_k^{\mathrm{H}}\overline{\boldsymbol{B}}_k\,\boldsymbol{G}_k\Big) - \sum_k \mathrm{tr}\{p_k\boldsymbol{G}_k^H\overline{\boldsymbol{A}}_k\boldsymbol{G}_k\} \tag{38}$$

$$\text{with } \overline{\boldsymbol{A}}_k = \sum_{i\neq k}\frac{\breve{\lambda}_i}{p_i\,\breve{\xi}_i}\left(\widehat{\boldsymbol{H}}_{i,b_k}^{\mathrm{H}}\breve{\boldsymbol{W}}_i\widehat{\boldsymbol{H}}_{i,b_k} + \mathrm{tr}\{\breve{\boldsymbol{W}}_i\}\boldsymbol{C}_{i,b_k}\right). \tag{39}$$

For the optimal Tx BF $\boldsymbol{G}_k$, the gradient of $\sum_i \frac{\breve{\lambda}_i}{\breve{\xi}_i}\underline{f}_i^s - \mu_{b_k}\sum_{i:b_i=b_k} p_i\mathrm{tr}\{\boldsymbol{G}_i^{\mathrm{H}}\boldsymbol{G}_i\}$ with (38) (or (21)) yields

$$\frac{\breve{\lambda}_k}{p_k\,\breve{\xi}_k}\overline{\boldsymbol{B}}_k\boldsymbol{G}_k\,(\boldsymbol{I} + \boldsymbol{G}_k^H\overline{\boldsymbol{B}}_k\boldsymbol{G}_k)^{-1} - (\overline{\boldsymbol{A}}_k + \mu_{b_k}\boldsymbol{I})\,\boldsymbol{G}_k = 0\,. \tag{40}$$

The solution is the d_k maximal generalized eigenvectors

$$\boldsymbol{G}_k' = V_{1:d_k}(\overline{\boldsymbol{B}}_k, \overline{\boldsymbol{A}}_k + \mu_{b_k}\boldsymbol{I}), \boldsymbol{G}_k = \boldsymbol{G}_k'\overline{\boldsymbol{P}}_k^{1/2}, \boldsymbol{\mathcal{G}}_k = \boldsymbol{G}_k\sqrt{p_k} \tag{41}$$

where the $\overline{\boldsymbol{P}}_k = \mathrm{diag}(p_{k,1},\ldots,p_{k,d_k})$, $\mathrm{tr}\{\overline{\boldsymbol{P}}_k\} = 1$, are the relative stream powers. Indeed, (40) represents the definition of generalized eigenvectors. Consider

$$\Sigma_k^{(1)} = \boldsymbol{G}_k^{'\mathrm{H}}\overline{\boldsymbol{B}}_k\boldsymbol{G}_k',\; \Sigma_k^{(2)} = \boldsymbol{G}_k^{'\mathrm{H}}\overline{\boldsymbol{A}}_k\boldsymbol{G}_k' \tag{42}$$

then the generalized eigenvectors $\boldsymbol{G}_k'$ of $\overline{\boldsymbol{B}}_k, \overline{\boldsymbol{A}}_k + \mu_{b_k}\boldsymbol{I}$ lead to diagonal matrices $\Sigma_k^{(1)}$, $\Sigma_k^{(2)} + \mu_{b_k}\boldsymbol{G}_k^{'\mathrm{H}}\boldsymbol{G}_k'$. Note that the normalized $\boldsymbol{G}_k'$ are not orthogonal. Then (40) represents the generalized eigenvector condition with associated generalized eigenvalues in the diagonal matrix $\frac{p_k\,\breve{\xi}_k}{\breve{\lambda}_k}(\boldsymbol{I} + \Sigma_k^{(1)}\overline{\boldsymbol{P}}_k)$. Also, plugging in generalized eigenvectors into (38) reveals that one should choose the eigenvectors associated to d_k maximal eigenvalues to maximize (38). Now, premultiplying both sides of (40) by $p_k\boldsymbol{G}_k^{\mathrm{H}}$, summing over all users $k : b_k = c$, taking trace and identifying the last term with $\sum_{k:b_k=c} p_k\mathrm{tr}\{\boldsymbol{G}_k^{\mathrm{H}}\boldsymbol{G}_k\} = P_{max,c}$ allows to solve for

$$\mu_c = \frac{1}{P_{max,c}}\left\lfloor\sum_{k:b_k=c}\mathrm{tr}\{\frac{\breve{\lambda}_k}{\breve{\xi}_k}\Sigma_k^{(1)}\overline{\boldsymbol{P}}_k(\boldsymbol{I} + \Sigma_k^{(1)}\overline{\boldsymbol{P}}_k)^{-1} - p_k\Sigma_k^{(2)}\overline{\boldsymbol{P}}_k\}\right\rfloor_+. \tag{43}$$

The $\overline{\boldsymbol{P}}_k$ are themselves found from an interference leakage aware water filling (ILAWF) operation. Substituting $\boldsymbol{G}_k'$ into term k of (38), dividing by p_k, and accounting for the constraint $\mathrm{tr}\{\overline{\boldsymbol{P}}_k\} = 1$ by Lagrange multiplier ν_k, we get the Lagrangian

$$\begin{aligned}&\frac{\breve{\lambda}_k}{p_k\,\breve{\xi}_k}\ln\det\big(\boldsymbol{I} + \Sigma_k^{(1)}\overline{\boldsymbol{P}}_k\big) - \mathrm{tr}\{(\Sigma_k^{(2)} + \nu_k\boldsymbol{I})\overline{\boldsymbol{P}}_k\} = \\ &\frac{\breve{\lambda}_k}{p_k\,\breve{\xi}_k}\ln\det\big(\boldsymbol{I} + \Sigma_k^{(1)}\overline{\boldsymbol{P}}_k\big) - \mathrm{tr}\{(\mathrm{diag}(\Sigma_k^{(2)}) + \nu_k\boldsymbol{I})\overline{\boldsymbol{P}}_k\}.\end{aligned} \tag{44}$$

Maximizing w.r.t. $\overline{\boldsymbol{P}}_k$ leads to the ILAWF

$$\overline{\boldsymbol{P}}_k = \left\lfloor\frac{\breve{\lambda}_k}{p_k\,\breve{\xi}_k}\,(\mathrm{diag}(\Sigma_k^{(2)}) + \nu_k\boldsymbol{I})^{-1} - \Sigma_k^{-(1)}\right\rfloor_+ \tag{45}$$

where the Lagrange multiplier ν_k is adjusted (e.g. by bisection) to satisfy $\mathrm{tr}\{\overline{\boldsymbol{P}}_k\} = 1$. Elements in $\overline{\boldsymbol{P}}_k$ corresponding to zeros in $\Sigma_k^{(1)}$ should also be zero.

B. Power Minimization (PM)

The key idea of this design is to change the power control policy when the user rate targets are feasible. In fact we have the following: since the ESIP-based MMR problem is formulated as max-min weighted interference plus noise powers, the related power minimization problem is constrained by

$$t = \min_k \frac{\overline{r}_k}{r_k^o} = 1 \tag{46}$$

$$\Leftrightarrow 1/p_k\frac{\mathrm{tr}\{\breve{\boldsymbol{W}}_k\overline{\boldsymbol{R}}_{\overline{k}}\}}{\xi_k} = 1 \tag{47}$$

or,

$$\overline{r}_k = r_k^o \Leftrightarrow 1/p_k\mathrm{tr}\{\breve{\boldsymbol{W}}_k\overline{\boldsymbol{R}}_{\overline{k}}\} = \breve{\xi}_k, \forall k \tag{48}$$

As we consider IBC case, we proceed as follows:

1) when $t < 1$, optimize the rate balancing problem while fulfilling the per cell power constraints by the means of Lagrangian multipliers μ_c.
2) when $t \geq 1$, change the power allocation strategy $\boldsymbol{p}$ to meet the targets with equality, which minimizes the total transmit power $P = ||\boldsymbol{p}||_1$. In this case, the corresponding MMR problem is constrained only by this new total transmit power, thus, the Lagrangian depends on $\mu_o = \sum_c \mu_c$.

Collecting the per user weighted interference plus noise powers in a diagonal matrix $\breve{\boldsymbol{\epsilon}}_w$ as follows

$$[\breve{\boldsymbol{\epsilon}}_w]_{k,k} = 1/p_k\mathrm{tr}\{\breve{\boldsymbol{W}}_k\overline{\boldsymbol{R}}_{\overline{k}}\} \tag{49}$$

$$\breve{\boldsymbol{\epsilon}}_w\boldsymbol{1}_K = \mathrm{diag}(\boldsymbol{p})^{-1}\left[\breve{\boldsymbol{\Psi}}\boldsymbol{p} + \boldsymbol{\sigma}\right] \tag{50}$$

The corresponding optimal power allocation to achieve the targets $\breve{\boldsymbol{\xi}}$ is then

$$\boldsymbol{p} = (\breve{\boldsymbol{\xi}} - \breve{\boldsymbol{\Psi}})^{-1}\boldsymbol{\sigma},. \tag{51}$$

Then, we set the new power constraint for MMR optimization as $P_{\max} = P$ with

$$P = ||\boldsymbol{p}||_1. \tag{52}$$

which completes the optimization framework.

TABLE I: ESIPrate based Power Minimization Algorithm

1. For predefined r_k^o, initialize: $\boldsymbol{G}_k^{(0,0)} = (\boldsymbol{I}_{d_k} : \boldsymbol{0})^{\mathrm{T}}$, $\boldsymbol{p}_k^{(0,0)} = \boldsymbol{q}_k^{(0,0)} = \frac{P_{\max,c}}{K}$, $m = n = 0$ and fix $n_{\max}, m_{\max}, \breve{\xi}_k^{(0)}$, and $\breve{\boldsymbol{W}}_k^{(0)} = \boldsymbol{I}$, $t^{(0)} = 0$
2. **repeat**
   - 2.1 $m \leftarrow m + 1$
   - 2.2 update $\overline{\boldsymbol{A}}_k$ from (39)
   - 2.3 update $\boldsymbol{G}_k'$ from (41)
   - 2.4 update $\overline{\boldsymbol{P}}_k$ from (45)
     if $t^{(m-1)} < 1$
     update $\boldsymbol{p}$ and $\boldsymbol{q}$ as maximal eigenvectors of $\breve{\boldsymbol{\Lambda}}$ in (33)
     else
     update $\boldsymbol{p}$ with (51) and do $P = ||\boldsymbol{p}||_1$ (52)
     update $\boldsymbol{q}$ as maximal left eigenvector of $\breve{\boldsymbol{\Lambda}}(P_{\max} = P)$
     end if
   - 2.5 compute $\overline{\boldsymbol{B}}_k(\overline{\boldsymbol{T}}_k)$ and update $\breve{\boldsymbol{W}}_k$ from (24)
   - 2.6 compute $\overline{r}_k^{s\,(m)} = \mathrm{lndet}\Big(\boldsymbol{I} + \boldsymbol{G}_k^{\mathrm{H}}\,\overline{\boldsymbol{B}}_k(\frac{1}{p_k}\overline{\boldsymbol{R}}_{\overline{k}})\;\boldsymbol{G}_k\Big)$ and determine $t^{(m)} = \min_k \frac{\overline{r}_k^{s\,(m)}}{r_k^o}$
   - 2.7 **if** $t^{(m)} < 1$
     update $\breve{\xi}_k^{(m)} = \mathrm{tr}\{\breve{\boldsymbol{W}}_k^{(m)}\overline{\boldsymbol{T}}_k^{(m)}\} + \overline{r}_k^{s(m)} - t^{(m)} r_k^o$
     else
     update $\breve{\xi}_k^{(m)} = \mathrm{tr}\{\breve{\boldsymbol{W}}_k^{(m)}\overline{\boldsymbol{T}}_k^{(m)}\} + \overline{r}_k^{s(m)} - r_k^o$
     end if
3. **until** required accuracy is reached or $m \geq m_{\max}$

C. Per cell power minimization/balancing

Now, consider the following power minimization problem

$$\begin{aligned} \min_{\boldsymbol{p},\boldsymbol{G}} \;& P^s \\ \text{s.t.}\;& r_k(\boldsymbol{p},\boldsymbol{G})/r_k^o \geq 1, \;\; \forall k \\ & \sum_{k:b_k=c} \mathrm{tr}\{\boldsymbol{\mathcal{G}}_k^{\mathrm{H}}\boldsymbol{\mathcal{G}}_k\} \leq P^s, \;\; \forall c \\ & P_c \leq P_{\max,c}, \;\; \forall c \end{aligned} \tag{53}$$

where $P^s = \max_c P_c$ and $P_c = \boldsymbol{c}_c^T\boldsymbol{p} = \sum_{k:b_k=c}\mathrm{tr}\{\boldsymbol{\mathcal{G}}_k^{\mathrm{H}}\boldsymbol{\mathcal{G}}_k\}$.

Similar to the total power minimization case, the optimal power allocation to achieve the targets $\breve{\boldsymbol{\xi}}$ is again

$$\boldsymbol{p} = (\breve{\boldsymbol{\xi}} - \breve{\boldsymbol{\Psi}})^{-1}\boldsymbol{\sigma}.$$

The per cell transmit powers P_c are obtained as follows

$$P_c = \boldsymbol{c}_c^T\boldsymbol{p}.$$

The defined optimization problem aims to minimize the maximum transmit power among BSs, namely $P^s = \max_c P_c$. Therefore, we set the new power constraints for MMR optimization as $P_{\max,c} = P^s$, identical $\forall$ c. Doing so, $\mu_c(P_{\max,c} = P^s)$ will make sure that all transmit powers $\boldsymbol{c}_c^T\boldsymbol{p}$ do not exceed the minimized maximum P^s, i.e., $P_c = \boldsymbol{c}_c^T\boldsymbol{p} \leq P^s$. Of course, at convergence, we have $P_c = P^s$ $\forall$ c.

VI. Simulation Results

In this section, we numerically evaluate the performance of RESIP-based vs. ESIP-based approaches. We consider for the multipath channel model,

$$\boldsymbol{C}_t = \sum_{i=1}^{N_p} \frac{\alpha_i}{\boldsymbol{v}_i^H\boldsymbol{v}_i}\boldsymbol{v}_i\boldsymbol{v}_i^H \tag{54}$$

with $\mathrm{tr}\{\boldsymbol{C}_t\} = \sum_{n=1}^{N_p}\alpha_i = M_c$, $\alpha_i = c^{i-1}\alpha_1$ and the $\boldsymbol{v}_i$ are i.i.d. vectors of M_c i.i.d. elements $\mathcal{CN}(0,1)$. We take $N_p = M_c/K$ and $c = 0.5$. In Figure 1, we plot the achieved average rate and total transmit power using Table I, for BC. We set identical user targets $r_k^o = 4$, $\forall k$ and $P_{\max} = 10^{\mathrm{SNR}\sigma_n^2/10}$. We can see that, when the rate targets are feasible, (i.e., $r_k(P_{\max})/r_k^o \geq 1$ with MMR optimization), the user rates using perfect CSIT and ESIPrate UB meet the targets with equality and the total transmit power is minimized accordingly. ESIPrate UB (Upper Bound) is the ESIP expected rate approximation that we optimize. ESIPrate is the actual average rate that this approach yields. From the figure zoom, one can see that the ESIP approximation is a tight upper bound. Also, the same total minimized power P is achieved $\forall$ $P_{\max}$. In this figure, this case corresponds to SNR> 10dB. When the targets are infeasible (SNR below 10dB), Table I acts as a MMR algorithm since the target rates are no longer feasible.

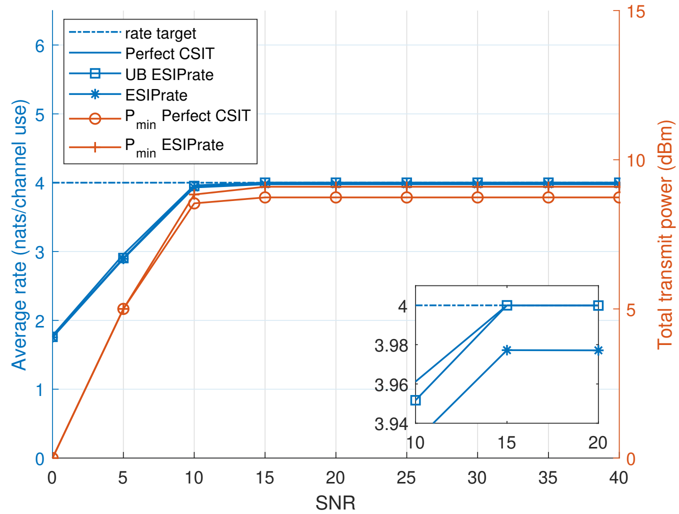

Fig. 1: Total Power Minimization for BC via ESIP, $C = 1, K = 3, M_c = 12, N_k = d_k = 2, \rho_D = 10$.

Figure 2 plots the achieved rate using per cell power minimization for IBC scenario, and Figure 3 illustrates the corresponding transmitted power per cell. We can see that, when the rate targets are feasible, the transmit power is minimized within each cell with equality while fulfilling the rate targets. Also, the power required reduces drastically with an increase in number of antennas, which was one of the driving forces behind massive MIMO. Finally, Figure 4 compares the evolution in power requirements vs. the number of BS antennas M, for given rate requirements. It can be seen that the power required diminishes with a factor slightly larger than the increase in M. Also, imperfect CSIT requires slightly larger power than perfect CSIT, but the gap diminishes with M.

VII. Conclusions

In this paper, we have investigated the total power minimization problem subject to a set of user rate targets. An iterative strategy was derived via the ergodic user rate balancing to optimize the problem, w.r.t. ESIP rate approximation. Simulation results showed that, for appropriate (feasible) user rate targets, arbitrary points within the achievable rate region can be achieved with minimal expense of transmission power.

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References

[1] A. Goldsmith, *Wireless Communications*.

[2] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, no. 3, p. 311–335, 1998.

[3] R. K. Mueller and G. J. Foschini, "The capacity of linear channels with additive gaussian noise," *The Bell System Technical Journal*, vol. 49, no. 1, pp. 81–94, 1970.

[4] E. Telatar, "Capacity of multi-antenna gaussian channels," *European Transactions on Telecommunications*, vol. 10, no. 6, p. 585–595, 1999.

[5] E. A. Jorswieck, E. G. Larsson, and D. Danev, "Complete characterization of the pareto boundary for the miso interference channel," *IEEE Transactions on Signal Processing*, vol. 56, no. 10, pp. 5292–5296, 2008.

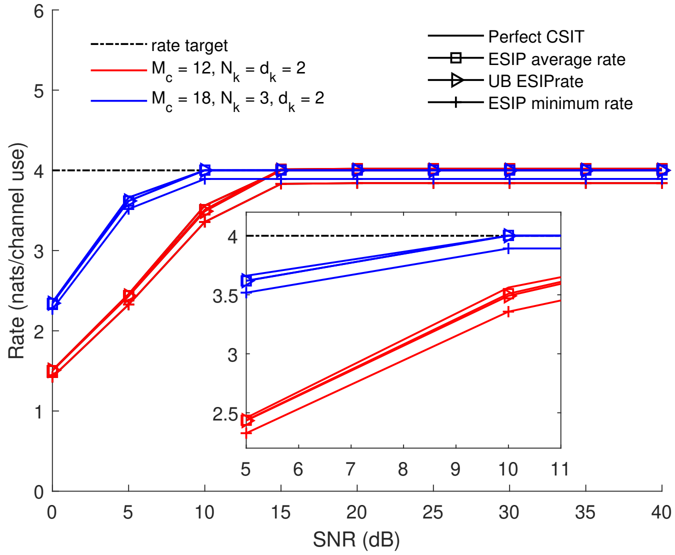

Fig. 2: Achieved rate vs. SNR using per cell Power Minimization via ESIP: $C = 2, K_c = 3, \rho_D = 10$, and $r_k^o = 4 \forall k$.

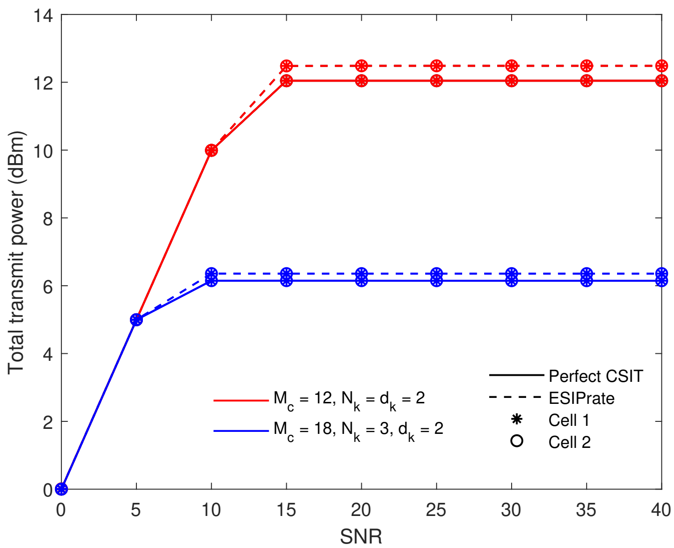

Fig. 3: Per cell transmit power vs. SNR using per cell Power Minimization via ESIP: $C = 2, K_c = 3, \rho_D = 10$, and $r_k^o = 4 \forall k$.

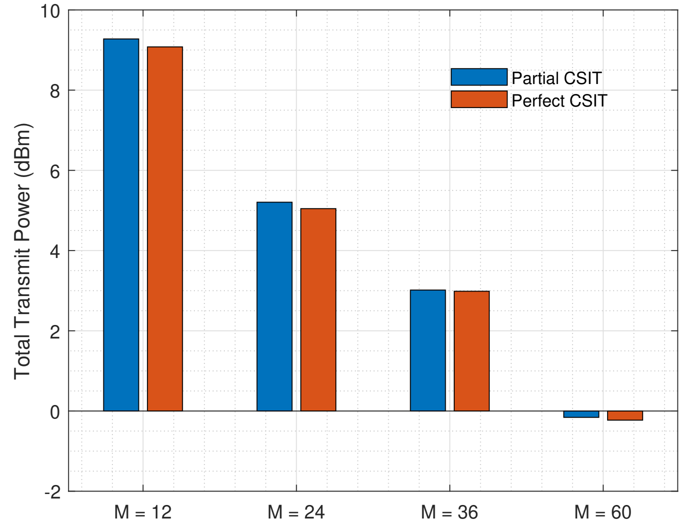

Fig. 4: Total Power Minimization for BC via ESIP: $C = 1, K_c = 3, N_k = d_k = 2, \rho_D = 10$, and $r_k^o = 4 \forall k$.

[6] L. P. Qian, Y. J. Zhang, and J. Huang, "Mapel: Achieving global optimality for a non-convex wireless power control problem," *IEEE Transactions on Wireless Communications*, vol. 8, no. 3, pp. 1553–1563, 2009.

[7] E. Björnson, G. Zheng, M. Bengtsson, and B. Ottersten, "Robust monotonic optimization framework for multicell miso systems," *IEEE Transactions on Signal Processing*, vol. 60, no. 5, pp. 2508–2523, 2012.

[8] P. C. Weeraddana, M. Codreanu, M. Latva-aho, A. Ephremides, and C. Fischione, 2012.

[9] S. S. Christensen, R. Agarwal, E. de Carvalho, and J. M. Cioffi, "Weighted Sum-Rate Maximization using Weighted MMSE for MIMO-BC Beamforming Design," in *Weighted Sum-Rate Maximization using Weighted MMSE for MIMO-BC Beamforming Design*, Dec 2008.

[10] Q. Shi, M. Razaviyayn, Z.-Q. Luo, and C. He, "An Iteratively Weighted MMSE Approach to Distributed Sum-Utility Maximization for a MIMO Interfering Broadcast Channel," in *IEEE Trans. Signal Processing*, Sept. 2011.

[11] P. Komulainen, A. Tölli, and M. Juntti, "Effective csi signaling and decentralized beam coordination in tdd multi-cell mimo systems," *IEEE Transactions on Signal Processing*, vol. 61, no. 9, pp. 2204–2218, 2013.

[12] D. H. N. Nguyen and T. Le-Ngoc, "Sum-rate maximization in the multicell mimo multiple-access channel with interference coordination," *IEEE Transactions on Wireless Communications*, vol. 13, no. 1, pp. 36–48, 2014.

[13] L. Tran, M. F. Hanif, A. Tolli, and M. Juntti, "Fast converging algorithm for weighted sum rate maximization in multicell miso downlink," *IEEE Signal Processing Letters*, vol. 19, no. 12, pp. 872–875, 2012.

[14] L. Zheng, D. W. H. Cai, and C. W. Tan, "Max-min Fairness Rate Control in Wireless Networks: Optimality and Algorithms by Perron-Frobenius Theory," *IEEE Transactions on Mobile Computing*, Jan. 2018.

[15] M. Schubert and H. Boche, "Solution of the Multiuser Downlink Beamforming Problem with Individual SINR Constraints," *IEEE Trans. Vehic. Tech.*, Jun 2004.

[16] W. Yu and T. Lan, "Transmitter Optimization for the Multi-antenna Downlink with Per-antenna Power Constraints," in *IEEE Trans. Signal Processing*, June 2007.

[17] L. Zhang, R. Zhang, Y. C. Liang, Y. Xin, and H. V. Poor, "On Gaussian MIMO BC-MAC Duality with Multiple Transmit Covariance Constraints," *IEEE Trans. Inform. Theory*, Apr. 2012.

[18] K. Cumanan, L. Musavian, S. Lambotharan, and A. B. Gershman, "SINR Balancing Technique for Downlink Beamforming in Cognitive Radio Networks," *IEEE Signal Process. Lett.*, Feb. 2010.

[19] S. Shi, M. Schubert, and H. Boche, "Downlink MMSE Transceiver Optimization for Multiuser MIMO Systems: Duality and sum-MSE Minimization," in *IEEE Trans. Signal Process*, vol. 55, no. 11, Nov 2007.

[20] ——, "Capacity Balancing for Multiuser MIMO Systems," in *Proc. IEEE ICASSP*, Apr 2007.

[21] R. Hunger, M. Joham, and W. Utschick, "On the MSE-Duality of the Broadcast Channel and the Multiple Access Channel," in *IEEE Trans. Signal Processing*, vol. 57, no. 2, Feb 2009, p. 698–713.

[22] M. Razaviyayn, M. Hong, and Z.-Q. Luo, "Linear Transceiver Design for a MIMO Interfering Broadcast Channel Achieving Max-min Fairness," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Nov 2011.

[23] I. Ghamnia, D. Slock, and Y. Yuan-Wu, "Rate balancing for multiuser multicell downlink MIMO Systems," in *27th European Signal Processing Conference (EUSIPCO), A Coruna, Spain*, 2019.

[24] ——, "Rate Balancing for Multiuser MIMO Systems," in *IEEE 20th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, Cannes, France, Jul 2019.

[25] ——, "MIMO user rate balancing in multicell networks with per cell power constraints," in *IEEE 91st Vehicular Technology Conference (VTC2020-Spring), Antwerp, Belgium*, 2020.

[26] ——, "Multi-cell mimo user rate balancing with partial CSIT," in *IEEE 93rd Vehicular Technology Conference (VTC2021-Spring), Virtual Conference*, 2021.

[27] C. Meyer, *Matrix Analysis and Applied Linear Algebra*. SIAM, 2000.