On Linear Channel-based Noise Subspace Parameterizations for Blind Multichannel Identification

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Abstract — **In a multichannel context, the problem of blind estimation of the channel can be parameterized either by the channel impulse response or by the noisefree multivariate prediction error filter and the first vector coefficient of the vector channel. The noise subspace, spanned by a set of vectors that are orthogonal to the signal subspace, can be parameterized according to different linear parameterizations. In the first part of this paper, we begin with the resaons due to which second-order-statistics-based estimation techniques give accurate channel estimates. In the second part, we focus on the different noise subspace parameterizations in terms of blocking equalizers and classify them. We present linear (in terms of subchannel impulse responses) noise subspace parameterizations and we prove that using a specific parameterization, which is minimal in terms of the number of rows, leads to span the overall noise subspace.**

I. Introduction

In blind channel identification, a multichannel framework can be obtained from oversampling a received signal and leads to a Single Input Multiple Output (SIMO) vector channel representation. The multiple FIR channels we obtain in this representation can also be obtained from multiple received signals from an array of antennas (in the context of mobile digital communications [1],[2],[3]) or from a combination of both. To further develop the case of oversampling, consider a linear digital modulation over a linear channel with additive noise so that the received signal $y(t)$ has the following form

$$y(t) \;=\; \sum_k h(t-kT)a(k) \;+\; v(t). \tag{1}$$

In (1) $a(k)$ are the transmitted symbols, T is the symbol period and $h(t)$ is the channel impulse response. The channel is assumed to be FIR with length NT. If the received signal is oversampled at the rate $\frac{m}{T}$ (or if m different samples of the received signal are captured by m sensors every T seconds, or a combination of both), the discrete input-output relationship can be written as:

$$\boldsymbol{y}(k) = \sum_{i=0}^{N-1} \boldsymbol{h}(i)a(k-i) + \boldsymbol{v}(k) = \boldsymbol{H}A_N(k) + \boldsymbol{v}(k) \tag{2}$$

where $\boldsymbol{y}(k) = [y_1^H(k)\cdots y_m^H(k)]^H$, $\boldsymbol{h}(i) = \left[h_1^H(i)\cdots h_m^H(i)\right]^H$, $\boldsymbol{v}(k) = [v_1^H(k)\cdots v_m^H(k)]^H$, $\boldsymbol{H} = [\boldsymbol{h}(N{-}1)\cdots \boldsymbol{h}(0)]$, $A_N(k) = \left[a(k{-}N{+}1)^H \cdots a(k)^H\right]^H$ and superscript ^{H} denotes Hermitian transpose. Let $\mathbf{H}(z) = \sum_{i=0}^{N-1}\boldsymbol{h}(i)z^{-i} = [\mathrm{H}_1^H(z)\cdots \mathrm{H}_m^H(z)]^H$ be the SIMO channel transfer function, and $\boldsymbol{h} = \left[\; \boldsymbol{h}^H(N{-}1)\cdots \boldsymbol{h}^H(0)\;\right]^H$. Consider additive independent white Gaussian circular noise $\boldsymbol{v}(k)$ with $r_{\boldsymbol{vv}}(k{-}i) = \mathrm{E}\,\boldsymbol{v}(k)\boldsymbol{v}(i)^H = \sigma_v^2 I_m\,\delta_{ki}$. Assume we receive M samples:

$$\boldsymbol{Y}_M(k) \;=\; \mathcal{T}_M(\boldsymbol{h})\,A_{M+N-1}(k) + \boldsymbol{V}_M(k) \tag{3}$$

where $\boldsymbol{Y}_M(k) = [\boldsymbol{y}^H(k{-}M{+}1)\cdots \boldsymbol{y}^H(k)]^H$ and similarly for $\boldsymbol{V}_M(k)$, and $\mathcal{T}_M(\boldsymbol{h})$ is a block Toepltiz matrix with M block rows and $[\boldsymbol{H}\;\; 0_{m\times(M-1)}]$ as first block row. We shall simplify the notation in (3) with $k = M{-}1$ to

$$\boldsymbol{Y} \;=\; \mathcal{T}(\boldsymbol{h})\,A + \boldsymbol{V}\;. \tag{4}$$

We assume that $mM > M{+}N{-}1$ in which case the channel convolution matrix $\mathcal{T}(\boldsymbol{h})$ has more rows than columns. If the $\mathrm{H}_i(z)$, $i = 1,\ldots,m$ have no zeros in common, then $\mathcal{T}(\boldsymbol{h})$ has full column rank (which we will henceforth assume). For obvious reasons, the column space of $\mathcal{T}(\boldsymbol{h})$ is called the signal subspace and its orthogonal complement the noise subspace. The signal subspace is parameterized linearly by $\boldsymbol{h}$, and the quality of its estimation, with a finite amount of data, is illustrated in the following section.

II. Accuracy of the Second-order Statistics in Blind Estimation of Multiple FIR Channels

Recently, there has been an explosion of work dealing with blind channel estimation and/or equalization based on (sample) Second-Order Statistics (SOS) of the received data. This attention is generally justified by the lesser complexity of the second-order moment which makes the use of this class of blind channel estimation methods more desirable compared to e.g. Higher-Order Statistics (HOS)-based techniques. The fact that SOS can be sufficient for channel identification is due to the multichannel aspect. In the sequel, we show that in the case of SOS, the fact that the channel can be estimated fairly accurately using relatively few data (a must for mobile communications) is due to the perfect estimation of the signal subspace with this few amount of data whereas the second-order moment can not be perfectly estimated.

The structure of the covariance matrix of the received signal, $\boldsymbol{Y}$, is

$$R_{YY} = \mathrm{E}\boldsymbol{Y}\boldsymbol{Y}^H = \mathcal{T}(\boldsymbol{h})R_{AA}\mathcal{T}^H(\boldsymbol{h}) + \sigma_v^2 I_{mM}, \tag{5}$$

where R_{AA} is the symbols covariance matrix $\mathrm{E}AA^H > 0$. The covariance matrix R_{YY} can be decomposed into signal and noise subspace contributions as follows:

$$\begin{aligned} R_{YY} = \mathrm{E}\boldsymbol{Y}\boldsymbol{Y}^H &= \sum_{i=1}^{M+N-1} \lambda_i V_i V_i^H + \sum_{i=M+N}^{mM} \lambda_i V_i V_i^H \\ &= \mathcal{V}_S \Lambda_S \mathcal{V}_S^H + \mathcal{V}_N \Lambda_N \mathcal{V}_N^H. \end{aligned} \tag{6}$$

In the eigen decomposition of the covariance matrix, R_{YY}, given in (6), the real nonnegative eigenvalues λ_i are ordered in descending order, $\lambda_i > \sigma_v^2$ for $i = 1, \cdots, M+N-1$; $\Lambda_N = \sigma_v^2 I_{(m-1)M-N+1}$ and the sets of the eigenvectors $\mathcal{V}_S$ and $\mathcal{V}_N$ are orthonormal: $\mathcal{V}_S^H \mathcal{V}_N = 0$. Since *Range* $\{\mathcal{T}(\boldsymbol{h})\}$ = *Range* $\{\mathcal{V}_S\}$, both $\mathcal{T}(\boldsymbol{h})$ and $\mathcal{V}_S$ should span the signal subspace [3], [2]. An analysis of the singular value decomposition of this noisy matrix and the associated subspaces is presented in [10]. In order to study the accuracy of the estimated signal subspace we use the asymptotic distribution of the signal eigen vectors of the sample covariance matrix. The signal and noise subspaces can be consistently estimated from the eigen decomposition of the sample covariance matrix

$$\widehat{R}_{YY} = \frac{1}{T}\sum_{i=1}^{T} Y_M(i) Y_M^H(i) = \widehat{\mathcal{V}}_S \widehat{\Lambda}_S \widehat{\mathcal{V}}_S^H + \widehat{\mathcal{V}}_N \widehat{\Lambda}_N \widehat{\mathcal{V}}_N^H. \tag{7}$$

From the asymptotic theory of principal components [11], we have the following result: *the $M+N-1$ largest eigen vectors of $\widehat{R}_{YY}$ are asymptotically (for large T) normally distributed with means and covariances given by*

$$\begin{aligned} \mathrm{E}\widehat{V}_k &= V_k + O(T^{-1}) \\ \mathrm{E}(\widehat{V}_k - V_k)(\widehat{V}_l - V_l)^H &= \delta_{kl}\frac{\lambda_k}{T}\sum_{i=1, i\neq k}^{mM} \frac{\lambda_i}{(\lambda_i - \lambda_k)^2} V_i V_i^H \\ &\quad + o(T^{-1}) \\ \mathrm{E}(\widehat{V}_k - V_k)(\widehat{V}_l - V_l)^T &= (1-\delta_{kl})\frac{-\lambda_k \lambda_l}{T(\lambda_k - \lambda_l)^2} V_l V_k^T \\ &\quad + o(T^{-1}). \end{aligned} \tag{8}$$

A measure of the error on the estimated signal subspace can be obtained by considering the metric $\mathrm{E}\|\widehat{\mathcal{V}}_S - \mathcal{V}_S\|_F^2 = \sum_k \mathrm{E}(\widehat{V}_k - V_k)(\widehat{V}_k - V_k)^H$ (where $\|A\|_F^2$ is the Frobenius norm of the matrix A). It is clear from (8) that $\mathrm{E}\tilde{V}_k \tilde{V}_k^H = \mathrm{E}(\widehat{V}_k - V_k)(\widehat{V}_k - V_k)^H$ can be rewritten as the sum of two components: the first is due to the signal subspace $\epsilon_{\tilde{V}_k}^S$ and the second is related to the noise subspace $\epsilon_{\tilde{V}_k}^N$

$$\begin{aligned} \mathrm{E}\tilde{V}_k \tilde{V}_k^H &= \frac{\lambda_k}{T}\sum_{i=1, i\neq k}^{mM} \frac{\lambda_i}{(\lambda_i-\lambda_k)^2} V_i V_i^H + o(T^{-1}) \\ &= \frac{\lambda_k}{T}\sum_{i=1, i\neq k}^{M+N-1} \frac{\lambda_i}{(\lambda_i-\lambda_k)^2} V_i V_i^H \\ &\quad + \frac{\lambda_k}{T}\sum_{i=M+N}^{mM} \frac{\lambda_i}{(\lambda_i-\lambda_k)^2} V_i V_i^H + o(T^{-1}) \\ &= \epsilon_{\tilde{V}_k}^S + \epsilon_{\tilde{V}_k}^N + o(T^{-1}). \end{aligned} \tag{9}$$

In (9), the contribution of the noise subspace component $\epsilon_{\tilde{V}_k}^N$ goes to zero as the SNR$\rightarrow \infty$, whereas the contribution of the signal subspace component $\epsilon_{\tilde{V}_k}^S$ does not influence the estimated signal subspace since the component $\tilde{V}_k^S = (\widehat{V}_k - V_k)^S$ remains in the signal subspace. This shows that the signal subspace of the sample covariance matrix (which is computed with a finite amount of data) is consistent in SNR. This conclusion is illustrated by simulation results in section §V. In the sequel, we focus on the noise subspace and different parameterizations for it.

III. Different Noise Subspace Parameterizations

In [4], we introduced different choices for the Noise Subspace Parameterization (NSP) based on blocking equalizers. To begin with, consider the case of two channels: $m = 2$. One can observe that for noise-free signals, we have $\mathrm{H}_2(z)y_1(k) - \mathrm{H}_1(z)y_2(k) = 0$, which can be written in a matrix form as $[\mathrm{H}_2(z) \;\; -\mathrm{H}_1(z)]\,\boldsymbol{y}(k) = \mathbf{H}^{\perp\dagger}(z)\boldsymbol{y}(k) = 0$. The matrix $\mathbf{H}^{\perp\dagger}(z)$ is parameterized by the channel impulse response and satisfies $\mathbf{H}^{\perp\dagger}(z)\mathbf{H}(z) = 0$. For $m > 2$, blocking equalizers $\mathbf{H}^{\perp\dagger}(z)$ can be constructed by considering the channels in pairs. The choice of $\mathbf{H}^{\perp\dagger}(z)$ is far from unique. To begin with, the number of pairs to be considered, which is the number of rows in $\mathbf{H}^{\perp\dagger}(z)$, is not unique. The minimum number is $m-1$ whereas the maximum number is $\frac{m(m-1)}{2}$. We shall call $\mathbf{H}^{\perp\dagger}(z)$ balanced if $\mathrm{tr}\,\{\mathbf{H}^{\perp\dagger}(z)\mathbf{H}^{\perp}(z)\} = \alpha \mathbf{H}^{\dagger}(z)\mathbf{H}(z)$ for some real scalar α and $\mathbf{H}^{\dagger}(z) = \mathbf{H}^H(1/z^*)$. People usually take the maximum number of rows, which corresponds to a balanced $\mathbf{H}^{\perp\dagger}(z)$: $\mathbf{H}^{\perp\dagger}_{bal,max}(z)$. The minimum number of rows in $\mathbf{H}^{\perp\dagger}(z)$ to be balanced is m. We get for instance

$$\mathbf{H}^{\perp\dagger}_{min}(z) = \begin{bmatrix} -\mathrm{H}_2(z) & \mathrm{H}_1(z) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\mathrm{H}_m(z) & 0 & \cdots & \mathrm{H}_1(z) \end{bmatrix} \tag{10}$$

$$\mathbf{H}^{\perp\dagger}_{bal,min}(z) = \begin{bmatrix} -\mathrm{H}_2(z) & \mathrm{H}_1(z) & 0 & \cdots & 0 \\ 0 & -\mathrm{H}_3(z) & \mathrm{H}_2(z) & \cdots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ \mathrm{H}_m(z) & 0 & \cdots & 0 & -\mathrm{H}_1(z) \end{bmatrix}. \tag{11}$$

By construction, the NSP introduced in [2] produces the exact set of independent required vectors to span the noise subspace. The number of these vectors is equal to $mM - (M + N - 1)$ which is the dimension of the noise subspace. The minimum NSP, for example the one given by (10), produces $(m-1)(M-N+1)$ independent vectors of the noise subspace. This means that by using this kind of parameterization, we need $(m-2)(N-1)$ extra independent vectors to span the totality of the noise subspace. A linear NSP, $\mathbf{H}^{\perp\dagger}(z)$, can be written as the z transform of the $p \times m$ sequence: $\left\{\boldsymbol{h}^{\perp}(0), \cdots, \boldsymbol{h}^{\perp}(N-1)\right\}$ $(m-1 \leq p \leq m(m-1)/2)$, as

$$\mathbf{H}^{\perp\dagger}(z) = \sum_{i=0}^{N-1} \boldsymbol{h}^{\perp}(i) z^{-i}. \tag{12}$$

In (12), the $p \times m$ elements $\boldsymbol{h}^{\perp}(i)$, $i = 0, \cdots, N-1$ are written as a function of the elements of the vector $\boldsymbol{h}(i)$. For example, for the NSP $\mathbf{H}^{\perp\dagger}_{bal,min}(z)$ given by (11), where $p = m$, we have

$$\boldsymbol{h}^{\perp}(i) = \begin{bmatrix} -h_2(i) & h_1(i) & 0 & \cdots & 0 \\ 0 & -h_3(i) & h_2(i) & \cdots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ h_m(i) & 0 & \cdots & 0 & -h_1(i) \end{bmatrix}, i = 0, \cdots, N-1. \tag{13}$$

Let $\boldsymbol{h}^{\perp}$ be the $p \times mN$ matrix defined as $\boldsymbol{h}^{\perp} = \left[\begin{array}{ccc} \boldsymbol{h}^{\perp}(N-1) & \cdots & \boldsymbol{h}^{\perp}(0) \end{array}\right]$. We have $\boldsymbol{h}^{\perp}\boldsymbol{h} = 0$ and the relationship $\mathbf{H}^{\perp\dagger}(z)\mathbf{H}(z) = 0$ becomes in the temporal domain

$$\mathcal{T}_{M-N+1}(\boldsymbol{h}^{\perp})\mathcal{T}(\boldsymbol{h}) = 0. \tag{14}$$

In the sequel, we present the proof of the following result: *the use of a $\mathbf{H}^{\perp\dagger}_{bal,min}(z)$ as a NSP leads to span the overall noise subspace.* The reasoning is developed for $\mathbf{H}^{\perp\dagger}_{bal,min}(z)$ given by (11) but it holds also for any other choice of $\mathbf{H}^{\perp\dagger}_{bal,min}(z)$.
For $m = 2$, $\mathbf{H}^{\perp\dagger}_{bal,min}(z)$ and $\mathbf{H}^{\perp\dagger}_{min}(z)$ are identical, the corresponding matrix $\mathcal{T}_{M-N+1}(\boldsymbol{h}^{\perp})$ has $M-N+1$ independent rows that span the noise subspace (of dimension $2M-(M+N-1) = M-N+1$). For $m \geq 3$, the basic idea is to count the number of dependencies between the rows of the matrix $\mathcal{T}_{M-N+1}(\boldsymbol{h}^{\perp})$. Consider the case of $m = 3$ channels, we have

$$\begin{aligned} &\left[\begin{array}{ccc} \mathrm{H}_3(z) & \mathrm{H}_1(z) & \mathrm{H}_2(z) \end{array}\right] \mathbf{H}^{\perp\dagger}_{bal,min}(z) \\ &= \left[\begin{array}{ccc} \mathrm{H}_3(z) & \mathrm{H}_1(z) & \mathrm{H}_2(z) \end{array}\right] \left[\begin{array}{ccc} -\mathrm{H}_2(z) & \mathrm{H}_1(z) & 0 \\ 0 & -\mathrm{H}_3(z) & \mathrm{H}_2(z) \\ \mathrm{H}_3(z) & 0 & -\mathrm{H}_1(z) \end{array}\right] \\ &= \left[\begin{array}{ccc} 0 & 0 & 0 \end{array}\right]. \end{aligned} \tag{15}$$

In the case of $m = 4$ channels, we have

$$\mathbf{H}^{\perp\dagger}_{bal,min}(z) = \left[\begin{array}{cccc} -\mathrm{H}_2(z) & \mathrm{H}_1(z) & 0 & 0 \\ 0 & -\mathrm{H}_3(z) & \mathrm{H}_2(z) & 0 \\ 0 & 0 & -\mathrm{H}_4(z) & \mathrm{H}_3(z) \\ \mathrm{H}_4(z) & 0 & 0 & -\mathrm{H}_1(z) \end{array}\right]. \tag{16}$$

When we multiply the following row vector

$$\left[\begin{array}{cccc} \mathrm{H}_3(z)\mathrm{H}_4(z) & \mathrm{H}_4(z)\mathrm{H}_1(z) & \mathrm{H}_1(z)\mathrm{H}_2(z) & \mathrm{H}_2(z)\mathrm{H}_3(z) \end{array}\right] \tag{17}$$

by (16), we obtain $0_{1\times 4}$. For $m \geq 3$, let's call the row vector multiplying $\mathbf{H}^{\perp\dagger}_{bal,min}(z)$ the vector $\mathrm{g}^{\perp}(z)$. This vector satisfies

$$\mathrm{g}^{\perp}(z)\mathbf{H}^{\perp\dagger}_{bal,min}(z) = 0_{1\times m}. \tag{18}$$

In the temporal domain, the expression (18) gives the exact number of dependencies between the rows of the matrix $\mathcal{T}(\boldsymbol{h}^{\perp})$. Each element of the vector $\mathrm{g}^{\perp}(z)$ is a product of $(m-2)$ subchannel impulse responses $\mathrm{H}_i(z)$, with order equal to $(m-2)N-(m-3) = (m-2)(N-1)+1 = K$. Hence, the expression given by (18) can be written in the temporal domain as

$$\mathcal{T}_{M-N+1-K+1}(\boldsymbol{g}^{\perp})\mathcal{T}_{M-N+1}(\boldsymbol{h}^{\perp}) = 0. \tag{19}$$

Equation (19) gives the number of dependencies between the rows of the matrix $\mathcal{T}_{M-N+1}(\boldsymbol{h}^{\perp})$. This number is equal to $M-N+1-K+1 = M-(m-1)(N-1)$. Hence the number of linearly independent rows in the matrix $\mathcal{T}_{M-N+1}(\boldsymbol{h}^{\perp})$ is

$$m(M-N+1)-M+(m-1)(N-1) = mM-(M+N-1) \tag{20}$$

which is the dimension of the noise subspace, and hence the columns of $\mathcal{T}^{H}_{M-N+1}(\boldsymbol{h}^{\perp})$ span the noise subspace. This proves that using the NSP: $\mathbf{H}^{\perp\dagger}_{bal,min}(z)$ leads to span the overall noise subspace.

IV. Discussion

Since the rows of $\mathcal{T}(\boldsymbol{h}^{\perp})$ corresponding to $\mathbf{H}^{\perp\dagger}_{bal,min}(z)$ are a subset of the rows $\mathcal{T}(\boldsymbol{h}^{\perp})$ corresponding to $\mathbf{H}^{\perp\dagger}_{bal,max}(z)$, and since $\mathbf{H}^{\perp\dagger}_{bal,min}(z)$ spans the noise subspace then $\mathbf{H}^{\perp\dagger}_{bal,max}(z)$ also spans the noise subspace. This constitutes a proof simpler than the one given in [7] to show that taking the maximum number of rows in $\mathbf{H}^{\perp\dagger}(z)$ leads to span the overall noise subspace.

Indeed, the mecanism according to which the noise subspace is spanned can be described as follows: a NSP $\mathbf{H}^{\perp\dagger}_{bal,min}(z)$ includes a parameterization $\mathbf{H}^{\perp\dagger}_{min}(z)$. It is this last parameterization that produces the $(m-1)(M-N+1)$ first independent vectors of the noise subspace. The extra line in $\mathbf{H}^{\perp\dagger}_{bal,min}(z)$ compared to $\mathbf{H}^{\perp\dagger}_{min}(z)$ will produce a set of $M-N+1$ vectors. Any subset of $(m-2)(N-1)$ vectors selected from these last vectors will complete the previous $(m-1)(M-N+1)$ vectors in order to span the noise subspace, and the remaining $(M-N+1-(m-2)(N-1))$ vectors become immediately a linear combination of the vectors that have spanned the noise subspace.

In [8] and [9], the authors propose a new parameterization $\mathbf{H}^{\perp\dagger}(z)$ for the noise subspace, having $(2m-3)$ rows (and then a number of rows greater than the one of $\mathbf{H}^{\perp\dagger}_{bal,min}(z)$), that spans the noise subspace under the constarint that $\mathrm{H}_1(z)$ and $\mathrm{H}_2(z)$ dont share common zeros. From a frequency-domain point of view, this parameterization can be written, when $m > 2$, as

$$\begin{aligned} \mathbf{H}^{\perp\dagger}_{AH}(z) &= \left[\begin{array}{ccccc} -\mathrm{H}_2(z) & \mathrm{H}_1(z) & 0 & \cdots & 0 \\ -\mathrm{H}_3(z) & 0 & \mathrm{H}_1(z) & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -\mathrm{H}_m(z) & 0 & \cdots & 0 & \mathrm{H}_1(z) \\ \hline 0 & -\mathrm{H}_3(z) & \mathrm{H}_2(z) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -\mathrm{H}_m(z) & 0 & \cdots & \mathrm{H}_2(z) \end{array}\right] \\ &= \left[\begin{array}{c} \mathbf{H}^{\perp\dagger}_{min}(z) \\ \hline \mathbf{H}^{\perp\dagger}_{second}(z) \end{array}\right]. \end{aligned} \tag{21}$$

Note that for $m = 3$, the parameterization $\mathbf{H}^{\perp\dagger}_{AH}(z)$ is balanced. The convolution matrix corresponding to $\mathbf{H}^{\perp\dagger}_{AH}(z)$ is

$$\mathcal{T}(\boldsymbol{h}^{\perp}) = \left[\begin{array}{c} \mathcal{T}(\boldsymbol{h}^{\perp}_{min}) \\ \hline \mathcal{T}(\boldsymbol{h}^{\perp}_{second}) \end{array}\right], \tag{22}$$

where the matrices $\mathcal{T}(\boldsymbol{h}^{\perp}_{min})$ and $\mathcal{T}(\boldsymbol{h}^{\perp}_{second})$ correspond to $\mathbf{H}^{\perp\dagger}_{min}(z)$ and $\mathbf{H}^{\perp\dagger}_{second}(z)$ respectively. The authors propose to make a judicious choice of the rows of the matrix$\mathcal{T}(\boldsymbol{h}^{\perp})$ in order to span the noise subspace: they propose to consider all the rows of $\mathcal{T}(\boldsymbol{h}^{\perp}_{min})$ (their number is equal to $(m-1)(M-N+1)$), and then for each row of the $(m-2)$ rows of $\mathbf{H}^{\perp\dagger}_{second}(z)$, consider the $(N-1)$ corresponding first rows in $\mathcal{T}(\boldsymbol{h}^{\perp}_{second})$. Hence, the obtained total number is

$$(m-1)(M-N+1)+(m-2)(N-1) = mM-(M+N-1) \tag{23}$$

which is the dimension of the noise subspace. The authors show that this procedure of rows selection gives the exact set of independent vectors that span the noise subspace, provided that the two first subchannels $\mathrm{H}_1(z)$ and $\mathrm{H}_2(z)$ dont share common zeros. This seems to be of lesser interest to be adopted in a blind identification method using a NSP (such methods can be found in [5] and [6]), since our parameterization $\mathbf{H}^{\perp\dagger}_{bal,min}(z)$ is minimal and leads to span the overall noise subspace without any constraint required on the subchannel impulse responses.

V. Simulation Results

In the simulation presented here, our idea is to study the behaviour of the estimated signal subspace and the second-order moments (both estimated from a finite amount of data) as a function of SNR. The received signal covariance matrix is estimated from a burst of 200 symbols. The oversampling factor is $m = 3$ and the symbols are i.i.d. BPSK. The channel is complex, i.i.d. randomly generated with length $N = 3$:

$$\boldsymbol{H} = \begin{bmatrix} -0.9712 + 0.9603i & 0.6087 + 0.1392i & 0.0243 - 1.2639i \\ 1.0701 + 0.8570i & 0.0974 + 0.3818i & -0.9887 - 0.6548i \\ -0.1217 + 2.1088i & 1.0908 - 0.6647i & -0.6495 + 0.4929i \end{bmatrix}. \quad (24)$$

The channel covariance matrix is considered with $M = 10$. The Normalized Mean Square Error (NMSE) on the signal subspace is computed over 300 Monte-Carlo runs as follows:

$$\text{NMSE1} = \frac{1}{300}\sum_{i=1}^{300}\left(1 - \frac{\|\widehat{\mathcal{V}}_S^{(i)\,H}\mathcal{V}_S\|_F^2}{M+N-1}\right) \quad (25)$$

where $\widehat{\mathcal{V}}_S^{(i)}$ is the estimated signal subspace given by the $M + N - 1$ largest eigenvectors of the sample covariance matrix $\widehat{R}_{YY}$ in the i^{th} trial. In order to compare the accuracy of the estimated signal subspace to the one of the estimated second-order moments, we consider the first bloc element of the sample covariance matrix $\widehat{r}_{YY}^{(i)}(0)$ in the i^{th} trial and we define the NMSE as follows:

$$\text{NMSE2} = \frac{1}{300}\sum_{i=1}^{300}\|\widehat{r}_{YY}^{(i)}(0) - r_{YY}(0)\|_F^2 / \|r_{YY}(0)\|_F^2 \quad (26)$$

where $r_{YY}(0)$ is the first bloc element of the channel covariance matrix computed using the exact statistics $R_{YY} = \sigma_a^2\mathcal{T}(\boldsymbol{H})\mathcal{T}^H(\boldsymbol{H}) + \sigma_v^2 I_{mM}$. In Figure 1, we plot NMSE1 versus SNR: it is clear that with a finite amount of data we are capable of estimating accurately the signal subspace, wheras in Figure 2 the error remains constant from an SNR of about 20dB. This implies that with a finite amount of data we can not estimate perfectly the second-order moments at any value of the SNR. So the estimation of second-order statistics is not consistent in SNR, whereas the subspace estimation is.

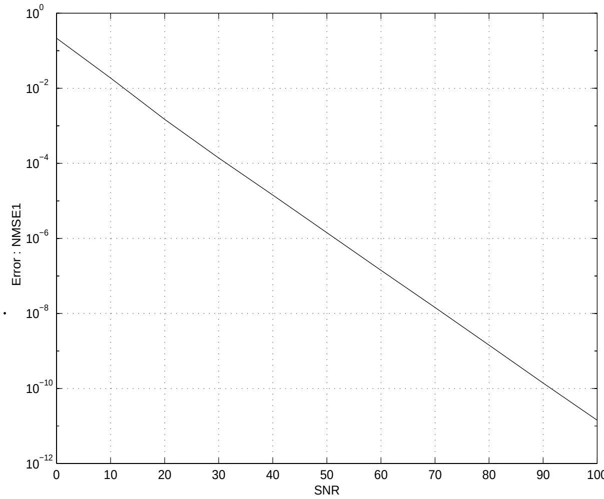

Figure 1: Accurate signal subspace estimation with a finite amount of data

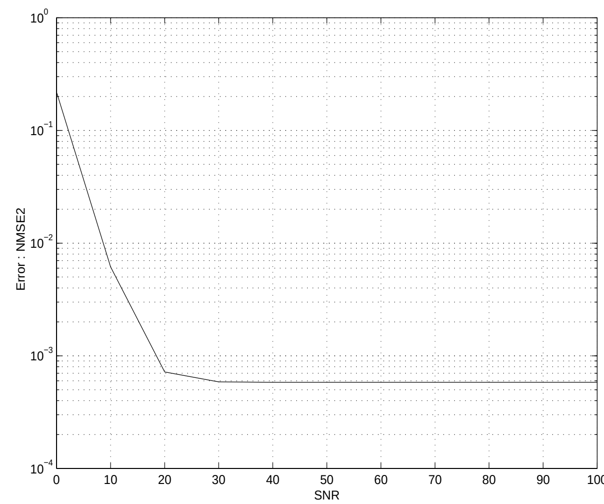

Figure 2: Estimation of the first second-order moment with a finite amount of data

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