MULTICHANNEL ESTIMATION BY BLIND MMSE ZF EQUALIZATION

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ABSTRACT

We investigate a new multichannel estimation method based on blind MMSE ZF Equalization. The recently proposed method by Tsatsanis *et al.* [4] corresponds to unbiased MMSE Equalization. We interpret this approach in terms of Two-Sided Linear Prediction (TSLP), also called smoothing by Tong [7]. We establish the links between MMSE, Minimum Output Energy (MOE) and MMSE ZF and we prove equivalence under the unbiasedness constraint in the noiseless case. Our analysis shows how to properly apply Capon's principle [3] for Linearly Constrained Minimum Variance (LCMV) beamforming to multichannel equalization. Furthermore, we show that Tsatsanis's application of Capon's principle becomes only correct, and Tong's channel estimate becomes only unbiased, at high SNR. Whereas the goal is to do MMSE ZF, it is easier to approach the problem via Unbiased MMSE (UMMSE) on noiseless data. Hence, the covariance matrix of the received signal has to be "denoised" before using it in the blind estimation method. We provide an approach without eigen decomposition that shows excellent performance. Simulation results are presented to support our claims.

1. INTRODUCTION

Blind single-user multichannel identification techniques exploit a multichannel formulation corresponding to a Single Input Multiple Output (SIMO) vector channel. The channel is assumed to have a finite delay spread NT. The multiple FIR channels can be obtained by oversampling a single received signal, but can also be obtained as multiple received signals from an array of antennas (in the context of mobile digital communications [1],[2]) or from a combination of both. For m channels the discrete-time input-output relationship can be written as:

$$y(k)=\sum_{i=0}^{N-1} h(i)a(k-i)+v(k)=\boldsymbol{H}A_N(k)+v(k) \qquad (1)$$

where $y(k)=[y_1^H(k)\cdots y_m^H(k)]^H$, $h(i)=\left[h_1^H(i)\cdots h_m^H(i)\right]^H$, $v(k)=[v_1^H(k)\cdots v_m^H(k)]^H$, $\boldsymbol{H}=[h(N-1)\cdots h(0)]$, $A_N(k)=\left[a(k-N+1)^H\cdots a(k)^H\right]^H$ and superscript ^{H} denotes Hermitian transpose. Let $\mathbf{H}(z)=\sum_{i=0}^{N-1} h(i)z^{-i}=[\mathrm{H}_1^H(z)\cdots \mathrm{H}_m^H(z)]^H$ be the SIMO channel transfer function. The channel coefficients vector is $\boldsymbol{h}=\left[\; \boldsymbol{h}^H(N-1)\cdots \boldsymbol{h}^H(0)\;\right]^H$. Consider the symbols i.i.d. if required and additive independent white Gaussian circular noise $\boldsymbol{v}(k)$ with $r_{\boldsymbol{vv}}(k-i)=\mathrm{E}\,\boldsymbol{v}(k)\boldsymbol{v}(i)^H=\sigma_v^2 I_m\,\delta_{ki}$. Assume we receive M samples:

$$\boldsymbol{Y}_M(k)=\mathcal{T}_M(\boldsymbol{h})\,A_{M+N-1}(k)+\boldsymbol{V}_M(k) \qquad (2)$$

where $\boldsymbol{Y}_M(k)=[\boldsymbol{y}^H(k-M+1)\cdots \boldsymbol{y}^H(k)]^H$ and similarly for $\boldsymbol{V}_M(k)$. $\mathcal{T}_M(\boldsymbol{h})$ is a block Toeplitz matrix filled with the channel coefficients. We shall simplify the notation in (2) with $k=M-1$ to:

$$\boldsymbol{Y}=\mathcal{T}A+\boldsymbol{V}\,. \qquad (3)$$

We assume that $mM>M+N-1$ in which case the channel convolution matrix $\mathcal{T}$ has more rows than columns. A channel will be said irreducible if the $\mathrm{H}_i(z)$, $i=1,\ldots,m$ have no zeros in common, and reducible otherwise. For obvious reasons, the column space of $\mathcal{T}$ is called the signal subspace and its orthogonal complement the noise subspace.

2. CAPON'S METHOD

A well known principle in array processing applications, when the direction of arrival and the signature of the user of interest is known (or estimated), is the Minimum Variance Distortionless Response (MVDR) beamformer. This beamformer suppresses the interfering users signals without distorting the signal of the user of interest. The Capon's method [3] starts from the MVDR principle to derive blind solutions (without having to know the signatures). Assume that we are interested in the user 1 and we have to determin an FIR linear equalizer F that provides its corresponding transmitted signal with a possible delay of d samples. In this case a linear estimator of the transmitted signal is given by

$$\widehat{a}_{1,k-d}=F^H\boldsymbol{Y}(k). \qquad (4)$$

Assume that N_jT is the finite delay spread of the j^{th} user, the received signal can be written as

$$\boldsymbol{Y}_M(k)=\mathcal{T}_{M,1}A_{1,M+N_1-1}(k)+\sum_{j>1}\mathcal{T}_{M,j}A_{j,M+N_j-1}(k)+\boldsymbol{V}_M(k), \qquad (5)$$

hence, the distortioneless response for the user of interest satisfies

$$F^H\mathcal{T}_{M,1}=[\underbrace{0\cdots0}_{d}10\cdots0]=e_d^H\,, \qquad (6)$$

which implies that the problem is equivalent to find a linear FIR ZF equalizer F that acts as a ZF for the Inter-Symbol Interference (ISI) of the user of interest.

3. BLIND UNBIASED MMSE EQUALIZATION

The method proposed by Tsatsanis *et al.* [4] is an unbiased MMSE Equalization point of view that consists in

$$F^H\mathcal{T}_{M,1}e_d=F^H\begin{bmatrix}0\\ \boldsymbol{h}\\ 0\end{bmatrix}=F^H\tilde{\boldsymbol{h}}=1. \qquad (7)$$

This implies that under the unbiased constraint, the MMSE criterion and the MVDR one (also known as Minimum Output Energy (MOE)) are equivalent. In the Tsatsanis's approach, the MOE criterion $\min_{F} F^H R_{YY} F$ is sloved according to a two steps algorithm.

- step1: the optimization criterion $\min_{F: F^H \tilde{h}=1} F^H R_{YY} F$ is solved via the following Lagrange multiplier equation:

$$\min_{F,\lambda} F^H R_{YY} F + \lambda(F^H \tilde{h} - 1). \tag{8}$$

The solution is given by $F = -\lambda R_{YY}^{-1} \tilde{h}$ and $\lambda = -\frac{1}{\tilde{h}^H R_{YY}^{-1} \tilde{h}}$.

Hence $F = \frac{1}{\tilde{h}^H R_{YY}^{-1} \tilde{h}} R_{YY}^{-1} \tilde{h}$: unbiased MMSE equalizer.

- step2: this step coincides with the Capon's method, in which a max/min idea arises:

$$\begin{aligned} &\max_{h:\|h\|=1} \min_{F: F^H \tilde{h}=1} F^H R_{YY} F \\ &= \max_{h:\|h\|=1} \left(\tilde{h}^H R_{YY}^{-1} \tilde{h}\right)^{-1} \text{ or } \min_{h:\|h\|=1} \tilde{h}^H R_{YY}^{-1} \tilde{h} \end{aligned} \tag{9}$$

Since $\tilde{h} = \begin{bmatrix} 0 \\ h \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix} h = T_1 h$, the optimization problem given by (9) becomes

$$\min_{h:\|h\|=1} h^H \left(T_1^H R_{YY}^{-1} T_1\right) h \Rightarrow h = V_{min}\left(T_1^H R_{YY}^{-1} T_1\right), \tag{10}$$

where $V_{min}(A)$ denotes the eigenvector corresponding to the minimum eigenvalue of A.

4. INTERPRETATION IN TERMS OF TWO-SIDED LINEAR PREDICTION (TSLP)

Whereas in the forward linear prediction we predict linearly a given sample from his previous samples, and in the backward linear prediction the sample is linearly predicted from the samples that come immediatly afterward; the TSLP approach predicts linearly a sample vector simultaniously from his past and future samples.

4.1. Interpretation for step2

Consider the matrix T_2 defined as $T_2 = \begin{bmatrix} I & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix}$. T_2 can be interpreted as $T_2 = T_1^{\perp}$: the orthogonal complement of T_1 since $T_2^H T_1 = 0$ and $P_{T_2} + P_{T_1} = I$ ($P_{T_2} = P_{T_1}^{\perp}$), where $P_X^{\perp} = I - P_X = I - X(X^H X)^{-1} X^H$. From the partitioned Matrix Inversion Lemma (MIL), the matrix $T_1^H R_{YY}^{-1} T_1$ appearing in (10) can be written as

$$\left(T_1^H R_{YY} T_1 - T_1^H R_{YY} T_2 \left(T_2^H R_{YY} T_2\right)^{-1} T_2^H R_{YY} T_1\right)^{-1} \tag{11}$$

Consider now $\tilde{Y} = T_1^H Y - Q T_2^H Y$, where the matrix $Q = \left(T_1^H R_{YY} T_2\right)\left(T_2^H R_{YY} T_2\right)^{-1}$ is optimized to minimize $R_{\tilde{Y}\tilde{Y}}$. The expression given in (11) becomes

$$T_1^H R_{YY}^{-1} T_1 = R_{\tilde{Y}\tilde{Y}}^{-1}, \text{ and hence (10)} \Rightarrow h = V_{max}\left(R_{\tilde{Y}\tilde{Y}}\right). \tag{12}$$

where $V_{max}(A)$ denotes the eigenvector corresponding to the maximum eigenvalue of A.

Analysis in the Noiseless Case

In the noiseless case $Y = \mathcal{T} A$, the vetors $T_1^H Y$ and $T_2^H Y$ are illustrated in Fig 1. We analyse the TSLP approach in two symbols cases: the i.i.d. symbols case and the correlated symbols case.

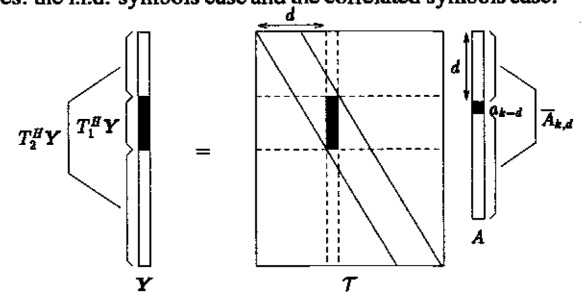

Figure 1: TSLP

- i.i.d. symbols case: in this case the estimation of $T_1^H Y$ from $T_2^H Y$ is the same as the estimation of $T_1^H \mathcal{T}$ from $T_2^H \mathcal{T}$. If each of the matrices $\begin{bmatrix} I & 0 & 0 \end{bmatrix} \mathcal{T}$ and $\begin{bmatrix} 0 & 0 & I \end{bmatrix} \mathcal{T}$ is full column rank then the estimation error in estimating $T_1^H \mathcal{T}$ from $T_2^H \mathcal{T}$ is $h e_d^H$ (note that $\begin{bmatrix} I & 0 & 0 \end{bmatrix} \mathcal{T}$ and $\begin{bmatrix} 0 & 0 & I \end{bmatrix} \mathcal{T}$ are also talls but only of smaller dimension). Since $T_2 = T_1^{\perp}$, we have

$$\begin{aligned} &T_1^H \mathcal{T} - Q T_1^{\perp H} \mathcal{T} \\ &= T_1^H - \left(T_1^H \mathcal{T} \mathcal{T}^H T_1^{\perp}\right)\left(T_1^{\perp H} \mathcal{T} \mathcal{T}^H T_1^{\perp}\right)^{-1} T_1^{\perp H} \mathcal{T} \\ &= T_1^H \mathcal{T} P_{\mathcal{T}^H T_1^{\perp}}^{\perp} = \left(P_{\mathcal{T}^H T_1^{\perp}}^{\perp} \mathcal{T}^H T_1\right)^H = h e_d^H. \end{aligned} \tag{13}$$

This leads to the following equivalences

$$\begin{aligned} &span\left\{\mathcal{T}^H\right\} \cap span\left\{\mathcal{T}^H T_1\right\} = span\left\{e_d\right\} \\ &\Rightarrow span\left\{\mathcal{T}^H\right\} = span\left\{\mathcal{T}^H T_1^{\perp}\right\} \oplus span\left\{e_d\right\}. \end{aligned} \tag{14}$$

- Correlated symbols case: if each of the matrices $\begin{bmatrix} I & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 & I \end{bmatrix} \mathcal{T}$ is full column rank, the estimation of $T_1^H Y$ from $T_2^H Y = T_2^H \mathcal{T} A$ is equivalent to the estimation from $\overline{A}_{k,d}$ (illustrated in Fig. 1). Hence,

$$\tilde{Y} = h \tilde{a}_{k-d}|_{\overline{A}_{k,d}} \Rightarrow R_{\tilde{Y}\tilde{Y}} = \sigma_{\tilde{a}}^2 h h^H. \tag{15}$$

$\sigma_{\tilde{a}}^2$ is replaced by σ_a^2 in the uncorrelated symbols case.

As a conclusion, in the noiseless case, in any symbols context case the channel is given by $h = V_{max}(R_{\tilde{Y}\tilde{Y}})$.

4.2. Alternative Interpretation for step1

- A first interpretation of step1 is deduced from the following equalitites:

$$\text{Unbiased MMSE} = \text{Unbiased MOE} = \max \text{SINR}. \tag{16}$$

The signal part of Y is given by $Y_s = \tilde{h} a_{k-d}$ ($\tilde{h} = \mathcal{T} e_d$) and the interference plus noise part $Y_{IN} = \overline{\mathcal{T}}\,\overline{A} + V$ (where the matrix $\overline{\mathcal{T}}$ is the matrix $\mathcal{T}$ without $\tilde{h}$). Hence, the output Signal to Interference plus Noise Ratio (SINR) is given by

$$\text{SINR} = \frac{F^H R_{Y_s Y_s} F}{F^H R_{Y_{IN} Y_{IN}} F} \tag{17}$$

Assuming now that the symbols are uncorrelated, equation (17) becomes

$$\begin{aligned} \text{SINR} &= \frac{\sigma_a^2 F^H \tilde{h}\tilde{h}^H F}{F^H\left(R_{YY} - \sigma_a^2 \tilde{h}\tilde{h}^H\right)F} \\ \Rightarrow \text{SINR}^{-1} &= \frac{\sigma_a^2 F^H \tilde{h}\tilde{h}^H F}{\sigma_a^2 |F^H\tilde{h}|^2} - 1. \end{aligned} \quad (18)$$

This leads to the following equivalences

$$\begin{aligned} &\max_F \text{SINR} \Leftrightarrow \min_F \text{SINR}^{-1} \\ &\Leftrightarrow \min_F \frac{\sigma_a^2 F^H R_{YY} F}{\sigma_a^2 |F^H \tilde{h}|^2} \Leftrightarrow \min_{F:F^H\tilde{h}=1} F^H R_{YY} F. \end{aligned} \quad (19)$$

Note that the SINR is insensitive to the phase of F, hence one can choose the phase of F such that not only $|F^H\tilde{h}| = 1$ but $F^H\tilde{h} = 1$.

- A second interpretation of step1 comes from the fact that the unbiased MMSE is identical to an unbiased constrained MOE. This is proved as follows

$$\begin{aligned} \text{MSE} &= \sigma_{\tilde{a}}^2 = E|a_{k-d} - \hat{a}|^2 = E|a_{k-d} - F^H Y|^2 \\ &= \sigma_a^2 - F^H R_{Ya} - R_{aY}F + F^H R_{YY} F \\ &= \sigma_a^2 - \sigma_a^2 F^H \tilde{h} - \sigma_a^2 \tilde{h}^H F + \underbrace{F^H R_{YY} F}_{\text{OE}} \\ \Rightarrow &\min_{F^H\tilde{h}=1} \text{MSE} = \text{MOE s.t. } F^H\tilde{h} = 1, \end{aligned} \quad (20)$$

where s.t. refers to "such that".

5. BLIND MMSE ZF EQUALIZATION

In principle, The MMSE criterion (unbiased or not) gives a MMSE ZF equalizer in the noiseless case. When noise is present, one have to "denoise" the covariance matrix and hence derive MMSE ZF equalizers via MMSE with the denoised covariance matrix R_{YY}^d (superscript ^{d} denotes the obtained matrix after the denoising operation).

Direct Derivation

The ZF constraint is given by $F^H\mathcal{T} = e_d^H$. The MMSE ZF equalizer is derived from the following minmizing criterion, where $R_{YY} = \sigma_a^2\mathcal{T}\mathcal{T}^H + R_{VV} = \sigma_a^2\mathcal{T}\mathcal{T}^H + \sigma_v^2 I$:

$$\begin{aligned} \min_{F:F^H\mathcal{T}=e_d^H} F^H R_{YY} F &= \min_{F:F^H\mathcal{T}=e_d^H} \left\{\sigma_a^2 + F^H R_{VV} F\right\} \\ &\Leftrightarrow \min_{F:F^H\mathcal{T}=e_d^H} F^H F. \end{aligned} \quad (21)$$

- One approach consists in considering $\mathcal{T}^\perp$: the orthogonal complement of the matrix $\mathcal{T}$ which verifies $P_{\mathcal{T}^\perp} = P_{\mathcal{T}}^\perp$. The equallizer $F = \left[\begin{array}{cc} F_1^H & F_2^H \end{array}\right]$ can be written as $F = \mathcal{T}F_1 + \mathcal{T}^\perp F_2$. The ZF constraint is $F^H\mathcal{T} = e_d^H = F_1^H\mathcal{T}^H\mathcal{T}$ implies that $F_1 = \mathcal{T}\left(\mathcal{T}^H\mathcal{T}\right)^{-1}e_d$, and hence $F = \mathcal{T}\left(\mathcal{T}^H\mathcal{T}\right)^{-1}e_d + \mathcal{T}^\perp F_2$, where F_2 is unconstrained. Now applying the MMSE ZF criterion

$$\begin{aligned} &\min_{F:F^H\mathcal{T}=e_d^H} F^H F = \min_{F_2} e_d^H\left(\mathcal{T}^H\mathcal{T}\right)^{-1}e_d + F_2^H\mathcal{T}^{\perp H}\mathcal{T}^\perp F_2 \\ &\Rightarrow F_2 = 0 \Rightarrow F = \mathcal{T}\left(\mathcal{T}^H\mathcal{T}\right)^{-1}e_d \\ &\text{and } F^H F = e_d^H\left(\mathcal{T}^H\mathcal{T}\right)^{-1}e_d. \end{aligned} \quad (22)$$

- Other approach consists in writing the minimization criterion $\min_{F:F^H\mathcal{T}=e_d^H} F^H R_{YY} F$ as

$$\min_{F^H R_{YY}^{\frac{1}{2}} R_{YY}^{-\frac{1}{2}}\mathcal{T}=e_d^H} \left(F^H R_{YY}^{\frac{1}{2}}\right)\left(R_{YY}^{\frac{H}{2}}F\right) \quad (23)$$

Let's define $D \triangleq R_{YY}^{\frac{H}{2}}F$, the minimization criterion (23) becomes $\min_{D:D^H R_{YY}^{-\frac{1}{2}}\mathcal{T}=e_d^H} D^H D$, hence

$$\begin{aligned} &D = R_{YY}^{-\frac{1}{2}}\mathcal{T}\left(\mathcal{T}^H R_{YY}^{-1}\mathcal{T}\right)^{-1}e_d \\ &\Rightarrow F = R_{YY}^{-\frac{H}{2}}D = R_{YY}^{-1}\mathcal{T}\left(\mathcal{T}^H R_{YY}^{-1}\mathcal{T}\right)^{-1}e_d. \end{aligned} \quad (24)$$

Hence

$$\min_{F:F^H\mathcal{T}=e_d} F^H R_{YY} F = e_d^H\left(\mathcal{T}^H R_{YY}^{-1}\mathcal{T}\right)^{-1}e_d. \quad (25)$$

If $R_{YY} = \sigma_a^2\mathcal{T}\mathcal{T}^H + \sigma_v^2 I$, using the MIL leads to write $\left(\mathcal{T}^H R_{YY}^{-1}\mathcal{T}\right)^{-1} = \sigma_a^2 I + \sigma_v^2\left(\mathcal{T}^H\mathcal{T}\right)^{-1}$ and hence $F^H R_{YY} F = e_d^H\left(\mathcal{T}^H R_{YY}^{-1}\mathcal{T}\right)^{-1}e_d = \sigma_a^2 + \sigma_v^2 e_d^H\left(\mathcal{T}^H\mathcal{T}\right)^{-1}e_d$ which is the same expression as the first approach. Now apllying the Capon's method leads to

$$\max_{\hat{h}:\|\hat{h}\|=1} \min_{F:F^H\mathcal{T}=e_d^H} F^H R_{YY} F = \max_{\hat{h}:\|\hat{h}\|=1} e_d^H\left(\mathcal{T}^H R_{YY}^{-1}\mathcal{T}\right)^{-1} \quad (26)$$

In any case, the maximum is obtained when $\hat{h}$ is h. To conclude, we have

$$\begin{aligned} &F_{MMSEZF} = R_{YY}^{-1}\mathcal{T}\left(\mathcal{T}^H R_{YY}^{-1}\mathcal{T}\right)^{-1}e_d \\ &= R_{VV}^{-1}\mathcal{T}\left(\mathcal{T}^H R_{VV}^{-1}\mathcal{T}\right)^{-1}e_d = \mathcal{T}\left(\mathcal{T}^H\mathcal{T}\right)^{-1}e_d, \\ &F_{UMMSE,noiseless} = \frac{1}{\tilde{h}^H R_{YY}^{-d}\tilde{h}} R_{YY}^{-d}\tilde{h} \\ &= \frac{1}{e_d^H\mathcal{T}^H\left(\mathcal{T}\mathcal{T}^H\right)^+\mathcal{T}e_d}\left(\mathcal{T}\mathcal{T}^H\right)^+\mathcal{T}e_d = \mathcal{T}\left(\mathcal{T}^H\mathcal{T}\right)^{-1}e_d, \end{aligned} \quad (27)$$

where superscript $^+$ denotes the Moore-Penrose pseudo-inverse.

So the goal is to do MMSE ZF but it's easier to approach the problem via UMMSE on noiseless data. In fact, MOE and MMSE are equivalent in the unbiased case (ZF also implies unbiazed).

6. RELATION WITH OTHER APPROACHES

In this section, we focus on the relation of our blind MMSE ZF equalization approach with other recently proposed approaches. In the previous section we had shown that our formulation constitutes the proper application of the Capon's principle for LCMV beamforming to multichannel equalization whereas the Tsatsanis's formulation becomes only correct at high SNR. Our approach and specifically the TSLP method corresponds to a one-step solution for the multi-step approach of Gesbert and Duhamel [8]. The Least-Squares Smoothing (LSS) introduced by Tong *et al.* [7] gives biased channel estimates: the method keeps only the signal subspace part (the noise subspace part is removed via the SVD operation which remove the noise subspace vectors) but the contribution of the noise eigenvalues is not removed. Our TSLP approach corresponds to a bias-removed version of Tong's smoothing method, and furthermore by using this method we avoid computing a SVD of the covariance matrix of the received signal as Tong's method requires. In fact, our demonstration of the presence of bias in Tong's method shows that the deterministic point of view on which his derivation is based is inappropriate.

7. SIMULATION RESULTS

We consider a burst length of $M = 200$, a complex channel H randomly generated, of length $N = 3$ with $m = 2$ subchannels. The input symbols is drawn from an i.i.d. QPSK symbols sequence. The SNR is defined as $(\|h\|^2\sigma_a^2)/(m\sigma_v^2)$. Blind estimation gives a channel estimate $\hat{\hat{h}}$ with $\|\hat{\hat{h}}\| = 1$, we adjust the right scale factor α so that $h_o^H(\alpha\hat{\hat{h}}) = h_o^H h_o$ where h_o is the true channel vector (see [6]): the final estimate is $\hat{h} = \alpha\hat{\hat{h}}$. The performance measure is the Normalized MSE (NMSE): NMSE, averaged over 100 Monte-Carlo runs and defined as

$$\text{NMSE} = \|h - \hat{h}\|^2/\|h\|^2 \tag{28}$$

In Fig 2, we compare our TSLP approach to other estimation methods and to its blind Cramer-Rao Bound (CRB) computed with constraint $h_o^H\hat{h} = h_o^H h_o$ [6], corresponding to the way we have previously adjusted the scale factor. We use a sample covariance matrix $\widehat{R}_{YY}$ of length $L = 3N$. The Tsatsanis's method [4] gives the worst performance at low SNRs and improvement in its performance can be noticed as the SNR increases (this supports our theoritical claim that says the Tsatsanis's formulation becomes only correct at high SNR). The Tong's LSS method gives comparable performance as the Tsatsanis's method due to the bias in the channel estimate. We perform two kinds of denoising on the covariance matrix. In the first denoising operation, we remove the noise contribution estimated as the minimum eigenvalue of $\widehat{R}_{YY}$ and we keep its positive definite part $\lfloor\widehat{R}_{YY} - \lambda_{min}(\widehat{R}_{YY})I\rfloor_+$, significant improvement in performance compared to both Tsatsanis's and Tong's methods can be noticed. Whereas this improvement supports our theoritical claims concerning the TSLP operating on noiseless data, one can notice that we still did not reach the performance of the Pseudo-Quadratic Maximum Likelihood (PQML) [9] initialized by the Subchannel Response Matching (SRM)[5], which was shown to be asymptotically optimal (asymptotically the PQML estimate reachs its CRB). This is justified by the fact that the minimum eigenvalue of $\widehat{R}_{YY}$ underestimates the noise power and hence the noise contribution that we remove from $\widehat{R}_{YY}$ is not sufficient enough to completly denoise the covariance matrix. Hence we propose, in the second denoising operation, to estimate the noise power by the SRM method [5]. With a finite amount of data this method will underestimate the noise power but the estimte is close to the true value of the noise power (the underestimation error dissapears asymptoticolly). Hence, in order to completly denoise $\widehat{R}_{YY}$, we propose to replace $\widehat{R}_{YY}$ by $\lfloor\widehat{R}_{YY} - \alpha\widehat{\sigma_v^2}I\rfloor_+$ where $\widehat{\sigma_v^2}$ is the SRM noise power estimate and α is a choosen scalar s.t. $\alpha\widehat{\sigma_v^2} \gtrapprox \sigma_v^2$. We plot the curves corresponding to $\alpha = 1.5$ and $\alpha = 2$. It is clear that with $\alpha = 2$ we reach the same performance as the PQML method which means that $\widehat{R}_{YY}$ is perfectly denoised and hence the TSLP performance is optimal. Note that eliminating the proper noise contribution from the eigenvalues in the LSS method leads also to the same optimal performance.

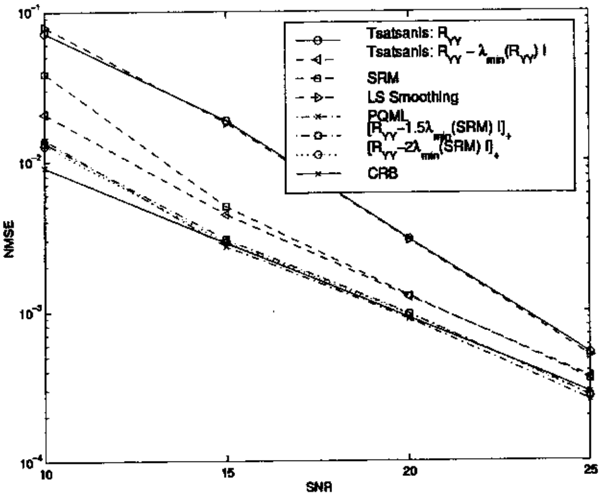

Figure 2: Performance of the different blind estimation algorithms.

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