

SUBSPACE FITTING WITHOUT EIGENDECOMPOSITION

Luc Deneire¹, Jaouhar Ayadi and Dirk T.M. Slock

Institut EURECOM, 2229 route des Crêtes, B.P. 193, F-06904 Sophia Antipolis Cedex, FRANCE
 {deneire, ayadi, slock}@eurecom.fr

Abstract : Subspace fitting has become a well known method to identify FIR Single Input Multiple Output (SIMO) systems, only resorting to second-order statistics. The main drawback of this method is its computational cost, due to the eigendecomposition of the sample covariance matrix. We propose a scheme that solves the subspace fitting problem without using the eigendecomposition of the cited matrix. The approach is based on the observation that the signal subspace is also the column space of the noise-free covariance matrix. We suggest a two-step procedure. In the first step, the column space is generated by arbitrary combinations of the columns. In the second step, this column space estimate is refined by optimally combining the columns using the channel estimate resulting from the first step. Our method only requires computation of two eigenvectors of a small matrix and of two projection matrices, although yielding the same performance as the usual subspace fitting.

1. INTRODUCTION

Subspace fitting algorithms have been applied to the multi-channel identification problem. In [8], it was shown that oversampled and/or multiple antenna received signals may be modeled as low rank processes and thus lend themselves to subspace methods, exploiting the orthogonality property between the noise subspace of the covariance matrix and the convolution matrix of the channel. Recent papers [1] [10] provide performance analysis of these methods. The huge majority of algorithms recently proposed to perform subspace fitting resort to SVD (singular value decomposition), which make them of little use for real-time implementations. On the other hand, literature on other, computationally less demanding rank revealing decompositions [3] has lead to some fast subspace estimation methods (see a.o. [5]). The usual scheme is to use a rank revealing decomposition (the SVD being the best, but the less cost-effective one) to determine the so called signal subspace. Possible candidates for this decomposition are the URV decomposition, the rank revealing QR decomposition and the HQR factorization in a Schur-type method (see [5] and references therein).

Recently, using the circularity property of the noise in a real symbol constellation based communication system, Kristensson, Ottersten and Slock proposed in [6] an alternative subspace fitting algorithm. In this paper, we show that this method can be used in the general case, leading to a consistent estimate, and that the performance is similar to that of the usual subspace fitting algorithms. Our method does not require the eigendecomposition of the covariance matrix. Nevertheless, as in the usual subspace fitting method, the computation of a projection matrix is required which may remain computationally demanding. Although we do not discuss this topic in detail here, fast algorithms can be used to alleviate this problem. In this paper, we consider the channel identification problem, but the ideas presented here apply to any subspace fitting problem.

2. DATA MODEL

We consider a communication system with one emitter and a receiver consisting of an array of M antennas. The received signals are oversampled by a factor m w.r.t. the symbol rate. We furthermore consider linear digital modulation over a linear channel with additive noise, so that the received signal $\mathbf{y}(t) = [y_1(t) \dots y_M(t)]^T$ has the following form

$$\mathbf{y}(t) = \sum_k \mathbf{h}(t - kT) a(k) + \mathbf{v}(t)$$

where $a(k)$ are the transmitted symbols, T is the symbol period and $\mathbf{h}(t) = [h_1(t) \dots h_M(t)]^T$ is the channel impulse response. The channel is assumed to be FIR with duration NT . If the received signals are oversampled at the rate $\frac{m}{T}$, the discrete input-output relationship can be written as :

$$\mathbf{y}(k) = \sum_{i=0}^{N-1} \mathbf{h}(i) a(k - i) + \mathbf{v}(k) = \mathbf{H}\mathbf{A}_N(k) + \mathbf{v}(k)$$

where

$$\begin{aligned} \mathbf{y}(k) &= [\mathbf{y}(kT)^H \mathbf{y}(kT + \frac{1}{mT})^H \dots \mathbf{y}(kT + \frac{m-1}{mT})^H]^H, \\ \mathbf{h}(k) &= [\mathbf{h}(kT)^H \mathbf{h}(kT + \frac{1}{mT})^H \dots \mathbf{h}(kT + \frac{m-1}{mT})^H]^H, \\ \mathbf{v}(k) &= [\mathbf{v}(kT)^H \mathbf{v}(kT + \frac{1}{mT})^H \dots \mathbf{v}(kT + \frac{m-1}{mT})^H]^H, \\ \mathbf{H} &= [\mathbf{h}(0), \dots, \mathbf{h}(N-1)] \text{ and } \mathbf{A}_N(k) = \\ &= [a(kT) \dots a((k-N+1)T)], \text{ superscript } ^H \text{ denoting conjugate transpose.} \end{aligned}$$

So we get a SIMO system with Mm channels. We consider additive temporally and spatially white Gaussian circular noise $\mathbf{v}(k)$ with $\mathbf{r}_{v_v}(k - i) = \mathbf{E} \{ \mathbf{v}(k) \mathbf{v}(i)^H \} = \sigma_v^2 \mathbf{I}_{Mm} \delta_{ki}$. Assume we receive L samples :

$$\mathbf{Y}_L(k) = \mathcal{T}_L(\mathbf{H})\mathbf{A}_{L+N-1}(k) + \mathbf{V}_L(k)$$

where $\mathcal{T}_L(\mathbf{H})$ is the convolution matrix of \mathbf{H} , $\mathbf{Y}_L(k) = [\mathbf{y}^H(k) \dots \mathbf{y}^H(k - L + 1)]^H$ and similarly for $\mathbf{V}_L(k)$. In an obvious shorthand notation, we will use the following expression :

$$\mathbf{Y} = \mathcal{H}\mathbf{A} + \mathbf{V}.$$

¹The work of Luc Deneire is supported by the EC by a Marie-Curie Fellowship (TMR program) under contract No ERBFMBICT950155

We assume that $mML > L + N - 1$, so that the convolution matrix \mathcal{H} is ‘‘tall’’ and we assume \mathcal{H} to have full column rank (which leads to the usual identifiability conditions). The covariance matrix of \mathbf{Y} is

$$\mathbf{R}_{YY} = \mathbb{E}\{\mathbf{Y}\mathbf{Y}^H\} = \mathcal{H}\mathbf{R}_{AA}\mathcal{H}^H + \sigma_v^2\mathbf{I}$$

3. SUBSPACE FITTING

3.1. Signal Subspace Fitting

One can write the eigendecomposition of the covariance matrix $\mathbf{R}_{YY} = \mathbb{E}\{\mathbf{Y}\mathbf{Y}^H\} = \mathbf{V}_S\Lambda_S\mathbf{V}_S^H + \mathbf{V}_N\Lambda_N\mathbf{V}_N^H$ in which \mathbf{V}_S has the same dimensions as \mathcal{H} and $\Lambda_N = \sigma_v^2\mathbf{I}$. The signal subspace can be expressed as:

$$\text{range}\{\mathbf{V}_S\} = \text{range}\{\mathcal{H}\}$$

We can then formulate the classical subspace fitting problem :

$$\min_{\mathbf{H}, \mathbf{Q}} \|\mathcal{H} - \mathbf{V}_S\mathbf{Q}\|_F^2$$

Since \mathbf{V}_N spans the noise subspace, this leads to

$$\min_{\mathbf{H}} \mathbf{H}^t \left[\sum_{i=D^\perp}^{LMm} \mathcal{T}_N(\mathbf{v}_i^{Ht}) \mathcal{T}_N^H(\mathbf{v}_i^{Ht}) \right] \mathbf{H}^{Ht}$$

where \mathbf{v}_i is column i of $\mathbf{V} = [\mathbf{V}_S \mathbf{V}_N]$, $D^\perp = N + L$ and superscript t denotes the transposition of the blocks of a block matrix. Under constraint $\|\mathbf{H}\| = 1$, $\hat{\mathbf{H}}^t$ is then the eigenvector corresponding to the minimum eigenvalue of the matrix between the brackets. One can lower the computational burden by using $D^\perp > N + L$, loosing some performance (see a.o. [7],[9]).

Obviously, the projection on the noise subspace satisfies :

$$\mathbf{P}_{\mathbf{V}_N} = \mathbf{P}_{\mathbf{V}_S}^\perp = \mathbf{I} - \mathbf{P}_{\mathbf{V}_S} = \mathbf{I} - \mathbf{V}_S(\mathbf{V}_S^H\mathbf{V}_S)^{-1}\mathbf{V}_S^H$$

which leads to the equivalent maximization :

$$\max_{\mathbf{H}} \mathbf{H}^t \left[\sum_{i=1}^{D^\perp-1} \mathcal{T}_N(\mathbf{v}_i^{Ht}) \mathcal{T}_N^H(\mathbf{v}_i^{Ht}) \right] \mathbf{H}^{Ht}$$

3.2. Noise Subspace Fitting

Similarly to signal subspace fitting, \mathbf{V}_N spans the noise subspace and $\mathcal{T}^H(\mathbf{H}^\perp)$ spans most of it, where \mathbf{H}^\perp is such that $\mathbf{H}^{\perp t}(z)y(k) = 0$ and $\mathbf{H}^t(z) = \mathbf{H}^H(1/z^*)$ (for more details, see [2]). Hence, the following noise subspace fitting (NSF) can be introduced :

$$\min_{\mathbf{H}, \mathbf{Q}} \|\mathcal{T}^H(\mathbf{H}^\perp) - \mathbf{V}_N\mathbf{Q}\|_F^2 \quad (1)$$

4. ALTERNATIVE SIGNAL SUBSPACE FITTING

4.1. The method

In the absence of noise, we have :

$$\mathbf{R}_{YY} = \mathcal{H}\mathbf{R}_{AA}\mathcal{H}^H = \mathbf{R} = \mathbf{V}_S\Lambda'_S\mathbf{V}_S^H + \mathbf{V}_N\Lambda'_N\mathbf{V}_N^H$$

where $\Lambda'_S = \Lambda_S - \sigma_v^2\mathbf{I}$ and $\Lambda'_N = 0$. From this expression, we observe that the column spaces of \mathcal{H} and \mathbf{R} are

the same, leading us to introduce the following subspace fitting criterion :

$$\min_{\mathbf{H}, \mathbf{Q}} \|\mathcal{H} - \hat{\mathbf{R}}\mathbf{B}\mathbf{Q}\|_F^2 \quad (2)$$

where $\|\cdot\|_F$ denotes Frobenius norm and $\hat{\mathbf{R}}$ is a consistent estimate of \mathbf{R} . The matrix \mathbf{B} has the same dimensions as \mathcal{H} and is fixed; we will see later how its choice influences the performance. Note that the range of $\mathbf{F} = \hat{\mathbf{R}}\mathbf{B}$ provides an estimate for the signal subspace. We can take $\hat{\mathbf{R}}$ as $\hat{\mathbf{R}} = \hat{\mathbf{R}}_{YY} - \hat{\sigma}_v^2\mathbf{I} = \hat{\mathbf{R}}_{YY} - \lambda_{\min}(\hat{\mathbf{R}}_{YY})\mathbf{I}$ where $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue (a rank revealing decomposition of $\hat{\mathbf{R}}_{YY} - \lambda_{\min}(\hat{\mathbf{R}}_{YY})\mathbf{I}$ would lead to a better estimate of \mathbf{R}). We note that the simulations below show that even simply $\hat{\mathbf{R}} = \hat{\mathbf{R}}_{YY}$ can work well also. The criterion (2) is separable in \mathbf{H} and \mathbf{Q} . Minimizing w.r.t. \mathbf{Q} first yields

$$\mathbf{Q} = (\mathbf{F}^H\mathbf{F})^{-1}\mathbf{F}^H\mathcal{H}.$$

Substitution in (2) yields :

$$\min_{\mathbf{H}} \|\mathbf{P}_{\mathbf{F}}^\perp\mathcal{H}\|_F^2 = \min_{\mathbf{H}} \text{trace} \{ \mathcal{H}^H \mathbf{P}_{\mathbf{F}}^\perp \mathcal{H} \}.$$

With the constraint $\|\mathbf{H}\| = 1$, we get:

$$\hat{\mathbf{H}} = \arg \max_{\|\mathbf{H}\|=1} \{ \text{trace} \{ \mathcal{H}^H \mathbf{P}_{\mathbf{F}} \mathcal{H} \} = \mathbf{H}^t \mathcal{F} \mathbf{H}^{Ht} \}$$

where \mathcal{F} is a sum of submatrices of block size N of $\mathbf{P}_{\mathbf{F}}$. The solution is thus $V_{\max}(\mathcal{F})$, the eigenvector of \mathcal{F} corresponding to $\lambda_{\max}(\mathcal{F})$. Given that $\mathbf{R} = \mathcal{H}\mathbf{R}_{AA}\mathcal{H}^H$, not every choice for \mathbf{B} is acceptable. For instance, if the columns of \mathbf{B} are in the noise subspace, then $\mathbf{F} = 0$ for $\hat{\mathbf{R}} = \mathbf{R}$. Intuitively, the best choice for \mathbf{B} should be $\mathbf{B} = \mathcal{H}$, which corresponds to matched filtering \mathcal{H}^H with \mathcal{H} (postmultiplication of \mathbf{B} with a square non-singular matrix does not change anything since that matrix can be absorbed in \mathbf{Q}). These considerations lead to the following *two-step procedure*:

step 1: at first, \mathbf{B} is chosen to be a fairly arbitrary selection matrix. The first step yields a consistent channel estimate (if $\mathcal{H}^H\mathbf{B}$ is non-singular).

step 2: in this step, the consistent channel estimate of the first step is used to form $\hat{\mathcal{H}}$ and we solve (2) again, but now with $\mathbf{B} = \hat{\mathcal{H}}$.

For the first step, for instance the choice $\mathbf{B}^H = [I \ 0]$ leads to something that is quite closely related to the ‘‘rectangular Pisarenko’’ method of Fuchs [4]. We found however that a \mathbf{B} of the same block Toeplitz form as \mathcal{H} but filled with a randomly generated channel works fairly well (this choice will be the one used in the simulations).

4.2. Asymptotics: exact estimation

Asymptotically, $\hat{\mathbf{R}} = \mathbf{R}$. We get $\mathbf{F} = \mathbf{R}\mathbf{B} = \mathcal{H}\mathbf{R}_{AA}\mathcal{H}^H\mathbf{B}$. Assuming $\mathbf{R}_{AA} > 0$, then if $\mathcal{H}^H\mathbf{B}$ is non-singular, we get

$$\mathbf{P}_{\mathbf{F}} = \mathbf{P}_{\mathcal{H}} = \mathbf{P}_{\mathbf{V}_S}. \quad (3)$$

If furthermore we have a consistent channel estimate, then we can take asymptotically $\mathbf{B} = \mathcal{H}$. In that case,

the use of $\mathbf{R} = \mathbf{R}_{YY}$ and hence $\mathbf{F} = \mathbf{R}_{YY}\mathcal{H}$ also leads to (3). Pursuing this issue further, and applying a perturbation analysis similar to the one hereunder, we will have a consistent estimate of the channel with $\hat{\mathbf{R}} = \hat{\mathbf{R}}_{YY}$ as $\text{SNR} \rightarrow \infty$.

4.3. Perturbation analysis

For the first step of the algorithm, we get a consistent channel estimate if $\mathcal{H}^H \mathbf{B}$ is non-singular. We can furthermore pursue the following asymptotic (first order perturbation) analysis. This analysis is based on the perturbation analysis of a projection matrix. If $\hat{\mathbf{F}} = \mathbf{F} + \Delta \mathbf{F}$, then up to first order in $\Delta \mathbf{F}$, $\mathbf{P}_{\hat{\mathbf{F}}} = \mathbf{P}_{\mathbf{F}} + \Delta \mathbf{P}_{\mathbf{F}}$ where

$$\Delta \mathbf{P}_{\mathbf{F}} = 2 \text{Sym}(\mathbf{P}_{\mathbf{F}}^\perp \Delta \mathbf{F} (\mathbf{F}^H \mathbf{F})^{-1} \mathbf{F}^H) \quad (4)$$

where $2 \text{Sym}(\mathbf{X}) = \mathbf{X} + \mathbf{X}^H$.

a. Optimality of $\mathbf{B} = \mathcal{H}$

Let $\hat{\mathbf{R}} = \mathbf{R} + \Delta \mathbf{R}$, then using the eigendecomposition of \mathbf{R} , we get up to first order

$$\Delta \mathbf{R} = \Delta \mathbf{V}_S \Lambda'_S \mathbf{V}_S^H + \mathbf{V}_S \Delta \Lambda'_S \mathbf{V}_S^H + \mathbf{V}_S \Lambda'_S \Delta \mathbf{V}_S^H + \Delta \mathbf{V}_N \Lambda'_N \mathbf{V}_N^H + \mathbf{V}_N \Delta \Lambda'_N \mathbf{V}_N^H + \mathbf{V}_N \Lambda'_N \Delta \mathbf{V}_N^H$$

Let $\mathbf{B} = \mathbf{V}_S \mathbf{B}_S + \mathbf{V}_N \mathbf{B}_N$ where we assume \mathbf{B}_S non-singular. Then using $\hat{\mathbf{F}} = \hat{\mathbf{R}} \mathbf{B}$ leads to

$$\Delta \mathbf{P}_{\mathbf{F}} = 2 \text{Sym}(\mathbf{P}_{\mathbf{V}_N} \left[\Delta \mathbf{V}_S \Lambda'_S + \mathbf{V}_N \Delta \Lambda'_N \mathbf{B}_N \mathbf{B}_S^{-1} \right] (\mathcal{H}^H \mathbf{V}_S)^{-1} \mathbf{R}_{AA}^{-1} (\mathcal{H}^H \mathcal{H})^{-1} \mathcal{H}^H)$$

This shows that $\mathbf{B}_N = 0$ is optimal.

b. Asymptotic equivalence of the two SSF

Using $\hat{\mathbf{F}} = (\hat{\mathbf{R}}_{YY} - \lambda_{\min}(\hat{\mathbf{R}}_{YY}) \mathbf{I}) \hat{\mathcal{H}}$ with $\hat{\mathbf{R}}_{YY}$ and $\hat{\mathcal{H}}$ consistent estimates, one can show that $\Delta \mathbf{P}_{\mathbf{F}}$ is the same as with $\hat{\mathbf{F}} = \hat{\mathbf{V}}_S$. Hence we get up to first order

$$\mathbf{P}_{(\hat{\mathbf{R}}_{YY} - \lambda_{\min}(\hat{\mathbf{R}}_{YY}) \mathbf{I}) \hat{\mathcal{H}}} = \mathbf{P}_{(\hat{\mathbf{R}}_{YY} - \sigma_v^2 \mathbf{I}) \mathcal{H}} = \mathbf{P}_{\hat{\mathbf{V}}_S}$$

This shows that the alternative signal subspace fitting method gives asymptotically exactly the same performance as the original SSF method. Furthermore, as long as consistent estimates are used for σ_v^2 and \mathcal{H} , the corresponding estimation errors have no influence up to first order.

c. Simplified method

When we use simply $\hat{\mathbf{F}} = \hat{\mathbf{R}}_{YY} \hat{\mathcal{H}}$, then we get

$$\Delta \mathbf{P}_{\mathbf{F}} = 2 \text{Sym}(\mathbf{P}_{\mathbf{V}_N} (\Delta \mathbf{V}_S + \Delta \mathcal{H} (\mathbf{V}_S^H \mathcal{H})^{-1} \Lambda_N \Lambda_S^{-1}) \mathbf{V}_S^H)$$

The use of $\hat{\mathbf{R}}_{YY}$ instead of $\hat{\mathbf{R}}$ leads to the appearance of the second term, the relative importance of which is proportional to $\Lambda_N \Lambda_S^{-1}$. Hence this term is negligible at high SNR.

5. ALTERNATIVE NOISE SUBSPACE FITTING

Another possibility to formulate the NSF criterion is :

$$\min_{\mathbf{H}} \left\| \mathcal{T}(\mathbf{H}^\perp) \mathbf{R}_{YY}^{\frac{1}{2}} \right\|_F^2 \quad (5)$$

where the matrix square-root is of the form $\mathbf{R}_{YY}^{\frac{1}{2}} = \mathbf{V}_S \Lambda_B^{\frac{1}{2}} \mathbf{Q}$ for some unitary \mathbf{Q} . This NSF criterion can be written as $\min_{\mathbf{H}} \mathbf{H}$ of

$$\begin{aligned} & \text{tr}\{\mathcal{T}(\mathbf{H}^\perp) \mathbf{R}_{YY}^{\frac{1}{2}} \mathbf{R}_{YY}^{\frac{H}{2}} \mathcal{T}(\mathbf{H}^\perp)^H\} \\ &= \text{tr}\{\mathcal{T}(\mathbf{H}^\perp) \mathbf{R}_{YY} \mathcal{T}(\mathbf{H}^\perp)^H\} \\ &= \text{tr}\{\mathbf{H}^\perp \left(\sum_{k=N-1}^{M-1} \mathbf{R}_{YY}(k) \right) \mathbf{H}^{\perp H}\} \\ &= (M-N+1) \text{tr}\{\mathbf{H}^\perp \hat{\mathbf{R}}_{YY} \mathbf{H}^{\perp H}\} \end{aligned} \quad (6)$$

Furthermore, it is possible to write this NSF criterion as

$$\min_{\mathbf{H}} \mathbf{H}^H \mathbf{B} \mathbf{H}$$

This coincides with the Subchannel Response Matching (SRM) formula in [2]. This proves that the noise subspace fitting problem given by (5) is nearly equivalent to the SRM problem, apart from a weighting matrix $\Lambda_N^{\frac{1}{2}}$.

6. SIMULATIONS

6.1 Signal Subspace fitting

In our simulations, we use a randomly generated real channel of length $6T$, an oversampling factor of $m = 1$ and $M = 3$ antennas. We draw the NRMSE of the channel, defined as

$$\text{NRMSE} = \sqrt{\frac{1}{100} \sum_{l=1}^{100} \|\hat{\mathbf{H}}^{(l)} - \mathbf{H}\|_F^2 / \|\mathbf{H}\|_F^2}$$

where $\hat{\mathbf{H}}^{(l)}$ is the estimated channel in the l^{th} trial.

The SNR is defined as $(\|\mathbf{H}\|^2 \sigma_a^2) / (mM \sigma_v^2)$. The correlation matrix is calculated from a burst of 100 QAM-4 symbols. For these simulations, we used 100 Monte-Carlo runs.

We draw the NRMSE for the first step of the algorithm, the second step and the subspace fitting with eigendecomposition.

These curves show that the proposed algorithm yields the same performance as the subspace fitting algorithm with eigendecomposition (even slightly better at low SNR, but this is not relevant). It is to note that we made the simulation using \mathbf{R}_{YY} and $\hat{\mathbf{R}}_{YY} - \lambda_{\min}(\hat{\mathbf{R}}_{YY}) \mathbf{I}$, which gives the same performance.

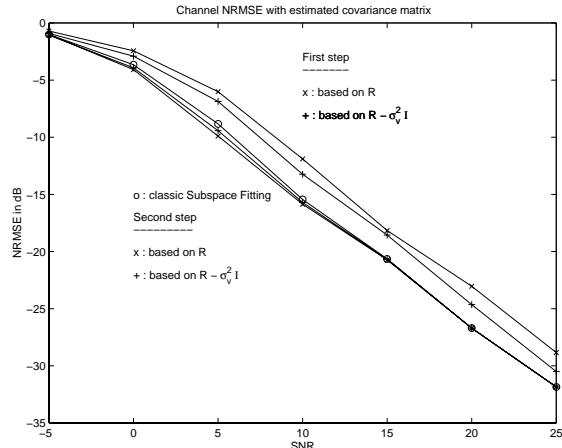


Figure 1: Subspace Fitting performance

Furthermore, we also include the NRMSE of channel estimate when using a perfect estimated covariance for the two steps. Comparison of the two graphs illustrates the preponderance of the covariance estimation error on the channel estimation error.

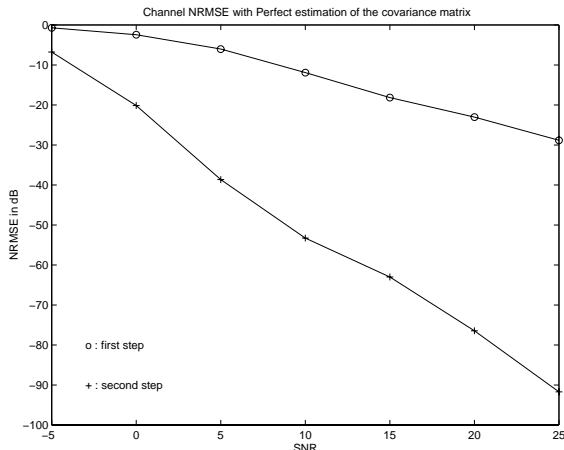


Figure 2: Subspace Fitting performance

6.2 Noise Subspace fitting

In these simulations, we use a randomly generated complex channel of length $3T$, an oversampling factor of $m = 1$ and $M = 3$ antennas and 300 Monte-Carlo runs.

In the figure hereunder, we plot the curves corresponding to NMSE versus SNR using the NSF technique with $\mathbf{R}_{Y^{\frac{1}{2}}Y}$ replaced by \mathbf{R}_{YY} . We also plot the normalized deterministic Cramer-Rao bound [9]. The obtained curves are identical to SRM curves. In order to study the influence of noise, on NSF performance, we consider NMSE computed with $\hat{\mathbf{R}}_{YY}$, $\mathbf{R}_{YY} - \lambda_{\min}(\mathbf{R}_{YY})\mathbf{I}$ and $\hat{\mathbf{R}}_{YY} - \sigma_v^2\mathbf{I}$: the obtained curves are superimposed which shows that noise has little influence on NSF (note that this was to be expected since the considered NSF is equivalent to SRM and it is well known that using $\hat{\mathbf{R}}_{YY}$ or $\hat{\mathbf{R}}_{YY} - \alpha\mathbf{I}$ leads to same performance for SRM).

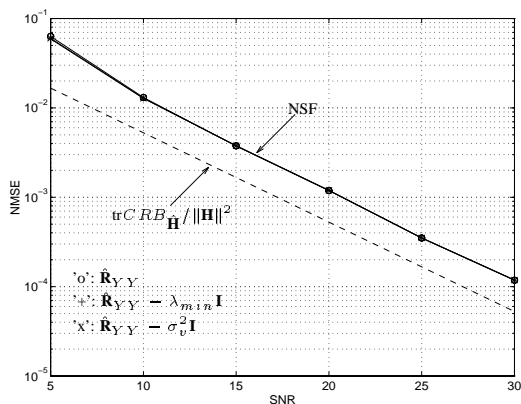


Figure 3: NSF performance

7. CONCLUSIONS

We have proposed a new two-step algorithm for solving the signal subspace fitting problem, in the channel identification context, which is computationally less demanding than the usual algorithms. Perturbation analysis shows the asymptotic equivalence of

the eigendecomposition-free approach to the original method. This equivalence was confirmed by simulation results. Further work should explore different rank revealing decompositions that can improve the finite sample performance and explore fast algorithms for the calculation of the projection matrix and associated performance if degraded. Continuing our investigations for the noise subspace fitting, we found a tight link between noise subspace fitting and SRM.

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