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Abstract: We consider a Spatial Division Multiple Access (S.D.M.A.) situation in which p users operate on the same carrier frequency and use the same linear digital modulation format. We consider $m > p$ antennas receiving mixtures of these signals through multi-path propagation (equivalently, oversampling of the received signals of a smaller number of antenna signals could be used). Current approaches to multiuser blind channel identification include subspace-fitting techniques [7], deterministic Maximum-Likelihood (DML) techniques [13] and linear prediction methods [13]. The two first techniques are rather closely related and give the channel apart from a triangular dynamical multiplicative factor (see [7]), moreover, they are not robust to channel length overestimation. The latter approach is robust to channel length overestimation and yields the channel estimate apart from a unitary static multiplicative factor, which can be determined by resorting to higher order statistics. On the other hand, Gaussian Maximum Likelihood (GML) methods have been introduced in [5] for the single user case and have given better performances than DML. Extending GML to the multiuser case, we can expect good performances, and, as will be shown in the identifiability section, we will get the channel apart from a unitary static multiplicative factor.

I Problem Formulation

Consider linear digital modulation over a linear channel with additive Gaussian noise. Assume that we have p transmitters at a certain carrier frequency and m antennas receiving mixtures of the signals. We shall assume that $m > p$. The received signals can be written in the baseband as

$$y_i(t) \;=\; \sum_{j=1}^{p}\sum_{k} a^j(k)h_i^j(t-kT) \;+\; v_i(t) \qquad (1)$$

where the $a^j(k)$ are the transmitted symbols from source j, T is the common symbol period, $h_i^j(t)$ is the (overall) channel impulse response from transmitter j to receiver antenna i. Assuming the $\{a^j(k)\}$ and $\{v_i(t)\}$ to be jointly (wide-sense) stationary, the processes $\{y_i(t)\}$ are (wide-sense) cyclostationary with period T. If $\{y_i(t)\}$ are sampled with period T, the sampled processes are (wide-sense) stationary. Sampling in this way leads to an equivalent discrete-time representation. We could also obtain multiple channels in the discrete-time domain by oversampling the continuous-time received signals, see [11],[14].

We assume the channels to be FIR. In particular, after sampling we assume the (vector) impulse response from source j to be of length N^j. Without loss of generality, we assume the first non-zero vector impulse response sample to occur at discrete-time zero. Let $N = \sum_{j=1}^{p} N^j$ and $N^1 = \max_j(N^j)$. The discrete-time received signal can be represented in vector form as

$$\begin{aligned}
\boldsymbol{y}(k) &= \sum_{j=1}^{p}\sum_{i=0}^{N^j-1} \boldsymbol{h}^j(i)a^j(k-i) + \boldsymbol{v}(k) \\
&= \sum_{i=0}^{N^1-1} \boldsymbol{H}(i)\boldsymbol{a}(k-i) + \boldsymbol{v}(k) \\
&= \sum_{j=1}^{p} \boldsymbol{H}^j A^j_{N^j}(k) + \boldsymbol{v}(k) \\
&= \boldsymbol{H}\boldsymbol{A}_N(k) + \boldsymbol{v}(k)
\end{aligned} \qquad (2)$$

$$\begin{aligned}
&\boldsymbol{y}(k) = \left[y_1^H(k)\cdots y_m^H(k)\right]^H, \boldsymbol{v}(k) = \left[v_1^H(k)\cdots v_m^H(k)\right]^H, \\
&\boldsymbol{h}^j(k) = \left[h_1^{jH}(k)\cdots h_m^{jH}(k)\right]^H, \\
&\boldsymbol{H}^j = \left[\boldsymbol{h}^j(N^j-1)\cdots \boldsymbol{h}^j(0)\right], \boldsymbol{H} = \left[\boldsymbol{H}^1\cdots\boldsymbol{H}^p\right], \\
&\boldsymbol{H}(k) = \left[\boldsymbol{h}^1(k)\cdots\boldsymbol{h}^p(k)\right], \boldsymbol{a}(k) = \left[a^{1H}(k)\cdots a^{pH}(k)\right]^H, \\
&A_n^j(k) = \left[a^{jH}(k-n+1)\cdots a^{jH}(k)\right]^H, \\
&\boldsymbol{A}_N(k) = \left[A_{N^1}^{1H}(k)\cdots A_{N^p}^{pH}(k)\right]^H.
\end{aligned} \qquad (3)$$

where superscript ^{H} denotes Hermitian transpose.

We consider additive temporally and spatially white Gaussian circular noise $\boldsymbol{v}(k)$ with $R_{vv}(k-i) = \mathrm{E}\left\{\boldsymbol{v}(k)\boldsymbol{v}^H(i)\right\} = \sigma_v^2 I_m\delta_{ki}$. Assume we receive M samples :

$$\boldsymbol{Y}_M(k) \;=\; \mathcal{T}_M^p\left(\boldsymbol{H}\right)\, \boldsymbol{A}_{N+p(M-1)}(k) + \boldsymbol{V}_M(k) \qquad (4)$$

where $\boldsymbol{Y}_M(k) = \left[\boldsymbol{Y}^H(k-M+1)\cdots\boldsymbol{Y}^H(k)\right]^H$ and $\boldsymbol{V}_M(k)$ is defined similarly whereas $\mathcal{T}_M^p(\boldsymbol{H})$ is the multichannel multiuser convolution matrix of $\boldsymbol{H}$, with M block lines. Therefore, the structure of the covariance matrix of the received signal $\boldsymbol{Y}(k)$ is

$$R_{YY} = \mathcal{T}_M^p\left(\boldsymbol{H}\right) R_{AA}\mathcal{T}_M^{pH}\left(\boldsymbol{H}\right) + \sigma_v^2 I_{mM} \qquad (5)$$

where $R_{AA} = \mathrm{E}\left\{\boldsymbol{A}_{N+p(M-1)}(k)\boldsymbol{A}^H_{N+p(M-1)}(k)\right\}$. From here on, we will assume mutually i.i.d. white sources with power σ_a^2 $(R_{AA} = \sigma_a^2 I)$.

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Likelihood.

In stochastic ML, the input symbols are modeled as Gaussian quantities. ML estimation with a Gaussian prior for the symbols has been introduced in [15] and [5] for the single user case and its robustness properties have been shown. Rewriting equation (4) in shorthand : $\boldsymbol{Y} = \mathcal{T}(\boldsymbol{H})\boldsymbol{A} + \boldsymbol{V}$, with Gaussian hypotheses on the noise and the symbols ($\boldsymbol{V} \sim \mathcal{N}(0, R_{VV})$ and $\boldsymbol{A} \sim \mathcal{N}(0, R_{AA})$). We want to maximize $f(\boldsymbol{Y}|\boldsymbol{H})$. Hence, $\boldsymbol{Y} \sim \mathcal{N}(0, R_{YY})$ and the corresponding log-likelihood function to be minimized is :

$$\min_{\boldsymbol{H}} \left\{ \ln(\det R_{YY}) + \boldsymbol{Y}^H R_{YY}^{-1} \boldsymbol{Y} \right\}. \tag{6}$$

A Identifiability conditions

Parameters are considered identifiable when they are determined uniquely by the probability distribution of the data (i.e. $\forall \boldsymbol{Y},\ f(\boldsymbol{Y}|\theta) = f(\boldsymbol{Y}|\theta') \Rightarrow \theta = \theta'$). In the models we will consider, data have a Gaussian distribution, so identifiability in this case means identifiability from the mean and the covariance of $\boldsymbol{Y}$, hence from the covariance of $\boldsymbol{Y}$, since its mean is zero. Another indicator of identifiability is regularity of the Fisher Information Matrix (FIM). This point of view is not equivalent however [10]. In particular, discrete valued ambiguities cause unidentifiability but don't lead to singularity of the FIM.

On the basis of the number of parameters (excluding a unitary mixture) compared to the number of equations, we arrive at :

Necessary condition $M > \frac{N}{m} + \frac{1}{2}$

Sufficient condition *In the Gaussian model, the m-channel $\boldsymbol{H}$ is identifiable blindly up to a unitary static mixture factor if*

1. *(i) The channel is irreducible and column reduced.*
2. *(ii) $M \geq \underline{L}$ $\quad (\underline{L} = \left\lceil \frac{N-p}{m-p} \right\rceil)$.*

Identifiability means identifiability from R_{YY}, which is equivalent to having enough data such that $\mathcal{T}(\boldsymbol{H})$ is tall, which leads to (ii), and full column rank, which leads to (i). We have $\sigma_v^2 = \lambda_{min}(R_{YY})$. $\boldsymbol{H}$ can be identified from the de-noised R_{YY} by linear prediction [13] or Schur triangularization (see here under). The unitary static mixture is in fact block diagonal, where the different blocks correspond to channels of the same length. Indeed an arbitrary unitary mixture can be undone by forcing the proper channel lengths for the different users with different channel lengths.

Sufficient condition *Any minimum-phase channel (i.e. $\mathbf{H}(z) = \underline{\mathbf{H}}(z)R(z)$ with $\underline{\mathbf{H}}(z)$ irreducible and column reduced, and $R(z)$ minimum-phase) can be identified up to a static mixture for a larger M.* Indeed, linear prediction allows to identify $\underline{\mathbf{H}}(z)$ and the correlation sequence of $R(z)$ [13, 14] from which $R(z)$ can be identified up to a unitary mixture by spectral factorization.

A nice feature of Blind GML methods is their robustness to channel length overestimation. Indeed, let

$$\mathbf{S}_{yy}(z) = \sigma_a^2 \mathbf{H}(z)\mathbf{H}^\dagger(z) + \sigma_v^2 I_m \tag{7}$$

where $\boldsymbol{H}$ is of length N^1 (with some users having possibly shorter channels).

Now, take $\overline{\boldsymbol{H}}$ of length $\overline{N}^1 \geq N^1$, then, asymptotically, GML gives $\overline{\boldsymbol{H}}$ such that :

$$\begin{array}{ll} & \overline{\mathbf{H}}(z)\overline{\mathbf{H}}^\dagger(z) = \mathbf{H}(z)\mathbf{H}^\dagger(z) \\ \Rightarrow & \overline{\mathbf{H}}(z) = \mathbf{H}(z)\Phi(z) \quad , \quad \Phi(z)\Phi^\dagger(z) = I_p \end{array} \tag{8}$$

and $\Phi(z)$ is a $p \times p$ FIR filterof length $\overline{N}^1 - N^p$ (assuming $N^1 \geq N^2 \geq \cdots \geq N^p$). Now, a lossless FIR square transfer function is necessarily of the form

$$\Phi(z) = diag\{z^{-n_1}, \ldots, z^{-n_p}\}\Psi \tag{9}$$

$$0 \leq n_j \leq \overline{N}^1 - N^j - 1, \qquad j = 1, \ldots, p$$

where $\Psi^H\Psi = I_p$. This means that GML based methods (as well as Linear Prediction methods and the Schur method developped here under) yield consistent channel estimates, even when the channel lengths have been overestimated.

C Prediction Based GML

Let $\mathbf{P}(z) = \sum_{i=0}^{L} \boldsymbol{p}(i) z^{-i}$ with $\boldsymbol{p}(0) = I_m$ be the MMSE multivariate prediction error filter of order L for the noise-free received signal $\boldsymbol{Y}(k)$. If $L \geq \underline{L} = \left\lceil \frac{N-p}{m-p} \right\rceil$, then it can be shown [13] that $\mathcal{T}(\boldsymbol{P})\mathcal{T}(\boldsymbol{H}) = \mathcal{T}(\boldsymbol{h}(0))$, or equivalently $\mathbf{P}(z)\mathbf{H}(z) = \boldsymbol{h}(0)$. From this expression, it is clear that $\mathbf{H}(z)$ and $\{\mathbf{P}(z), \boldsymbol{h}(0)\}$ are equivalent parameterizations. Expressing $\mathcal{T}(\boldsymbol{P})\mathbf{Y} \sim \mathcal{N}(0, \sigma_a^2 \mathcal{T}(\boldsymbol{h}(0))\mathcal{T}^H(\boldsymbol{h}(0)) + \sigma_v^2 \mathcal{T}(\boldsymbol{P})\mathcal{T}^H(\boldsymbol{P}))$, we can apply the GML procedure described here above, and minimize w.r.t. $\{\boldsymbol{P}, \boldsymbol{h}(0)\}$ [5]. A consistent estimate to initialize the IQML procedure can be obtained by a linear prediction algorithm (e.g. the multichannel Levinson algorithm).

D Channel Based GML

a Initialization

A consistent estimate to initialize the IQML procedure could be derived from the linear prediction algorithm, and then deriving the channel estimate from the prediction quantities. Another idea is to resort to the Schur algorithm which gives, as part of the triangular factor of $\mathcal{R} = R_{YY} - \sigma_v^2 I$, a consistent estimate of the channel.

noise-free received signal $\boldsymbol{Y}$. Let $\boldsymbol{Y}$ denote $\boldsymbol{Y}$ the prediction errors, then $\boldsymbol{Y}$ can be perfectly predicted from $\tilde{\boldsymbol{Y}}$, thus, the covariance matrix of the error in estimating $\boldsymbol{Y}$ from $\tilde{\boldsymbol{Y}}$ is zero :

$$\mathcal{R} = R_{Y\tilde{Y}} R^{\#}_{\tilde{Y}\tilde{Y}} R_{\tilde{Y}Y} = U^H D U \tag{10}$$

where $^{\#}$ denotes some pseudo-inverse. If the triangularization is performed block-wise, D is a block-diagonal whose first $\underline{L}-1$ blocks are full rank, the $\underline{L}^{th}$ block has rank $(1-\underline{L})(m-p)+N$ and subsequent blocks are of rank p. From prediction considerations, $\tilde{\boldsymbol{Y}}$ contains, from the $(\underline{L}+1)^{th}$ block onwards, the transmitted symbols $\boldsymbol{a}(k)$, apart from an instantaneous unitary mixture. Hence, since $U^H = R_{Y\tilde{Y}} R^{\#}_{\tilde{Y}\tilde{Y}}$, U^H contains, from the $(\underline{L}+1)^{th}$ block onwards, the channel impulse response (again, apart from an instantaneous unitary mixture).

A modification of the generalized Schur Algorithm The generalized Schur Algorithm provides an efficient way of performing LU triangularization of Toeplitz or Near-Toeplitz matrices, based on their low displacement rank. Let $\Delta\mathcal{R} = \mathcal{R} - Z\mathcal{R}Z^H = G^H \Sigma G$ be the displacement of the block Toeplitz correlation matrix, which is of rank (at most) $2m$, where G is called the generator (of size $2m \times Km$) and Σ the signature matrix. The Schur algorithm then proceeds by applying Σ-unitary transformations to perform partial triangularization of $\mathcal{R}$. After $\underline{L}-1$ steps, we will encounter singularities, which could be treated by perturbations techniques [6] (indeed, look-ahead techniques as in [6] do not apply in our case), but, apart from the precision problems generated, these techniques do not provide the channel estimate. Indeed, the matrix being singular, the triangularization is not unique any more, and we have to force D and U to have the structure of $R^{\#}_{\tilde{Y}\tilde{Y}}$ and $R_{\tilde{Y}Y}$, which, from the cross- and auto-correlation properties of the prediction error $\tilde{\boldsymbol{Y}}$ and $\boldsymbol{Y}$, can be determined as below. Consider we perform a scalar triangularization, then for D, the structure is :

$$D = \text{diag}[* * \cdots *, * * * 0, * * 00, * * 00, \cdots] \tag{11}$$

with the rank profile described above, and U has zero lines corresponding to the zeros in D. Considering that the non-zero lines in U repeat themselves from the $\underline{L}^{th}$ block onwards, we concentrate on the steps $\underline{L}-1$ and $\underline{L}$. At step $\underline{L}-1$, we can just put zeros when we encounter the singularity. After this step, the Schur complement with respect to the $\underline{L}$ principal minor is not any more strictly related to the generators of the preceding step, so we have to compute it's generator anew, knowing that it's displacement rank is now $2p$, which completes the algorithm.

b Channel Complement Based GML

Apart from the first procedure described, one can base the minimization on an orthogonal channel complement, introduced, a.o. in [4, 9]. Consider the single user case, with $m = 2$ channels, one can observe that for noise-free signals, we have $[\mathbf{H}_2(z) \;\; -\mathbf{H}_1(z)]\boldsymbol{Y}(k) = 0$, which leads to $\mathbf{H}^{\perp\dagger}(z)\mathbf{H}(z) = 0$, where $\mathbf{H}^{\perp\dagger}(z) = [\mathbf{H}_2(z) \;\; -\mathbf{H}_1(z)]$. This can be extended to more than two channels [3].

ment :

$$\mathcal{T}(\boldsymbol{H}^{1\perp\dagger})\boldsymbol{Y} \sim \mathcal{N}(0, R)$$
$$R = \mathcal{T}(\boldsymbol{H}^{1\perp\dagger})\left[\sigma_a^2 \sum_{i=2}^p \mathcal{T}(\boldsymbol{H}^i)\mathcal{T}^H(\boldsymbol{H}^i) + \sigma_v^2 I\right]\mathcal{T}^H(\boldsymbol{H}^{1\perp\dagger}) \tag{12}$$

This will lead to the following ML minimization :

$$\min_{\boldsymbol{H}^1}(\ln(\det R) + \boldsymbol{Y}^H \mathcal{T}^H(\boldsymbol{H}^{1\perp\dagger})R^{-1}\mathcal{T}(\boldsymbol{H}^{1\perp\dagger})\boldsymbol{Y}) \tag{13}$$

where we consider the channels of the other users as known (estimated separately) . Hence, we can pursue an approximate ML procedure, using consistent estimates (determined in the initialization step described here above) to initialize R, and repeat this procedure for each user.

It's rather obvious that, for non-minimal channel complements [3], R is low rank, which means the the Gaussian pdf is only present in a subspace. One possibility to get around this is to use Tikhonov regularization ($R+\epsilon I$), the other one is to use a projection on the subspace (which is computationally more intensive). Both approaches give very similar results. Use of the first regularization in (13) shows that the $\ln(\det R)$ can be neglected, which leads to the use of an IQML (Iterative Quadratic ML) algorithm [5, 2].

c Combined DML/GML approach

Let user 1 be the user of interest, we will perform Deterministic Maximum Likelihood (DML) on the first user and GML on the others. That is, if we consider the first source to be deterministic and the other to be white Gaussian sources of power σ_a^2 :

$$\boldsymbol{Y} \sim \mathcal{N}(\mathcal{T}(\boldsymbol{H}^1)A^1, \underbrace{\sigma_a^2 \sum_{i=2}^p \mathcal{T}(\boldsymbol{H}^i)\mathcal{T}^H(\boldsymbol{H}^i) + \sigma_v^2 I}_{=R}) \tag{14}$$

which leads to the following approximate ML minimization :

$$\min_{\boldsymbol{H}^1, A^1} \left(\boldsymbol{Y} - \mathcal{T}(\boldsymbol{H}^1)A^1\right)^H R^{-1}\left(\boldsymbol{Y} - \mathcal{T}(\boldsymbol{H}^1)A^1\right) \tag{15}$$

which can be sequentially solved in $A^1, \boldsymbol{H}^1$. Plugging $\widehat{A}^1 = (\mathcal{T}^H(\boldsymbol{H}^1)R^{-1}\mathcal{T}(\boldsymbol{H}^1))^{-1}\mathcal{T}^H(\boldsymbol{H}^1)R^{-1}\boldsymbol{Y}$ in (15), and taking into account that $\mathbf{P}^{\perp}_{R^{-\frac{1}{2}}\mathcal{T}(\boldsymbol{H}^1)} = \mathbf{P}_{R^{\frac{H}{2}}\mathcal{T}^H(\boldsymbol{H}^{1\perp\dagger})}$, since $\mathcal{T}(\boldsymbol{H}^{1\perp\dagger})R^{\frac{1}{2}}R^{-\frac{1}{2}}\mathcal{T}(\boldsymbol{H}^1) = 0$, we get the same criterion as (13) apart from the $\ln(\det R)$.

d Global Channel Complement Based GML

Consider $\boldsymbol{H}^{\perp\dagger} = \begin{bmatrix} \boldsymbol{H}^{1\perp\dagger} \\ \vdots \\ \boldsymbol{H}^{p\perp\dagger} \end{bmatrix}$, we can consider the data filtered by this collection of chqnnel complements :

$$\mathcal{T}(\boldsymbol{H}^{\perp\dagger})\boldsymbol{Y} \sim \mathcal{N}(0, R)$$
$$R = \mathcal{T}(\boldsymbol{H}^{\perp\dagger})\left[\sigma_a^2 \sum_{i=1}^p \mathcal{T}(\boldsymbol{H}^i)\mathcal{T}^H(\boldsymbol{H}^i) + \sigma_v^2 I\right]\mathcal{T}^H(\boldsymbol{H}^{\perp\dagger}) \tag{16}$$

$$\min_{\boldsymbol{H}}(\ln(\det R) + \boldsymbol{Y}^H \mathcal{T}^H(\boldsymbol{H}^{\perp\dagger}) R^{-1} \mathcal{T}(\boldsymbol{H}^{\perp\dagger})\boldsymbol{Y}) \qquad (17)$$

which is a way of doing the minimization in one shot rather than sequentially. R is again singular and requires regularization.

E Iterative Quadratic GML (IQGML) Approach

The Iterative Quadratic ML algorithm (IQML) can be used to solve (13) and (17), where the $\ln(\det R)$ is neglected. The denominator R, computed thanks to the previous iterationm is considered as constant and hece the criteria (13) and (17) become quadratic. It is proved to be consistent at high SNR and requires a very good initialization. At low SNR conditions, a Denoised Iterative Quadratic ML (DIQML) has been develeopped in [2], which can be applied here.

F Pseudo-Quadratic GML (PQGML) Approach

The principle of PQML has been first applied to sinusoids in noise estimation [12] and to single user channel identification in [8, 2]. The gradient of the approximated GML cost function may be arranged as $\mathcal{P}(h)h$, where $h = \text{Vec}(\boldsymbol{H})$ and $\mathcal{P}(h)$ is ideally positive semi-definite. The ML solution verifies $\mathcal{P}(h)h = 0$, which is solved under a non triviality condition by the PQML strategy as follows: in a first step, $\mathcal{P}(h)$ is considered constant, and as $\mathcal{P}(h)$ is positive semi-definite, h is chosen as the eigenvector corresponding to the smallest absolute eigenvalue of $\mathcal{P}(h)$. This solution is used to reevaluate $\mathcal{P}(h)$ and other iterations may be done. The PQ approach can be applied to any of the previous promlem formulation. For the basic GML problem in (6) however, it leads to an uninteresting result since in that casem the expected value of $\mathcal{P}(h)$ is zero. The quantities involved in the sequential channel complement method in (13) are, for user i :

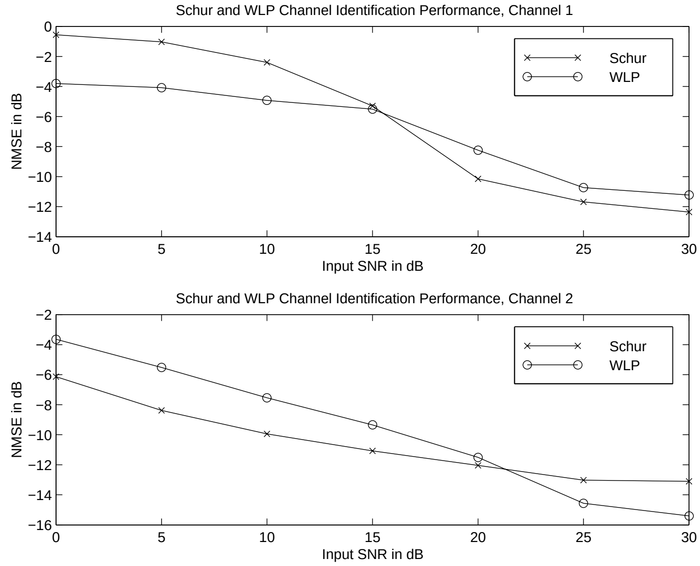
$$\mathcal{P}(h_i) = \mathcal{Y}^H R^{\#} \mathcal{Y} - \mathcal{B}^H \mathcal{B} \qquad (18)$$

where $\mathcal{Y}h_i = \mathcal{T}(\boldsymbol{H}^{i\perp\dagger})\boldsymbol{Y}$, $\mathcal{B}^* h_i^* = \mathcal{T}^H(\boldsymbol{H}^{i\perp\dagger})\boldsymbol{B}$ and $\boldsymbol{B} = R^{\#}\mathcal{T}(\boldsymbol{H}^{i\perp\dagger})$.

III Simulations

In order to evaluate the performance of the algorithms, we have computed the Normalized MSE (NMSE) on the estimated channels, averaged over 100 Monte Carlo runs. We have used a randomly generated channel with $p = 2$ users, $N_1 = 3$ and $N_2 = 4$, and $m = 4$ subchannels. The symbols are i.i.d. BPSK and the data length is $M = 250$. Due to the different channel lengths, we did not need to use higher order statistics, which means that the NMSE is not affected by separation problems.

We first evaluate the performance of the Schur algorithm compared to the Weighted Linear Prediction algorithm [1]. The performance of both algorithms is comparable. Experience with the Schur algorithm shows that some refinements could be done conditioned.



Comparison between Schur and WLP algorithms.

The IQGML procedure applied to the channel complement formulation gave poor results and are not reported here.

We have simulated the PQGML algorithm base on the complement channel. It turns out that this algorithm gives relatively poor results for short data bursts and suffers from the near far effect. This can be explained as follows. In (13),

$$\min_{\boldsymbol{H}^1} \boldsymbol{Y}^H \mathcal{T}^H(\boldsymbol{H}^{1\perp\dagger}) R^{-1} \mathcal{T}(\boldsymbol{H}^{1\perp\dagger})\boldsymbol{Y}, \qquad (19)$$

$R = \mathcal{T}(\boldsymbol{H}^{1\perp\dagger}) \left[\sigma_a^2 \sum_{i=2}^{p} \mathcal{T}(\boldsymbol{H}^i)\mathcal{T}^H(\boldsymbol{H}^i) + \sigma_v^2 I\right] \mathcal{T}^H(\boldsymbol{H}^{1\perp\dagger})$. The term between square brackets represents the Interference plus Noise term and is processed globally in the PQGML approach, which leads, even at relatively high SNR, to a sort of equivalent noise of high power, but with a lower dimensionality. Indeed, despite this noise-like behavior of the interference, the PQGML worked fairly well for SNR's above 20 dB (and a SINR of 0 dB, i.e. two equal power users) , but rather poorly below that. Furthermore, the ”noise-like” nature of the interference gives rise to a near-far effect (the highest power user completely shadowing the lower power users, resulting in break-downs for negative SINR's). This gives clues for better algorithms, in which the subspace structure of the term between brackets should be used.

IV Conclusions

Deterministic approaches (such as subspace fitting) for the blind estimation of multiple channels, especially in the multi-user case, only allow to estimate the channel modulo a large class of ambiguities. Using a Gaussian model for the sources, which exploits their second-order moments and especially the white and decorrelated nature of the sources, allows to reduce the ambiguities in channel estimation to (at most) an instantaneous mixture.

GML problem and some algorithms to obtain consistent initializations. The IQGML family of algorithm fails to work for the complement channel approach, and experience with IQML for the single user case, where it is proved to be inferior to PQML, leads to abandon this track. The PQGML approach applied to the channel complement asymptotically gives a consistent channel estimate, and, despite the noise-like behavior of the interference in (13) leads to fair results at high SNR's. This leads to at least two new future approaches, where the structure of the interference should be used and the global complement channel approach studied.

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