Doppler Compensation and Beamforming for High Mobility OFDM Transmissions in Multipath

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Abstract. The paper focuses on the use of receive beamforming (BF) for high speed train (HST) scenario under independent Doppler for the different multipath components in a Rician Fading environment. To combat ICI, we not only null out the ICI in the frequency domain, but do a pre-processing in the time domain via frequency correction (demodulation) to maximise the signal part at the output of the FFT. To obtain a suitable demodulation frequency, location aware and location agnostic approaches are considered. Cyclic prefix (CP) based estimation method is also considered as part of location information agnostic approach. In the location-aware approach, a technique that uses both the LoS and dominant scatterer information is also proposed. The paper then provides the optimal weights for the maximisation of the SINR criterion from a theoretical and practical perspective. In the case of linear approximation of the channel variation, the ICI is shown to be a rank 1 interferer and hence can be nulled out with just 2 receiver antennas. Finally, all the methods are compared via simulations. We conclude that in an LTE OFDM system simple, low complex, location agnostic BF schemes are very effective against ICI even with just two receive antennas.

Key words: ICI, Doppler,beamforming,SINR, OFDM

1 Introduction

The current LTE design has been done to support speeds of up to 500kmph but this is yet to be seen in practical demonstrations [1]. As is well known, the high Doppler in these environments violates the orthogonality requirement for the OFDM, resulting in ICI. While the lower data rate transmissions are not impacted by Doppler, the higher data rates are severely impacted. An analysis of the SINR due to ICI is clearly useful to bring out this dependency ([2],[3]). In [4], one can clearly see the presence of multipath components with independent Doppler values. Interestingly, these multipath components appear with the same delay (based on the sampling rates) and have amplitudes comparable to that of the LoS path. In [5], the Digital Video Broadcast (DVB) scenario is considered with multiple LoS paths from adjacent base stations. Here, individual LoS paths are extracted by beamforming on the spatial signatures, and then individually

corrected for the Doppler. The results are shown for strong LoS path. In practice, the multipath components would also have significant power and would arrive with their independent Doppler values. In another recent publication [6], ICI is analyzed using a two path model, but does not attempt to improve the SINR due to ICI. In [7], Taylor series approximation of the time varying channels is exploited for Doppler compensation with multiple receive antennas. In this paper, we consider a Rician fading channel with multipath components, each of which have individual Doppler frequencies. The optimal beamforming weights that maximise the SINR in the frequency domain is derived assuming full knowledge of the individual multipaths. It is then shown how the theoretical analysis can be effectively used to obtain the optimal weights in a practical scenario.

Using the simple time-varying nature of the channel it is shown that to maximise the SINR, it is important to maximise the signal part at the output of the FFT at the receiver. Hence, to combat ICI, we not only null out the ICI in the frequency domain, but do a pre-processing in the time domain via frequency correction (demodulation) to maximise the signal part at the output of the FFT. Several criterion are considered to obtain the suitable choice for this frequency. One of them is to choose a demodulation frequency that maximises the signal power in the frequency domain. We also derive an approximate expression to derive this frequency. A well known technique to estimate the carrier frequency offset in OFDM is to cross-correlate the cyclic prefix(CP) of a symbol with it's repeated segment. Intuitively, this technique tries to minimize the time variance of the channel, and hence we analyze this correlation technique as a candidate to estimate the demodulation frequency. These methods are contrasted against the location aided approach of using the LoS frequency. We also suggest a multiple demodulation approach based on the Doppler frequencies corresponding to the strongest paths, and derive the SINR expression in this case.

The rest of the paper is organized as follows. We first present the system model in Section 2. Section 3 motivates the need for an optimal demodulation frequency and details the various approaches to obtain an appropriate demodulation frequency. The theoretical SINR analysis and a practical beamformer for the same is given in section 4.1. Section 4.2 derives the SINR for the case of multiple demodulation frequencies. In section 4.3 we derive the beamformer weights when the channel variation due to ICI is approximately linear as in ([8],[9],[7]). This is followed by simulation results for multipath scenario in section 5. Finally, conclusions are given in section 6. In the following discussions, an underscore on a variable indicates that it is a vector. Capital letters are used for frequency domain representation.

2 System Model

Consider a single input multiple output (SIMO) system with N_r receive antennas on a train moving with velocity v. A typical LTE OFDM framework is chosen with N subcarriers and sampling rate f_s. We consider a Rician multipath channel with all the multipath components appearing in the same time sample as that of

the LoS path. In other words, there is negligible delay spread in the system. Let L be the total number of paths and θ_i be the direction of arrival (DoA) of each path (LoS and multipaths). The Doppler frequency of the individual multipaths is then given by

$$f_i = \frac{v\cos(\phi_i)}{c} f_c \tag{1}$$

where c is the velocity of light, f_c is the carrier frequency and ϕ_i is related to θ_i and direction of travel of the train. For every sample index n in the time domain,

$$\underline{y}_n = \underline{h}_n x_n + \underline{\nu}_n \tag{2}$$

The time varying channel h_n can be expressed as

$$\underline{h}_n = \sum_{i=0}^{L-1} A_i \underline{a}(\theta_i) e^{j2\pi\epsilon_i \frac{n}{N}} \tag{3}$$

where A_i, $\underline{a}(\theta_i)$ are the amplitudes and the spatial signature of the individual channel paths. ϵ_i refers to the Doppler frequency divided by the subcarrier spacing of the OFDM symbol.

$$\epsilon_i = \frac{f_i}{\frac{fs}{N}} \tag{4}$$

Assuming perfect symbol synchronization, the FFT of the received symbol is taken for each of the antennas.

FFT output for the l^{th} subcarrier, in the frequency domain is

$$\underline{Y}_l = \sum_{i=0}^{L-1} A_i \underline{a}(\theta_i) \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_n e^{-j2\pi(l-\epsilon_i)\frac{n}{N}} + \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \underline{\nu}_n e^{-j2\pi l \frac{n}{N}} \tag{5}$$

Let X_l be the data on subcarrier l. i.e.,

$$X_l = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_n e^{-j2\pi l \frac{n}{N}} \tag{6}$$

Let $\underline{G}$ be the weights used on the l^{th} subcarrier to extract the data on that subcarrier.

$$\hat{X}_l = \underline{G}^H \underline{Y_l}; \tag{7}$$

This can be expanded as,

$$\hat{X}_l = X_l \underline{G}^H \sum_{i=0}^{L-1} A_i \underline{a}(\theta_i) Q(\epsilon_i) + \underline{G}^H \sum_{i=0}^{L-1} A_i \underline{a}(\theta_i) \sum_{m \neq l} X_m Q(m-l+\epsilon_i) + \underline{G}^H \hat{\underline{\nu}}_n \tag{8}$$

The first term in equation (8) corresponds to the signal component . For obvious reasons, we now define

$$H = \sum_{i=0}^{L-1} A_i \underline{a}(\theta_i) Q(\epsilon_i) \tag{9}$$

The rest of the terms in equation (8) are the ICI components and the noise. The loss of orthogonality is captured by the function Q which is defined as

$$Q(m-l+\epsilon_i) = \frac{1}{N} \sum_{k=0}^{N-1} e^{j2\pi(m-l+\epsilon_i)\frac{k}{N}} \tag{10}$$

3 Choice of Demod Frequency

From equation (9), it is obvious that the signal power is dependent on the individual Doppler frequencies. We now provide an alternative argument for the same. The time varying channel model in the time domain can be expressed as

$$\underline{h}(t) = \underline{h}_0 + \underline{h}_1(t) \tag{11}$$

where h_0 is the (time-)average value of $h(t)$ over the OFDM time span, and $h_1(t) = h(t) - h_0$ has average value zero. h_0 (a constant signal) and $h_1(t)$ (with zero average) are orthogonal signals over the OFDM symbol duration. Upon sampling, we obtain

$$\underline{h}_n = \underline{h}_0 + \underline{h}_{1,n} \tag{12}$$

Orthogonality ensures that

$$\sum_{n=0}^{N-1} |\underline{h}_n|^2 = N|\underline{h}_0|^2 + \sum_{n=0}^{N-1} |\underline{h}_{1,n}|^2 \tag{13}$$

FFT output for the l^{th} subcarrier, in the frequency domain is

$$\underline{Y}_l = \underline{h}_0 \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_n e^{-j2\pi l \frac{n}{N}} + \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \underline{h}_{1,n} x_n e^{-j2\pi l \frac{n}{N}} + \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \underline{\nu}_n e^{-j2\pi l \frac{n}{N}} \tag{14}$$

Using equation (6),

$$\underline{Y}_l = X_l \underline{h}_0 + \sum_{k=0}^{N-1} X_k \Psi_{k,l} + \underline{\hat{\nu}_n} \tag{15}$$

where

$$\Psi_{k,l} = \frac{1}{N} \sum_{n=0}^{N-1} \underline{h}_{1,n} e^{j2\pi(k-l)\frac{n}{N}} \tag{16}$$

Thus, as the mean of $\underline{h}_{1,n}$ is zero, $\Psi_{k,k}$ is zero for all k. Hence, the desired signal part is a function of $\underline{h}_0$ and $\underline{h}_{1,n}$ contributes only to ICI. Thus $\underline{h}_0$ is a measure of the signal power and $\underline{h}_{1,n}$ is a measure of the ICI power. Now, applying a demod

frequency on the OFDM symbol is equivalent to multiplying the time domain channel in the following manner.

$$\tilde{\underline{h}}_n = \underline{h}_n e^{-j2\pi\epsilon\frac{n}{N}} \tag{17}$$

As this only impacts the phase of the channel, $\sum_{n=0}^{N-1} |\underline{h}_n|^2 = \sum_{n=0}^{N-1} |\tilde{\underline{h}_n}|^2$. But multiplication with the demod frequency impacts the signal power and ICI power. In addition, from equation 13, when the signal power increases, the ICI power decreases and vice versa. Thus, the demod frequency can be used effectively to improve the signal power to ICI ratio.

Let us assume that for every OFDM symbol, the receiver demodulates the time domain signal by multiplying sample n with a factor $e^{-j2\pi n\frac{f_d}{f_s}}$. The effective channel can now be represented as

$$\underline{h}_n = \sum_{i=0}^{L-1} A_i \underline{a}(\theta_i) e^{j2\pi(\epsilon_i-\epsilon)\frac{n}{N}} = \sum_{i=0}^{L-1} A_i \underline{a}(\theta_i) e^{j2\pi\acute{\epsilon}_i\frac{n}{N}} \tag{18}$$

where $\epsilon = \frac{f_d}{\frac{f_s}{N}}$ and $\acute{\epsilon}_i = \epsilon_i - \epsilon$.

Equation (9) also gets modified such that the terms ϵ_i get replaced by $\acute{\epsilon}$. The determination of the Demod frequency can be done from a location-aware stand point or from a location agnostic standpoint. We list different options here and later compare them in the section 5.

3.1 Signal Power Maximisation

At low SNR, the noise terms would dominate over the ICI. Hence, the beamforming weights would reduce to a maximal ratio combiner (MRC) and in this scenario, that the best ϵ would be the one that maximises the signal power. Let this be denoted by ϵ_{mrc}. The results of the following analysis will be used in simulations section to benchmark other techniques.

$$\max_{\epsilon} ||H^2|| = \max_{\epsilon} \sum_{p=0}^{Nr-1} \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} A_i A_j^* a_p(\theta_i) a_p^*(\theta_j) Q(\acute{\epsilon}_i) Q^*(\acute{\epsilon}_j) \tag{19}$$

Note that

$$\begin{aligned} Q(\acute{\epsilon}_i)Q^*(\acute{\epsilon}_j) &= e^{-j\pi\acute{\epsilon}_i(1-\frac{1}{N}))} \frac{sin(\pi\acute{\epsilon}_i)}{Nsin(\pi\frac{\acute{\epsilon}_i}{N})} e^{j\pi\acute{\epsilon}_j(1-\frac{1}{N})} \frac{sin(\pi\acute{\epsilon}_j)}{Nsin(\pi\frac{\acute{\epsilon}_j}{N})} \\ &= e^{-j\pi(\epsilon_i-\epsilon_j)(1-\frac{1}{N})} \frac{sin(\pi\acute{\epsilon}_i)}{Nsin(\pi\frac{\acute{\epsilon}_i}{N})} \frac{sin(\pi\acute{\epsilon}_j)}{Nsin(\pi\frac{\acute{\epsilon}_j}{N})} \end{aligned} \tag{20}$$

Thus, only the sinusoid terms involve ϵ. As the typical values of ϵ_i are small, using the first order Taylor series approximation ,

$$\frac{sin(\pi\acute{\epsilon}_i)}{Nsin(\pi\frac{\acute{\epsilon}_i}{N})} = \frac{\pi\acute{\epsilon}_i - (\pi\acute{\epsilon}_i)^3/6}{N * \pi\frac{\acute{\epsilon}_i}{N}} = 1 - (\pi\acute{\epsilon}_i)^2/6 \tag{21}$$

Thus, (20) becomes,

$$\begin{aligned} Q(\acute{\epsilon}_i)Q^*(\acute{\epsilon}_j) &\approx e^{(-j\pi(\epsilon_i-\epsilon_j)(1-\frac{1}{N})}(1-(\pi\acute{\epsilon}_i)^2/6)(1-(\pi\acute{\epsilon}_j)^2/6) \\ &\approx e^{(-j\pi(\epsilon_i-\epsilon_j)(1-\frac{1}{N})}(1-(\pi\acute{\epsilon}_i)^2/6-(\pi\acute{\epsilon}_j)^2/6) \end{aligned} \tag{22}$$

Taking the derivative of (22) with respect to ϵ gives

$$\begin{aligned} \frac{\partial(Q(\acute{\epsilon}_i)Q^*(\acute{\epsilon}_j))}{\partial\epsilon} &\approx e^{(-j\pi(\epsilon_i-\epsilon_j)(1-\frac{1}{N})}\left(-(\pi(\epsilon_i-\epsilon))/3-(\pi(\epsilon_j-\epsilon))/3\right) \\ &= \frac{\pi}{3}e^{(-j\pi(\epsilon_i-\epsilon_j)(1-\frac{1}{N})}(2\epsilon-\epsilon_i-\epsilon_j) \end{aligned} \tag{23}$$

Using this in (19) and equating to zero,

$$\epsilon_{mrc,approx} = \frac{\sum_{p=0}^{Nr-1}\sum_{i=0}^{L-1}\sum_{j=0}^{L-1}A_iA_j^*a_p(\theta_i)a_p^*(\theta_j)e^{-j\pi(\epsilon_i-\epsilon_j)(1-\frac{1}{N})}(\epsilon_i+\epsilon_j)}{2\sum_{p=0}^{Nr-1}\sum_{i=0}^{L-1}\sum_{j=0}^{L-1}A_iA_j^*a_p(\theta_i)a_p^*(\theta_j)e^{-j\pi(\epsilon_i-\epsilon_j)(1-\frac{1}{N})}} \tag{24}$$

3.2 Cyclic Prefix based Estimation

A well known technique to estimate the carrier frequency offset in OFDM is to cross-correlate the cyclic prefix(CP) of a symbol with it's repeated segment in the time domain. Intuitively, this technique tries to minimize the time variance of the channel, and hence we analyze this correlation technique as a candidate to estimate the demodulation frequency. The time domain correlation in the absence of noise is expressed as

$$\sum_{n=0}^{N_{cp}-1}\sum_{p=0}^{Nr-1}\sum_{i=0}^{L-1}A_ia_p(\theta_i)e^{j2\pi\epsilon_i\frac{n+N}{N}}\sum_{j=0}^{L-1}A_j^*a_p^*(\theta_j)e^{-j2\pi\epsilon_j\frac{n}{N}} \tag{25}$$

where N_{cp} is the length of the CP.

When the time variance is zero, equation (25) becomes real. Hence, the demodulation frequency ϵ is chosen such that the cross correlation becomes a real number.

3.3 LoS Doppler as demodulation frequency

Assuming sufficient number of receive antennas, we can exploit the knowledge of the Doppler frequency of the LoS component based on the location information. This method becomes optimal at high SNR, when the ICI becomes the dominant component compared to noise. In other words, the signal to interference ratio (SIR) is of interest here. It can be seen that, given sufficient number of antennas to remove the non-LoS multipath components, the best approach is to fully compensate for the Doppler of the LoS component. The optimal beamforming weights would essentially zero force the rest of the multi path components.

3.4 Doppler of LoS path and dominant scatterer as Demodulation frequencies

In scenarios where there is a significant LoS and a dominant scatterer with known Doppler, we propose a receiver with multiple Demod frequencies. Thus, the input at the receive antennas is first demodulated with the LoS frequency, then with the frequency of the dominant scatterer. The two sets of demodulated data are used to perform beamforming. The SINR analysis for this is given in section 4.2.

4 Receive Beamformer Design

In this section, we analyze the SINR for different scenarios and also derive the corresponding optimal beamforming weights.

4.1 Beamformer for Single Demod

The ICI on the l^{th} subcarrier from the m^{th} subcarrier would be $H_{l,m}^{ICI} X_m$, where

$$H_{l,m}^{ICI} = \sum_{i=0}^{L-1} A_i \underline{a}(\theta_i) Q(m - l + \epsilon_i) \tag{26}$$

The overall ICI power from all the subcarriers after weighting with G would be

$$\underline{G}^H \{ \sum_{m \neq l} H_{l,m}^{ICI} (H_{l,m}^{ICI})^H \} \underline{G} \sigma_x^2 \tag{27}$$

where $\sigma_x^2 = E[|X_m|^2]$. The SINR for the l^{th} carrier can now be written as

$$SINR_l = \frac{\underline{G}^H H H^H \underline{G} \sigma_x^2}{\underline{G}^H \{ \sum_{m \neq l} H_{l,m}^{ICI} (H_{l,m}^{ICI})^H \sigma_x^2 + \sigma_n^2 I_{Nr} \} \underline{G}} \tag{28}$$

where σ_n^2 is the noise variance and I_{Nr} is the identity matrix of size Nr

This the the well known Rayleigh quotient problem and the optimal weights G for this scenario are given by

$$\underline{G}_{opt,l} = R_l^{-1} H \tag{29}$$

where R_l is defined as

$$R_l = \{ \sum_{m \neq l} H_{l,m}^{ICI} (H_{l,m}^{ICI})^H \sigma_x^2 + \sigma_n^2 I_{Nr} \} \tag{30}$$

The ICI power in the above equations is dependent on the subcarrier location due to the presence of guard bands in the LTE symbol structure. For sake of simplicity, we can assume that all subcarriers of OFDM symbol are occupied. In that case, it can be easily seen that the impact of ICI is uniform across all subcarriers and we can drop the subscript l for R and the optimal $\underline{G}$. The optimal SINR is then given by

$$\text{SINR}_{opt} = H^H R^{-1} H \tag{31}$$

From Theory to Practice. The optimal weights were derived assuming full knowledge of the individual channel paths. However, we can take advantage of the form of the optimal weights to derive it in a practical receiver which would not have complete knowledge of all the individual channel taps.

Let us now look at the term $\underline{G}_{opt}$ more carefully. The factor $\{\sum_{i=0}^{L-1} A_i \underline{a}(\theta_i) Q(\acute{\epsilon}_i)\}$ is nothing but the channel value that will be observed on the subcarrier minus the impact of ICI and noise. Let $\widehat{H}$ be the estimated channel in the frequency domain with P pilot subcarriers. Then

$$\widehat{H} = H + \widetilde{H} \tag{32}$$

where $\widetilde{H} \sim \mathcal{CN}(0, \frac{1}{P}R)$. Hence, with sufficient number of pilots, $\widehat{H}$ becomes a good approximation for H. Now, if we can estimate R in (29), we would be able to compute the $\underline{G}_{opt}$. We now look at the relationship between R and $R_{yy} = E(\underline{y}_l \underline{y}_l^H)$.

$$R_{yy} = HH^H \sigma_x^2 + R \tag{33}$$

Now, by the Matrix Inversion Lemma [10], $R_{yy}^{-1} H \propto R^{-1} H$. Hence, R_{yy} can readily be obtained by averaging across the subcarriers as follows

$$R_{yy} = \frac{1}{N} \sum_{l=1}^{N} \underline{y}_l \underline{y}_l^H \tag{34}$$

Thus a practical receiver can use the following as optimal weights

$$\underline{G}_{opt,estimated} = R_{yy}^{-1} \hat{H}. \tag{35}$$

4.2 SINR analysis with Multiple Demodulation

There would be scenarios when the strength of one of the multi path components could become comparable to that of the LoS path. Again, with the knowledge of location, one can have information of both the LoS component as well as the dominant scatterer at a given location. This motivates using two different demodulation frequencies corresponding to the LoS component and the strongest scatterer. Thus, the input at the receive antennas is first demodulated with the LoS frequency, then with the frequency of the dominant scatterer. We now analyse this algorithm on the same lines as in section 4.

Let ϵ_a and ϵ_b be the demodulation frequencies being used for the LoS and dominant scatterer respectively. Let $\acute{\epsilon}_{i,a} = \epsilon_i - \epsilon_a$ and $\acute{\epsilon}_{i,b} = \epsilon_i - \epsilon_b$. For every subcarrier, we obtain $2Nr$ number of equations in the frequency domain as follows

$$\begin{bmatrix} \underline{Y_{l,a}} \\ \\ \underline{Y_{l,b}} \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{L-1} A_i \underline{a}(\theta_i) \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_n e^{-j2\pi(l-\acute{\epsilon}_{i,a})\frac{n}{N}} + \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \underline{\nu}_n e^{-j2\pi(l-\acute{\epsilon}_{i,a})\frac{n}{N}} \\ \\ \sum_{i=0}^{L-1} A_i \underline{a}(\theta_i) \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_n e^{-j2\pi(l-\acute{\epsilon}_{i,b})\frac{n}{N}} + \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \underline{\nu}_n e^{-j2\pi(l-\acute{\epsilon}_{i,b})\frac{n}{N}} \end{bmatrix} \tag{36}$$

Following the same steps as in section 4, we see that

$$\begin{aligned} H &= \begin{bmatrix} \sum_{i=0}^{L-1} A_i \underline{a}(\theta_i) Q(\acute{\epsilon}_{i,a}) \\ \sum_{i=0}^{L-1} A_i \underline{a}(\theta_i) Q(\acute{\epsilon}_{i,b}) \end{bmatrix} \\ H_{l,m}^{ICI} &= \begin{bmatrix} \sum_{i=0}^{L-1} A_i \underline{a}(\theta_i) Q(m-l+\acute{\epsilon}_{i,a}) \\ \sum_{i=0}^{L-1} A_i \underline{a}(\theta_i) Q(m-l+\acute{\epsilon}_{i,b}) \end{bmatrix} \end{aligned} \tag{37}$$

The AWGN noise terms across the two sets of demodulation output in the frequency domain are correlated. The correlation C_{2Nr} is given by

$$C_{2Nr} = \sigma_n^2 \begin{bmatrix} I_{Nr} & Q(\epsilon_b - \epsilon_a) I_{Nr} \\ Q(\epsilon_a - \epsilon_b) I_{Nr} & I_{Nr} \end{bmatrix} \tag{38}$$

The SINR for the l^{th} carrier can now be written as

$$SINR_l = \frac{\underline{G}^H H H^H \underline{G} \sigma_x^2}{\underline{G}^H \{ \sum_{m \neq l} H_{l,m}^{ICI} (H_{l,m}^{ICI})^H \sigma_x^2 + C_{2Nr} \} \underline{G}} \tag{39}$$

The optimal weights G for this scenario is given by

$$\underline{G}_{opt} = R^{-1} H \tag{40}$$

where $R = \{ \sum_{m \neq l} H_{l,m}^{ICI} (H_{l,m}^{ICI})^H \sigma_x^2 + C_{2Nr} \}$

4.3 Beamformer with linear approximation for channel variation

Equation (16) can be simplified if the channel variations are assumed to be linear. Several prior publications ([8],[9],[7]) have also done similar approximation.

$$\underline{h}_n = \underline{h}_0 + \left(n - \frac{N-1}{2} \right) \underline{h}_1 \tag{41}$$

Equation (15) can now be modified as

$$\underline{Y}_l = X_l \underline{h}_0 + \underline{h}_1 \sum_{k=0}^{N-1} X_k \Xi_{k,l} + \underline{\hat{\nu}_n} \tag{42}$$

where

$$\Xi_{k,l} = \frac{1}{N} \sum_{n=0}^{N-1} \left(n - \frac{N-1}{2} \right) e^{j2\pi(k-l)\frac{n}{N}} \tag{43}$$

As $\Xi_{k,l}$ is a constant, we see immediately that the ICI is approximately rank 1. Hence $N_r = 2$ receivers should be sufficient to cancel the ICI. Applying this to equation (29) (as in [7]),

$$G_{opt,linear} = \left(\underline{h}_1 \underline{h}_1^H \sum_{k=0}^{N-1} |\Xi_{k,l}|^2 + \sigma_n^2 I_{N_r} \right)^{-1} \underline{h}_0 \tag{44}$$

Using the matrix inversion lemma, this can be simplified to

$$G_{opt,linear} = \left(1 - \frac{\underline{h}_1 \underline{h}_1^H \frac{\sum_{k=0}^{N-1} |\Xi_{k,l}|^2}{\sigma_n^2}}{1 + \underline{h}_1^H \underline{h}_1 \frac{\sum_{k=0}^{N-1} |\Xi_{k,l}|^2}{\sigma_n^2}}\right)^{-1} \underline{h}_0 \tag{45}$$

5 Simulation Results

Here we consider an OFDM system with 1024 subcarriers and subcarrier spacing of 15KHz as in LTE. In addition to the LoS, 3 additional multipaths are considered. The Doppler offsets for all the paths in Hertz are [1080 -1080 758 220]. The relative tap strengths in dB are [0 0 -11 -0.7]. Such a channel with two equal strength paths but differing Doppler is motivated by the observations in [4]. Channel estimation is done using pilots with a spacing of 12 subcarriers. As a flat frequency channel is considered, the channel estimates are averaged across the pilots.

Figure 1 plots the theoretical SINR for $N_r = 2$ with CP based Demod frequency offset estimation (equation (29)) and compares this against a Maximum Ratio Combiner receiver that does not factor in the ICI. Also shown is a single antenna receiver (No BF). The huge performance gap clearly brings out the need for optimal Beam Forming with ICI in the SIMO receiver. The MRC receiver and the single antenna receiver do not apply the Demod frequency correction.

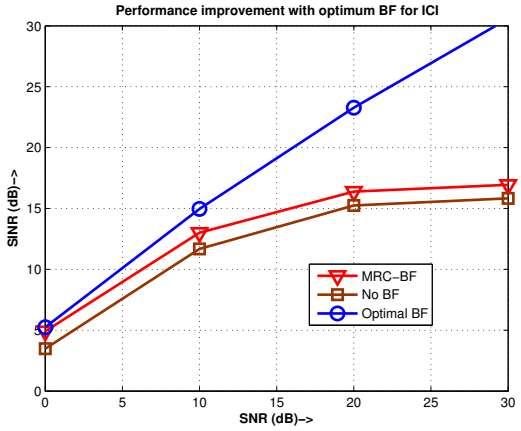

Fig. 1. Performance gains with optimum BF for $N_r = 2$

Figure 2 compares the different approaches to estimate the demodulation frequency over multiple channel realizations at an SNR of 30dB. The data points

corresponding to "Matlab Search for SINR" are obtained by determining the demodulation frequency that along with the corresponding optimal weights maximises the SINR. This was obtained by performing an exhaustive search in Matlab over the range of possible Doppler values. The data points "Matlab search for SIG PWR" were obtained by determining the demodulation frequencies that maximise the signal component in the frequency domain. This was again performed through exhaustive search in Matlab. The same was also obtained using equation (24) and is given by the data points "Approximate freq est". Finally, the demodulation frequencies obtained from CP correlation are also given. It can be seen that the CP based correlation method and the signal power maximisation method closely match the SINR maximisation criterion. The approximation for the signal power maximisation is limited by the first-order Taylor series approximation, but is still a useful approach. This establishes the CP based estimation as a useful practical approach to improve the SINR under ICI.

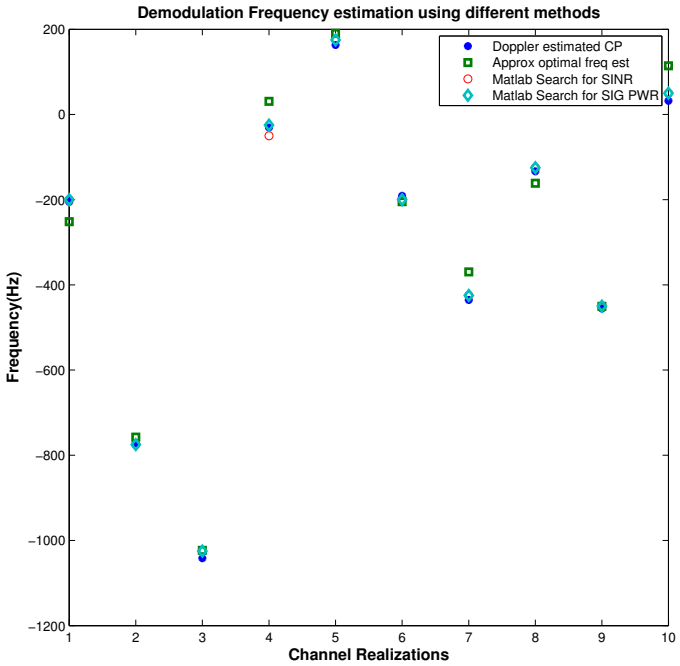

Fig. 2. Comparison of methods to estimate demodulation frequency

In Figure 3, for the same scenario as in Figure 1, different receivers are compared. The "Optimal BF" curve implements equation (29) with CP based demod frequency estimation. The "Estimated BF" curve implements equation (35) and the demod frequency is estimated using the CP . The "Approx BF" corresponds to the method in equation (45) with CP based demod. The $\underline{h}_1$ is computed as in [7]. The SINR with the above mentioned three methods almost overlap. The

"Approx BF,no Demod correction" refers to implementation of equation (45) without the application of the CP based Demod frequency correction to highlight the need for the Demod frequency correction. Also given are the curves with LoS Doppler used for Demod frequency correction. Clearly, in this scenario, the performance is suboptimal with this choice as there are scattered paths with significant signal strengths. The Multiple Demod, though computationally more expensive, performs slightly better than the optimal receiver with only a single Demod.

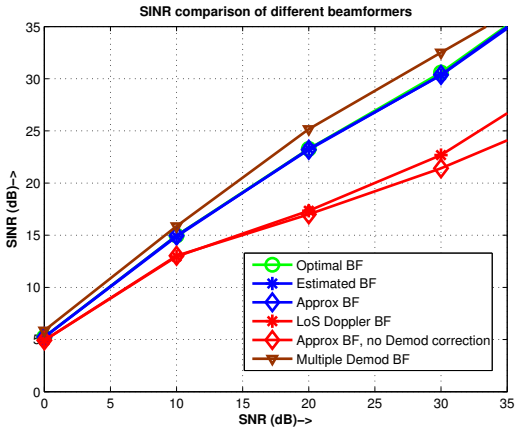

Fig. 3. Comparison of SINR with different receivers for $N_r = 2$

6 Conclusion

The paper focuses on ICI compensation for high Doppler scenarios. The need for an optimal Demod frequency correction is clearly motivated. This is followed by a theoretical analysis of the ICI for a multipath scenario and determines the optimal weights to maximise the SINR. We then proceed to show that these theoretical weights can indeed be obtained in a practical scenario and show the equivalence via simulations. While there have been publications on ICI for OFDM systems with multiple receivers ([8],[9],[7]), we focus on both the Demod frequency as well as the beamforming weights. We propose and analyse different approaches for obtaining the optimal demod frequency that include both location-aware and location information agnostic approaches. As a further extension, the performance is analyzed when two demodulation frequencies are used instead of one. It is shown through simulations that CP based Demod frequency estimation is a very practical way of obtaining the near-optimal Demodulation frequency. With the help of the simulations, we conclude that for a typical LTE OFDM scenario, good performance can be achieved with low com-

plexity beamforming techniques even at 450Kmph in the presence of significant scatterers with independent Doppler frequencies with just $N_r = 2$ receivers.

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