MATCHED FILTER BOUND OPTIMIZATION FOR MULTIUSER DOWNLINK TRANSMIT BEAMFORMING

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ABSTRACT

This paper is devoted to the optimization of the matched filter bounds (MFB) of different co-channel users, using adaptive antenna arrays at base stations for downlink transmit beamforming in cellular mobile communication systems. We mainly consider time division multiple access (TDMA) frequency division duplex (FDD) based systems. Note that in the case of time division duplex (TDD), under certain assumptions the downlink channel can be assumed to be practically the same as the uplink channel. On the contrary, when using FDD, the downlink channel can not be directly observed and estimated. That makes FDD based systems most difficult to deal with, although the proposed criteria and methods are general and suitable for both FDD and TDD. Problem formulations are provided for both spatial division multiple access (SDMA) and non-SDMA spectrum reuse techniques. Novel analytical solutions and algorithms are derived, implementation issues are discussed and simulations are provided in order to compare different approaches.

1. INTRODUCTION AND MOTIVATION

The use of adaptive antenna arrays at base stations allows increase the capacity of mobile radio networks by an improved spectrum efficiency, in the uplink as well as in the downlink. We investigate different approaches to optimize the weight vectors of adaptive antenna arrays at base stations for multiuser downlink transmit beamforming. We address the problem mainly in the context of time division multiple access (TDMA), frequency division duplex (FDD) based mobile communication systems. The main difficulty in transmission with antenna arrays in FDD systems consists in the limited knowledge at the base station of the downlink channel, since it can not be directly observed and therefore estimated. On the contrary, assuming the mobile velocity low enough and the receiver and transmitter appropriately calibrated, the uplink and the downlink channels can be considered to be practically the same in the case of time division duplex (TDD) based systems. When considering the FDD downlink, the base station needs feedback from the mobile about the downlink channel to operate in similar conditions as for the TDD downlink. Nevertheless such a feedback involves latency periods, due to the mobile–base station round-trip time and the processing time, resulting in a decrease of the spectrum efficiency. Moreover, in a typical outdoor propagation environment, due to the mobile velocity, such latency periods are usually not compatible with the feedback rate required to make the system reliable. This problem was addressed by Gerlach [5] who proposed to feedback from the mobile only the information related to the downlink channel covariance matrix instead of the channel impulse response, in order to reduce the necessary feedback rate. However, no current or conceived cellular standard is designed to support that feedback concept. On the other hand, if such feedback is not provided, the downlink channel characterization can only be based on the estimates of those parameters related to the uplink channel, which are relatively frequency independent and of which the changing rate is slow with respect to the uplink–downlink ping-pong time. In the presence of multipath such parameters are usually the directions of arrival (DOA), the powers and the delays associated to each path (e.g., see [2]). Unfortunately, the phases of the paths are strongly frequency dependent so that they cannot be estimated from the uplink channel. Actually in FDD mobile communication systems, in the absence of feedback, it is possible to estimate only the downlink channel covariance matrix averaged over the paths phases. That makes FDD transmission more difficult to deal with in practice, compared to TDD transmission. Nevertheless, since the proposed optimization criteria assume only the knowledge of the covariance matrix of the channel between each user and each base station (eventually averaged in the time or in the frequency domain in FDD case), they apply for both FDD and TDD.

Concerning the spectrum reuse technique we consider both spatial division multiple access (SDMA) and non-SDMA. For SDMA d co-channel users are allocated in the same cell in the presence of one base station, assuming interference from other cells is negligible. For non-SDMA the same users are allocated in d different cells, i.e., in the presence of d base stations with one user per cell. Then the optimization goal is to maximize the minimum matched filter bound (MFB) at each mobile receiver among all the d users in order to improve the mobile signal quality and/or spectrum efficiency.

2. GENERAL ASSUMPTIONS

We consider d co-channel users in both non-SDMA and SDMA. In the first case we shall consider d base stations, each one of them its weight vector to one user. In the second case only one base station transmits to all the users and optimizes all the corresponding weight vectors. In addition for SDMA, we neglect the co-channel interference. It would be possible though to mix the SDMA and non-SDMA cases.

The mobile is assumed to have a single antenna element whereas each base station has an array with m elements. We use the term downlink transmit beamforming since only spatial combining is used at a base station. However, the emphasis here is not really on forming beams to certain directions, but rather on exploiting the diversity between multiple sensors.

We assume multipath propagation but the channel not introducing inter-symbol interference (ISI) (i.e., the delay spread introduced by the channel is less than the duration of a symbol period). When this assumption holds the MFB reduces to the signal-to-interference plus noise-ratio (SINR). When the channel introduces ISI, we will just optimize the SINR instead of the MFB.

3. MFB OPTIMIZATION PROBLEM FORMULATION

3.1. Non-SDMA case

Let $\boldsymbol{R}_{ji}$ and $\boldsymbol{w}_j$ denote the covariance matrix of the channel between the jth base station and the ith user (including transmitter and receiver filters), and the weight vector of the jth base station respectively. The SINR for the ith user is given by

$$\mathrm{SINR}_i = \frac{\boldsymbol{w}_i^H \boldsymbol{R}_{ii} \boldsymbol{w}_i}{\sum_{j=1,\, j\neq i}^{d} \boldsymbol{w}_j^H \boldsymbol{R}_{ji} \boldsymbol{w}_j + \nu_i} \tag{1}$$

where $\nu_i = \sigma_{v_i}^2/\sigma_a^2$ and $\sigma_{v_i}^2$, σ_a^2 represent the noise power at the ith mobile receiver and the power of the transmitted symbols respectively. We denote $\mathrm{SINR}_i = \gamma_i$ for any i, throughout the paper.
Hence the general optimization problem is

$$\max_{\{\boldsymbol{w}_i\}} \min_i \{\gamma_i\} \tag{2}$$

or

$$\min_{\{\boldsymbol{w}_i\}} \max_i \{\gamma_i^{-1}\} \tag{3}$$

Then let $\boldsymbol{w}_i = \sqrt{p_i}\boldsymbol{u}_i$, with $\|\boldsymbol{u}_i\|_2 = 1$, the vector of the inverse MFB's $\boldsymbol{\gamma}^{-1} = [\gamma_1^{-1} \dots \gamma_d^{-1}]^T$ and the vector of the transmit powers $\boldsymbol{p} = [p_1, \dots, p_d]^T$, where ^{T} denotes transpose.
For non-SDMA we need to limit the maximum transmit power at each base station, i.e., $\|\boldsymbol{p}\|_\infty \leq p_{\max}$.
The criterion (3) can be reformulated as

$$\min_{\boldsymbol{p},\{\boldsymbol{u}_i\}} \|\boldsymbol{\gamma}^{-1}\|_\infty \quad \text{s.t. } \|\boldsymbol{p}\|_\infty \leq p_{\max}, \|\boldsymbol{u}_i\|_2 = 1\, \forall i \tag{4}$$

Then we define the normalized power delivered by the jth base station to the ith user as

$$c_{ji} = \boldsymbol{u}_j^H \boldsymbol{R}_{ji} \boldsymbol{u}_j \; .$$

For any i we have

$$\gamma_i^{-1} p_i c_{ii} = \sum_{j\neq i} p_j c_{ji} + \nu_i \; . \tag{5}$$

In order to account for all the users we introduce the matrix $\boldsymbol{D}_c = \mathrm{diag}(c_{11}, \dots, c_{dd})$, the matrix $\boldsymbol{C}^T$ defined as

$$[\boldsymbol{C}^T]_{ij} = \begin{cases} c_{ji} & \text{for } j \neq i \\ 0 & \text{for } j = i \end{cases}$$

the vector $\boldsymbol{\nu} = [\nu_1 \dots \nu_d]^T$ and the matrix $\boldsymbol{P} = \mathrm{diag}(\boldsymbol{p})$. Then we have the following equation

$$\boldsymbol{\gamma}^{-1} = \boldsymbol{D}_c^{-1} \boldsymbol{P}^{-1} [\boldsymbol{C}^T \boldsymbol{p} + \boldsymbol{\nu}] \; . \tag{6}$$

So the criterion (4) generally leads to a set of coupled problems which cannot be solved analytically.

3.2. SDMA case

In the SDMA case we have only one base station in the presence of d co-channel users. Then we shall replace $\boldsymbol{R}_{ji}$ with $\boldsymbol{R}_i$ for any i. So that the SINR definition (1) becomes

$$\mathrm{SINR}_i = \frac{\boldsymbol{w}_i^H \boldsymbol{R}_i \boldsymbol{w}_i}{\sum_{j=1,\, j\neq i}^{d} \boldsymbol{w}_j^H \boldsymbol{R}_i \boldsymbol{w}_j + \nu_i} \tag{7}$$

In the presence of only one base station we need to limit the overall transmitted power to be less than or equal to $p_{\max}$. Therefore the optimization criterion is

$$\min_{\boldsymbol{p},\{\boldsymbol{u}_i\}} \|\boldsymbol{\gamma}^{-1}\|_\infty \quad \text{s.t. } \|\boldsymbol{p}\|_1 \leq p_{\max}, \|\boldsymbol{u}_i\|_2 = 1\, \forall i \tag{8}$$

Then by redefining c_{ji} as

$$c_{ji} = \boldsymbol{u}_j^H \boldsymbol{R}_i \boldsymbol{u}_j$$

we obtain the expression of $\boldsymbol{\gamma}^{-1}$ in the same form as (6).
Due to the constraint on the powers in the case of SDMA it can be shown that the optimum (4) leads to the same γ for all the users. Indeed if the γ_i's, for $i = 1, \dots, d$ are not the same, then we can scale the powers $\{p_i\}$ to improve $\gamma_{\min}$ (refer to [1] for a detailed proof). On the contrary, the optimum generally does not yield the same γ for all the users in the case of non-SDMA because we cannot arbitrarily scale the powers p_i's (i.e., when any $p_i = p_{\max}$ no further increase of p_i is possible).

4. OPTIMIZATION IN THE ABSENCE OF NOISE

In the absence of noise the SINR becomes equal to the signal-to-interference ratio (SIR) which is insensitive to the absolute power level. Then the constraint on the maximum transmit power is irrelevant with respect to the SIR optimization in both non-SDMA and SDMA cases.
In addition we observe that the optimization problems for non-SDMA and SDMA have the same form and both of them lead to have the same SINR for all the users. So that in the sequel of this section we provide a detailed analysis only for the case of non-SDMA, and the case of SDMA will result by replacing the channel covariance matrices $\boldsymbol{R}_{ji}$ with $\boldsymbol{R}_i$. The SIR for the ith user in non-SDMA is defined as

$$\mathrm{SIR}_i = \frac{\boldsymbol{w}_i^H \boldsymbol{R}_{ii} \boldsymbol{w}_i}{\sum_{j=1,\, j\neq i}^{d} \boldsymbol{w}_j^H \boldsymbol{R}_{ji} \boldsymbol{w}_j} \tag{9}$$

and the equation (6) reduces to

$$\boldsymbol{\gamma}^{-1} = \boldsymbol{D}_c^{-1} \boldsymbol{P}^{-1} \boldsymbol{C}^T \boldsymbol{p} \tag{10}$$

where now $\gamma_i = \mathrm{SIR}_i$ for any i. Considering the criterion (4) and the definition (9) it is straightforward to see that the optimum is achieved when all the inter-user-interference (IUI) is zero so that $\gamma_i^{-1} = 0$ for all i's. Then the optimum approach in the absence of noise consists in forcing to zero the IUI, whenever possible.

4.1. Zero-Forcing (ZF) solution

In the absence of noise at the receiver the global optimum is achieved when the following ZF conditions are satisfied

$$\max_{\|\boldsymbol{u}_i\|_2=1} \boldsymbol{u}_i^H \boldsymbol{R}_{ii} \boldsymbol{u}_i \quad \text{s.t.} \quad \sum_{j\neq i} p_j \boldsymbol{u}_j^H \boldsymbol{R}_{ji} \boldsymbol{u}_j = 0 \tag{11}$$

Note that the second condition in (11) is equivalent to a set of ZF conditions of the form $\boldsymbol{u}_i^H \boldsymbol{R}_{ij} \boldsymbol{u}_i = 0$, i.e., $c_{ij} = 0$, for $j \neq i$. Practically that leads to nulling the IUI while maximizing the signal-to noise ratio (SNR) at the receiver. In that case $\boldsymbol{C}^T = \boldsymbol{0}_{d\times d}$ and the criterion (4) is satisfied when all the base stations transmit at the maximum power $p_{\max}$.
When considering SDMA the criterion (8) is satisfied when $p_i = p_{\max}/d$ for all i's.
Unfortunately to achieve conditions (11) we need a number of antennas greater than the sum of all the paths of all the users. In typical mobile propagation environments the number of all the paths of all the users is usually much larger than the number of antennas at a base station so that ZF generally cannot be performed with purely spatial processing.

4.2. Non-ZF solutions

When the ZF conditions (11) cannot be achieved, other approaches are possible but generally the optimization cannot be carried out analytically for both $\boldsymbol{p}$ and $\{\boldsymbol{u}_i\}$ at the same time. So we shall optimize $\boldsymbol{p}$ and $\{\boldsymbol{u}_i\}$ via a two-step procedure eventually seeking the global optimum using an iterative algorithm.

Because of the insensitivity to the absolute value of the powers we can simplify the optimization problem by requiring each vector $\boldsymbol{u}_i$ to have unit norm in the metric $\boldsymbol{R}_{ii}$, i.e., $\boldsymbol{u}_i^H \boldsymbol{R}_{ii} \boldsymbol{u}_i = 1$ for any i, instead of $\|\boldsymbol{u}_i\|_2 = 1$. Under that assumption, the power p_i corresponds to the power at the ith receiver whereas the actual power at the related transmitter is given by $p_i \|\boldsymbol{u}_i\|_2^2$, for any i. Also, $\boldsymbol{D}_c = \mathbf{I}_d$ in that case.

4.2.1. Power assignment optimization

We optimize the vector $\boldsymbol{p}$ assuming a given set of direction vectors $\{\boldsymbol{u}_i\}$. Note that since the optimum involves $\gamma_i = \gamma$ for any i, the equation (10) reduces to

$$\gamma^{-1} \boldsymbol{p} = \boldsymbol{A}^T \boldsymbol{p} \qquad (12)$$

where $\boldsymbol{A}^T = \boldsymbol{D}_c^{-1} \boldsymbol{C}^T = \boldsymbol{C}^T$ is a non-negative matrix. Moreover $\boldsymbol{p}$ has to be a non-negative vector and γ^{-1} has to be non-negative as well. On the basis of the following theorems ([8],[1])

Theorem 1
For a non-negative matrix, the eigenvalue of the largest norm is positive, and its corresponding eigenvector can be chosen to be non-negative.

Theorem 2
For a non-negative matrix $\boldsymbol{A}$, the non-negative eigenvector corresponding to the eigenvalue of the largest norm is positive.

Theorem 3
Given the matrix $\boldsymbol{A}$ there exists only one solution to equation (12).

we can claim that the only positive eigenvector of $\boldsymbol{A}^T$ is the one corresponding to its largest eigenvalue which is positive as well. In other words, that ensures the existence of a positive $\gamma^{-1} = \lambda_{\max}(\boldsymbol{A}^T)$ and a unique positive $\boldsymbol{p} = V_{\max}(\boldsymbol{A}^T)$.

4.2.2. Direction vectors optimization

Having an estimate of $\boldsymbol{p}$, we can optimize $\{\boldsymbol{u}_i\}$. Indeed the optimization criterion is given by

$$\min_{\{\boldsymbol{u}_i\}} \lambda_{\max}(\boldsymbol{A}^T) \qquad (13)$$

which is equivalent to

$$\min_{\{\boldsymbol{u}_i\}} \boldsymbol{q}^T \boldsymbol{A}^T \boldsymbol{p} \quad \text{s.t.} \quad \boldsymbol{u}_i^H \boldsymbol{R}_{ii} \boldsymbol{u}_i = 1 \qquad (14)$$

where $\boldsymbol{q} = V_{\max}(\boldsymbol{A})$. The criterion (14) leads to a set of d decoupled problems whose solution is given by $\boldsymbol{u}_i = \frac{\boldsymbol{e}_i}{\sqrt{\boldsymbol{e}_i^H \boldsymbol{R}_{ii} \boldsymbol{e}_i}}$, where $\boldsymbol{e}_i = V_{\max}(\boldsymbol{R}_{ii}, \sum_{j \neq i}^d q_j \boldsymbol{R}_{ij})$ for any i. The new set of direction vectors $\{\boldsymbol{u}_i\}$ can be used to re-optimize the powers $\boldsymbol{p}$ according to (12).

4.2.3. $\|\boldsymbol{A}^T\|_1$ minimization based solution

As initialization we can use the following criterion

$$\min_{\{\boldsymbol{u}_i\}} \|\boldsymbol{A}^T\|_1 \quad \text{s.t.} \quad \boldsymbol{u}_i^H \boldsymbol{R}_{ii} \boldsymbol{u}_i = 1 \qquad (15)$$

Indeed, $\lambda_{\max}(\boldsymbol{A}^T)$ will be small when $\boldsymbol{A}^T$ is small. This approach has the advantage of optimizing the direction vectors $\{\boldsymbol{u}_i\}$ independently from the powers $\boldsymbol{p}$. It leads to a set of d decoupled minimization problems whose solution is given by $\boldsymbol{u}_i = \frac{\boldsymbol{e}_i}{\sqrt{\boldsymbol{e}_i^H \boldsymbol{R}_{ii} \boldsymbol{e}_i}}$, where, in this case, $\boldsymbol{e}_i = V_{\max}(\boldsymbol{R}_{ii}, \sum_{j=1}^d \boldsymbol{R}_{ij})$ for any i.

Note that the criterion (15) corresponds to minimizing the power delivered to the undesired users while maximizing the power delivered to the desired user, by each base station in non-SDMA, by each each weight vector $\boldsymbol{w}_i$ in SDMA. Apart from a different normalization of $\|\boldsymbol{u}_i\|_2$, this criterion is equivalent to the one proposed in [3, 4].

4.2.4. $\lambda_{\max}(\boldsymbol{A}^T)$ minimization based algorithm

According to the previous argumentation, we propose the iterative procedure summarized in Table 1 to find the global optimum.

Table 1: $\lambda_{\max}(\boldsymbol{A}^T)$ minimization based algorithm

(i)	Initialize $\boldsymbol{u}_i$ using (15) for any i;
(ii)	Compute $\boldsymbol{q} = V_{\max}(\boldsymbol{A})$;
(iii)	Given $\boldsymbol{q}$, compute $\boldsymbol{e}_i = V_{\max}(\boldsymbol{R}_{ii}, \sum_{j \neq i} q_j \boldsymbol{R}_{ij})$;
(iv)	Compute $\boldsymbol{u}_i = \frac{\boldsymbol{e}_i}{\sqrt{\boldsymbol{e}_i^H \boldsymbol{R}_{ii} \boldsymbol{e}_i}}$;
(v)	Go back to (ii) until convergence;
(vi)	Compute $\boldsymbol{w}_i = \sqrt{p_i} \boldsymbol{u}_i$, where $\boldsymbol{p} = V_{\max}(\boldsymbol{A}^T)$;

4.2.5. $\|\boldsymbol{\gamma}^{-1}\|_1$ minimization based algorithm

Another generally sub-optimal criterion is the following

$$\min_{\boldsymbol{p}, \{\boldsymbol{u}_i\}} \|\boldsymbol{\gamma}^{-1}\|_1 \qquad (16)$$

We remark that the optimum $\boldsymbol{\gamma}$ yields $\|\boldsymbol{\gamma}^{-1}\|_1 = d \|\boldsymbol{\gamma}^{-1}\|_\infty$ so that near the optimum the criterion (16) is approximately equivalent to the criterion (4). With (16) as optimization criterion we can easily derive an iterative two-step optimization procedure, similar to the one previously described, summarized in Table 2. Unfortunately when not near the global optimum this algorithm can converge to a local minimum, resulting in worse performance than the algorithm described in Table 1.

Table 2: $\|\boldsymbol{\gamma}^{-1}\|_1$ minimization based algorithm

(i)	Initialize $\boldsymbol{u}_i$ using (15) for any i;
(ii)	Compute $\boldsymbol{p} = V_{\max}(\boldsymbol{A}^T)$;
(iii)	Given $\boldsymbol{p}$, compute $\boldsymbol{e}_i = V_{\max}(\boldsymbol{R}_{ii}, \sum_{j \neq i} \frac{1}{p_j} \boldsymbol{R}_{ij})$;
(iv)	Compute $\boldsymbol{u}_i = \frac{\boldsymbol{e}_i}{\sqrt{\boldsymbol{e}_i^H \boldsymbol{R}_{ii} \boldsymbol{e}_i}}$;
(v)	Go back to (ii) until convergence;
(vi)	Compute $\boldsymbol{w}_i = \sqrt{p_i} \boldsymbol{u}_i$.

An optimization criterion similar to (16) was proposed in [5] assuming a different cost function[1] to optimize the downlink beamforming weights at a base station in an SDMA context. Some variants are possible to improve the performance and the numerical robustness of the algorithm (see [5] for further details), but convergence to the global optimum remains not guaranteed.

[1] In [5] γ_i represents the ratio between the power delivered to the ith user and the interference generated to the other users by the weight vector $\boldsymbol{w}_i$.

5. OPTIMIZATION IN THE PRESENCE OF NOISE

We showed that when the noise is present, the different constraints on the transmit powers make the optimum still leading to the same SINR for all the users in the case of SDMA but not in the case of non-SDMA. Moreover, the noise makes the optimization of the direction vectors $\{\boldsymbol{u}_i\}$ a set of coupled problems that does not allow an analytical approach to find a solution.
Although one can observe that the criterion (16) for a given $\boldsymbol{p}$ still leads to the same optimization problem for the direction vectors $\{\boldsymbol{u}_i\}$ as in the noiseless case, nevertheless we shall consider that such criterion does not guarantee the convergence to the global optimum anyway. Therefore, we suggest to compute the vectors $\{\boldsymbol{u}_i\}$ by using the algorithm in Table 1 previously derived in the absence of noise and then optimize the power assignment according to the following criterion.

5.1. Power assignment optimization for SDMA

Assuming a given set $\{\boldsymbol{u}_i\}$ and all the γ_i's the same, the expression (6) can be arranged in order to include the constraint on the transmitted power as follows

$$\boldsymbol{B}\tilde{\boldsymbol{p}} = \gamma^{-1}\boldsymbol{G}\tilde{\boldsymbol{p}} \tag{17}$$

where $\tilde{\boldsymbol{p}} = [\boldsymbol{p}^T \; 1]^T$,

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{A}^T & \boldsymbol{\mu} \\ \boldsymbol{0}_{1\times d} & 0 \end{bmatrix} \qquad \boldsymbol{G} = \begin{bmatrix} \mathbf{I}_d & \boldsymbol{0}_{d\times 1} \\ \boldsymbol{g}^T & -p_{\max} \end{bmatrix}$$

where $\boldsymbol{\mu} = \boldsymbol{D}_c^{-1}\boldsymbol{\nu}$ and $\boldsymbol{g} = [\|\boldsymbol{u}_1\|_2^2 \; \dots \; \|\boldsymbol{u}_d\|_2^2]^T$ is defined in order to have $\boldsymbol{g}^T\boldsymbol{p} = p_{\max}$. Then as in to [1] since $\boldsymbol{G}$ is invertible we have

$$\boldsymbol{E}\tilde{\boldsymbol{p}} = \gamma^{-1}\tilde{\boldsymbol{p}}, \qquad \boldsymbol{E} = \boldsymbol{G}^{-1}\boldsymbol{B} = \begin{bmatrix} \boldsymbol{A}^T & \boldsymbol{\mu} \\ \frac{\boldsymbol{g}^T\boldsymbol{A}^T}{p_{\max}} & \frac{\boldsymbol{g}^T\boldsymbol{\mu}}{p_{\max}} \end{bmatrix} \tag{18}$$

which is a non-negative matrix. Hence according to theorems 1–3, $\tilde{\boldsymbol{p}}$ can only be the eigenvector associated to the maximum eigenvalue of $\boldsymbol{E}$. Further, note that we can always re-scale $\tilde{\boldsymbol{p}}$ in order to make its last element equal to one.

5.2. Power assignment optimization for non-SDMA

In the case of non-SDMA we cannot ensure that the optimum yields the same SINR for all the users. So that the optimization problem cannot be treated via any analytical approach. As a possible sub-optimal solution, given the vectors $\boldsymbol{u}_i$ computed in the absence of noise, we set $\boldsymbol{p} : \|\boldsymbol{p}\|_\infty \le p_{\max}$ in order to satisfy the criterion (4).

6. IMPLEMENTATION ISSUES

We remark that even though the computation of $\boldsymbol{u}_i$ reduces to a set of decoupled problems as in the absence of noise, the computation of $\boldsymbol{p}$ generally results in a coupled problem, unless ZF conditions (11) are satisfied. Hence in a non-SDMA scenario, different base stations need to communicate in order to choose the respective transmit powers. On the contrary this problem is not present in SDMA.
When the noise is present, since the base station cannot estimate the noise variance $\sigma_{v_i}^2$ at each receiver, unless such an estimate is provided by the mobile, the vector $\boldsymbol{\nu}$ cannot be estimated. To remedy this drawback we shall properly define the SNR at the receiver. A possible definition is given by

$$\mathrm{SNR}_i = \frac{p_i}{\nu_i}\lambda_{\max}(\boldsymbol{R}_{ii})$$

for any i. In practice we need

$$\min_i\{\mathrm{SNR}_i\} \ge \mathrm{SNR}_{\min} \tag{19}$$

where $\mathrm{SNR}_{\min}$ is a value necessary at the receiver to work with an outage probability below a specified maximum. Assuming all the users using the same receiver the worst case for the ith user occurs when $p_i = p_{\max}$ while $\nu_i = \nu_{\max} = \|\boldsymbol{\nu}\|_\infty$. Therefore a sufficient condition to satisfy the requirement (19) is given by setting

$$\mathrm{SNR}_{\min} = \frac{p_{\max}}{\nu_{\max}}\min_i\{\lambda_{\max}(\boldsymbol{R}_{ii})\} \tag{20}$$

Given $\mathrm{SNR}_{\min}$ and $p_{\max}$, $\nu_{\max}$ can be derived. Then setting $\nu_i = \nu_{\max}$ for all the i's the condition (19) is satisfied. Finally, note that for $p_{\max} \to \infty$ the optimum solution is the one in the absence of noise, for any $\nu_{\max} \neq 0$.

7. SIMULATIONS

In this section we consider an SDMA scenario in the presence of $d = 3$ co-channel users in the absence of channel delay spread but in the presence of angular spread due to multipath propagation. The number of all the paths of all the users is 14 (4 for the first two users and 6 for the third, respectively). An antenna array with $m = 8$ or $m = 6$ elements is assumed at the base station. Then ZF conditions (11) cannot be applied.
Figures 1 and 2 show the convergence curves of the first and the second proposed algorithms, for $m = 8$ and $m = 6$ respectively, in the presence of the same user scenario. As expected the second algorithm performs worse when not near the optimum. The variant proposed by Gerlach [5] in several cases makes the second algorithm perform more closely to the first one, but at the cost of an increased computational complexity and a lower convergence rate.

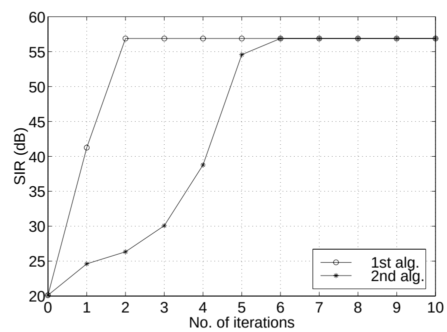

Figure 1: Convergence curves for the two proposed algorithms, $d = 3$ and $m = 8$

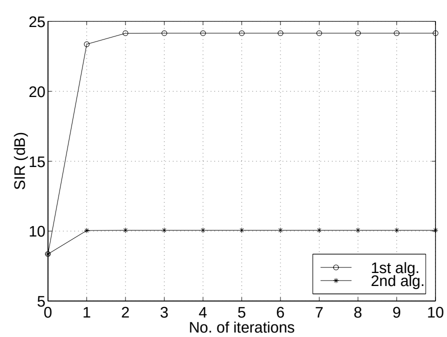

Figure 2: Convergence curves for the two proposed algorithms, $d = 3$ and $m = 6$

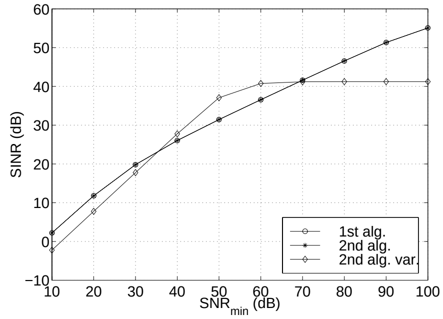

Figure 3: Optimum SINR vs. $\mathrm{SNR}_{\min}$ for the 1st, the 2nd and the variant in [5] of 2nd algorithm, for $d = 3$ and and $m = 8$

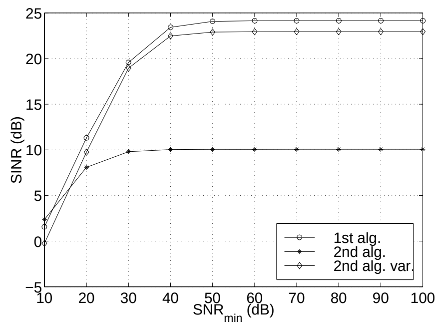

Figure 4: Optimum SINR vs. $\mathrm{SNR}_{\min}$ for the 1st, the 2nd and the variant in [5] of 2nd algorithm, for $d = 3$ and $m = 6$

Finally figures 3 and 4 show the performances of the two proposed algorithms and the Gerlach's variant of the second algorithm in the presence of noise, assuming the criterion (18) for the optimization of the power assignment. Once again we remark that none of the previous algorithms ensures the convergence to the global optimum in the presence of noise. That explains why the second algorithm variant (criterion (16), can perform better than the first one, as shown in figure 3.

8. RELATION TO PREVIOUS APPROACHES

Some similarities among the approaches proposed here and other ones already described in the literature have already been pointed out. Several approaches [2]–[6] assumed the cost function given by the SIR at the transmitter instead of at the receiver, i.e., SIR_i was defined as the the ratio between the power delivered to the ith user and the interference generated to the other co-channel users by the weight vector $\boldsymbol{w}_i$. That cost function was both optimized user by user [2, 3, 4, 6], without assuming any multiuser power assignment optimization, leading to the same criterion as (15), and considering the multiuser power assignment optimization [5]. Other authors [7] considered the possibility of applying ZF conditions to only the dominant path of each user.
A final remark concerns the power assignment optimization presented in [1]. Indeed in that paper the authors considered only the spatial signatures of the users instead of the channel covariance matrices for the formulation of the problem (18). Hence, in the presence of channel delay spread the formulation in [1] is not appropriate and does not lead to the optimization of the SINR of all the users.

9. CONCLUSIONS

In this paper we addressed the problem of the optimization of the MFB with respect to the beamforming weights at base stations, for multiuser downlink transmission in both non-SDMA and SDMA spectrum reuse techniques. A general problem formulation yielded the definition of a proper cost function to be minimized. Then we considered the optimization problem in the absence of noise. In that case the ZF solution represents the optimum, but it can be achieved only under certain conditions usually not verified in practice. Therefore a novel algorithm to find the global optimum in the absence of noise has been derived by an analytical approach which does not require the conditions necessary to the ZF solution. The novel algorithm has shown to outperform other sub-optimal algorithms based on different optimization criteria, in terms of optimum MFB, convergence rate, numerical robustness and computational complexity. We remarked that the latter sub-optimal are very similar to other ones already proposed in the literature [3, 4, 5], generally derived starting from different cost functions.
The optimization problem in the presence of noise has also been addressed, considering the constraints on the transmit power inherent to both SDMA and non-SDMA spectrum reuse techniques. At present, we are not aware of any analytical approach to find a global optimum in that case and we are currently investigating non-analytical optimization approaches. Only the assignment of the transmitted powers [1] can be easily optimized in the SDMA case for a given set of direction vectors. So that for that aim we propose to use the set of direction vectors optimized in the absence of noise through the algorithm we described in Table 1. Finally we analyzed via simulation how optimum and sub-optimum algorithms derived in the absence of noise, perform in the presence of noise with the appropriate power assignment optimization.

10. REFERENCES

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