Foresighted Delayed CSIT Feedback for Finite Rate of Innovation Channel Models and Attainable NetDoFs of the MIMO Interference Channel

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***Abstract*—Channel State Information at the Transmitter (CSIT) is of utmost importance in multi-user wireless networks, in which transmission rates at high SNR are characterized by Degrees of Freedom (DoF, the rate prelog). In recent years, a number of ingenious techniques have been proposed to deal with delayed and imperfect CSIT. In this paper we consider Finite Rate of Information (FRoI) channel models (CM) introduced earlier, which captures the DoF of the channel coefficient time series. Both the block fading model and the stationary bandlimited channel model are special cases of the FRoI CM. However, the fact that FRoI CMs model stationary channel evolutions allows to exploit one more dimension: arbitrary time shifts. In this way, the FroI CM allows Foresighted Channel Feedback (FCFB) which provides CSIT at all times, even in the presence of CSIT FB delay, by increasing the FB rate. DoF-optimal schemes in the presence of perfect CSIT then maintain optimal DoF when combined with FCFB. This is applicable to any multi-user network. In this paper we analyze NetDoFs, which account for training overhead and FB DoF consumption on the reverse link. We provide attainable NetDoF expressions in the MIMO Interference Channel (IC) with linear transceivers for the DoF-optimal schemes of Interference Alignment (IA) with FCFB and several simpler schemes. Numerical evaluations show that the attainable NetDoF may be far away from the DoF and that simpler schemes may be close to optimal in terms of NetDoF.**

I. Introduction

In this paper, Tx and Rx denote transmit/transmitter/ transmitting/transmission and receive/receiver/receiving/reception. Interference is undoubtedly the main limiting factor in multi-user wireless communication systems. Tx side or Rx side zero-forcing (ZF) beamforming (BF) or joint Tx/Rx ZF BF (signal space interference alignment (IA)) allow to obtain significant Degrees of Freedom (DoFs). These technique require very good Channel State Information at Tx and Rx (CSIT/CSIR). Especially CSIT is problematic since it requires feedback (FB) which involves delay, which may be substantial if FB Tx is slot based. We shall remark here up front that these observations advocate the design of wireless systems in which the FB delay is made as short as possible. In a TDD system this may be difficult but in a FDD system the FB delay can be made as short as the roundtrip delay! These considerations are independent of the fact that we can find ways to get around FB delay, as we elaborate below, because a reduction in FB delay always leads to improvements (be it in terms of DoF, or NetDoF or at finite SNR).

It came as a surprise that with totally outdated delayed CSIT (DCSIT), the MAT scheme [1] is still able to produce significant DoF gains for multi-antenna transmission compared to TDMA. In the DCSIT setting, (perfect) CSIT is available only after a FB delay T_{fb} (T_{fb} taken as the unit of time in a number of the following schemes). The channel correlation over T_{fb} can be arbitrary, possibly zero. Perfect overall CSIR is assumed (which leads to significant NetDoF reduction due to CSIR distribution overhead [2], [3]). The MISO BC (Broadcast Channel) and IC (Interference Channel) cases of [1] have been extended to some MIMO cases in [4]. Using a sophisticated variation of the MAT scheme, [5] was able to propose an improved scheme for the case where the FB delay T_{fb} is less than the channel coherence time T_c (defined as the inverse of the Doppler bandwidth (BW)). We use FRoI channel models [6] and exploit their approximately stationary character to propose a simple IA scheme based on Foresighted Channel FB (FCFB) for the MIMO IC as was done in [7] for ZF in the MISO BC.

II. Linear Finite Rate of Innovation (FRoI) Channel Models (CM)

The linear FroI channel model was introduced in [6]. The main characteristic of FRoI CMs is that they closely approximate stationary (BL) signals. *This means that if a FRoI CM is a good model, so is an arbitrary time shift of the FRoI model!* This can be exploited to overcome the FB delay as explained in Fig. 1. While the current coherence period is running, as the CSIT acquisition is going to induce a delay of T_{delay}, instead of waiting for the end of the current T_c, we start the next coherence period T_{delay} samples early. This means jumping from the subsampling grid of the FRoI model to the shifted subsampling grid of another instance of the same FRoI model. This involves recalculating the (finite number of past) FRoI parameters for the new grid from the past channel evolution on the old grid, plus a new channel estimate at the start of the T_c on the new grid. In this way the FB (sampling)

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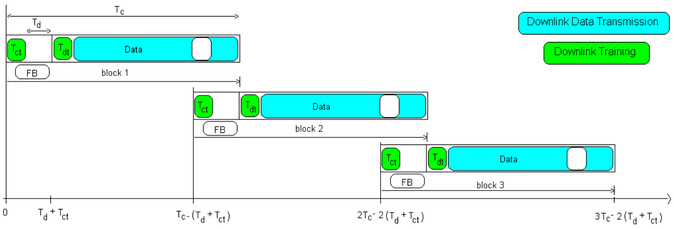

Fig. 1. Foresighted Channel Feedback (FCFB)

"rate" increases from $\frac{1}{T_c}$ to $\frac{1}{T_c - T_{\text{delay}}}$. But the CSIT is available at the Tx all the time, with a channel prediction error SNR proportional to the general SNR.

III. Attainable (sum) NetDoF in MIMO IC

In order to evaluate the performances that can be expected in actual systems we now account for training overhead as well as the DoF consumption due to the feedback on the reverse link. We consider the $(N_t, N_r, d)^K$ MIMO IC in which $d_k = d = \frac{N_t+N_r}{K+1}$ is the number of streams per user (attainable as long as $\min\{N_t, N_r\} \geq 2d$ according to [8]).

For the K Rxs to estimate their channel, a common training of length greater than or equal to N_t per Tx is needed resulting in a total training length $T_{ct} \geq KN_t$. To maximize the DoF we take the minimal $T_{ct} = KN_t$. According to [9], an additional dedicated training of d_k pilot is required in the end, to assure coherent reception at receiver k, resulting in $T_{tr} = KN_t + \sum_k d_k$ symbol periods per block devoted to training in order to perform IA.

Since we are interested in the DoF consumed by the FB, which is the scaling of the FB rate with $\log_2(P)$ as $P \to \infty$, the noise in the fed back channel estimate can be ignored in the case of analog FB or of digital FB of equivalent rate. The FB can then be considered accurate, suffering only from the delay. We consider analog FB and two FB strategies. First, channel feedback (CFB), in which the RXs estimate the channel state from the training sequences and feed back their channel estimate. Second, output feedback (OFB), in which the Rxs directly feed back the training signals they receive and the Txs perform the (downlink) channel estimation. The Txs only need the CSI on the cross links and not on the direct link with their user in order to perform IA. Therefore user k needs to feedback the coefficients of its $K-1$ channels with Tx $i, i \neq k$, i.e.; $(K-1)N_tN_r$ coefficient to feedback per user. The total FB is $K(K-1)N_tN_r$ symbols and consumes $T_{FB} = K(K-1)N_r$ channel uses on the reverse link for both feedback strategies.

The difference between CFB and OFB is the time it takes for the TX to have CSI after the training is done, with CFB it takes $T_{d,CFB} = T_{FB} + T_{fd}$ where T_{fd} is the delay in the feedback due to processing and propagation. With OFB the Rxs do not have to wait for all the training to be done to start the feedback and we have $T_{d,OFB} = \max(T_{FB} + T_{fd} - T_{tr}, T_{fd})$ as it cannot be less than T_{fd}. In order to have only one expression for the netDoF we will use the following notation, T_d, the dead time, the total time between the end of training and the moment CSI becomes available at the Txs, which will be equal to $T_{d,CFB}$ or $T_{d,OFB}$ depending on the FB strategy. The CSIT acquisition delay is then $T_{\text{delay}} = T_d + T_{tr}$.

Note that these FB length values are obtained assuming a distributed model: each Tx gets all the CSI from FB without the need for a central unit, to perform a complete IA Tx/Rx design from which to keep only its own Tx filter. Also each Rx learns the channel information between itself and all Txs so it can compute its signal and interference subspaces without any other overhead.

A. IA_{FCFB}

With FB every $T_c - T_d - T_{tr} = T_c - T_{\text{delay}}$ the Txs always have the current CSI which allows to perform IA without any dead time. The (sum) DoF achieved by IA is $\text{DoF(IA)} = K\frac{N_t+N_r}{K+1}$, together with the augmented frequency of training and FB it results in the following netDoF

$$\text{netDoF}(\text{IA}_{FCFB}) = \text{DoF(IA)}\left(1 - \frac{KN_t + \text{DoF(IA)} + K(K-1)N_r}{T_c - T_{\text{delay}}}\right) \quad (1)$$

as long as $T_c \geq T_{\text{delay}}$.

For sake of comparison we concisely derive the netDoF attained by other schemes in the MIMO IC with delayed CSIT. When FB is done only every T_c, there are always two parts in each block, a first part with outdated CSIT a second part with current CSIT.

B. TDMA

TDMA is the simplest strategy to avoid interferences and does not require CSIT, only one Tx transmits at a time. This reaches a sum DoF of $\min(N_t, N_r)$ and only requires CSIR, that can be obtained by the Rx after a training of $\min(N_t, N_r)$ channel uses thus achieving the following netDoF

$$\text{netDoF(TDMA)} = \min(N_t, N_r)\left(1 - \frac{\min(N_t, N_r)}{T_c}\right) \quad (2)$$

as long as $T_c \geq \min(N_t, N_r)$. (With an optimized number of active antennas we would get $\text{netDoF(TDMA)} = \min(N_t, N_r, T_c/2)$ as in [10].)

C. Classic IA

Waiting when CSIT is not available and performing IA only when CSIT is available achieves the following netDoF

$$\text{netDoF(IA)} = \text{DoF(IA)}\left(1 - \frac{T_d + KN_t + \text{DoF(IA)} + K(K-1)N_r}{T_c}\right) \quad (3)$$

as long as $T_c \geq T_{\text{delay}}$.

D. TDMA-IA

TDMA-IA is a direct extension of IA. The only difference being that while the transmitter is waiting for the CSI, and not sending training symbols it performs TDMA transmission since this does not require any CSIT, thus achieving

$$\text{netDoF(TDMA-IA)} = \text{netDoF(IA)} + \min(N_t, N_r)\frac{T_d}{T_c} \quad (4)$$

as long as $T_c \geq T_{\text{delay}}$.

IV. Numerical Results

In Fig. 2 we plot the netDof obtained by IA$_{FCFB}$, IA, TDMA-IA and TDMA in the MIMO IC with $K = 4$, $N_t = 4$, $N_r = 4$, $T_{fd} = 5$ and OFB, using (1) for IA$_{FCFB}$, (3) for IA, (4) for TDMA-IA and (2) for TDMA. We observe that for small values of T_c simple TDMA transmission performs the best as it necessitates very little overhead. Then for intermediate values of T_c, TDMA-IA is better as the increased overhead cost is compensated by being able to do IA and finally for larger values of T_c, IA$_{FCFB}$ outperforms the other schemes. The slight increase of overhead due to the increase of the training/FB frequency is made up for by having no dead time in the transmission.

A. Optimization of K, N_t and N_r

It was noticed in [11] that the number of active users and antennas in the MISO BC should be optimized. In the MIMO IC the numbers of active cells and active antennas N_t and N_r need to be optimized to find the right channel learning/using compromise because serving more users (or more active antennas) means a larger DoF but also larger overhead. All the net DoF of the schemes we reviewed reach a single maximum as a function of the number of antennas. To each scheme we associate its optimized version, in which the number of active cells K and antennas N_t, N_r are optimized, either analytically or empirically to assure the maximum net DoF.

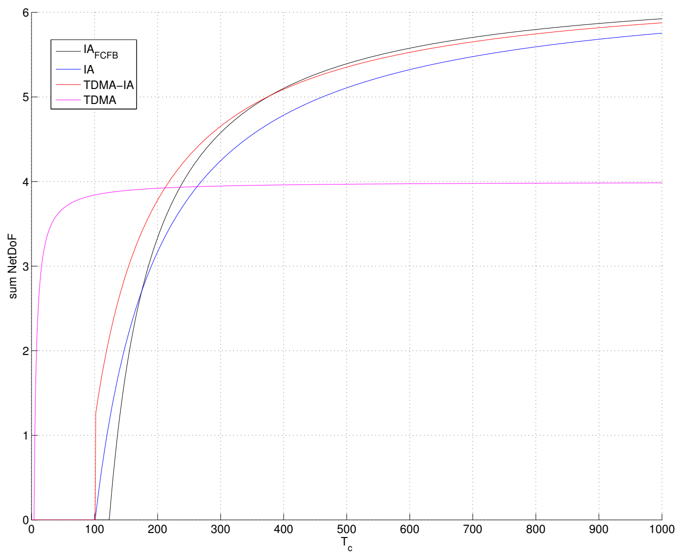

Fig. 2. NetDof attained by IA$_{FCFB}$, IA, TDMA-IA and TDMA in the MIMO IC with $K = 4$, $N_t = 4$, $N_r = 4$, $T_{fd} = 5$ and OFB.

In Fig. 3 we plot the net DoF of all considered schemes and of their optimized version for $K = 4$, $N_t = 6$, $N_r = 4$ and $T_{fd} = 5$ as a function of T_c. We notice that if the optimization results in a gain for all schemes it also confirms that with increasing values of T_c, successively TDMA, TDMA-IA and IA$_{FCFB}$ are the best strategies. TDMA corresponds to IA with K reduced to K = 1.

In fact, we see that one also needs to optimize the feedback sampling rate, with sampling period progressively shrinking from T_c to $T_c - T_{\text{delay}}$, evolving from TDMA-IA to IA$_{FCFB}$.

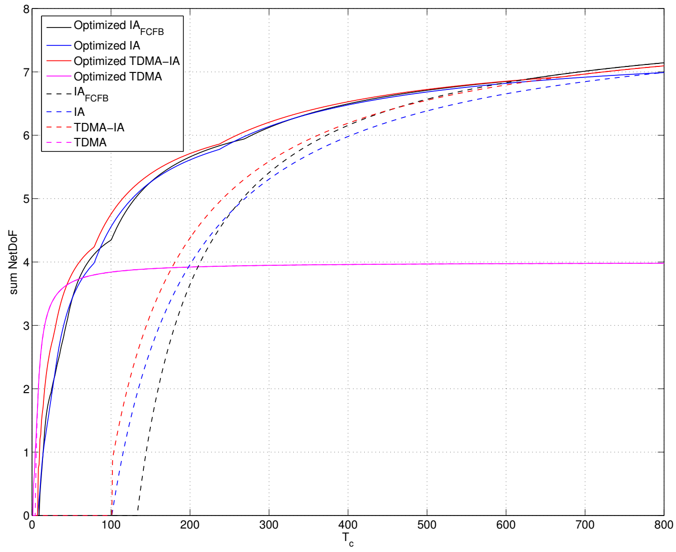

Fig. 3. NetDof attained by IA$_{FCFB}$, IA, TDMA-IA, TDMA and their optimized versions in the MIMO IC with $K = 4$, $N_t = 6$, $N_r = 4$, $T_{fd} = 5$ and OFB.

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