DETERMINISTIC QUADRATIC SEMI-BLIND FIR MULTICHANNEL ESTIMATION: ALGORITHMS AND PERFORMANCE

Elisabeth de Carvalho

Stanford University
Stanford, CA 94305-9515, USA
carvalho@dsl.stanford.edu

Dirk Slock

Institut Eurecom, B.P. 193
06904 Sophia Antipolis Cedex, FRANCE
slock@eurecom.fr

ABSTRACT

The purpose of semi-blind channel identification methods is to exploit the information used by blind methods and the information coming from known symbols. The main focus of this paper is the study of deterministic quadratic semi–blind algorithms which are of particular interest because of their low computational complexity. The associated criteria are formed as a linear combination of a blind and a training sequence based criterion. Through the examples of Subchannel Response Matching and Subspace Fitting based semi–blind criteria, we study how to construct properly such semi–blind criteria and how to choose the weights of the linear combination. We provide a performance study for these algorithms and give theoretical conditions for the semi–blind performance to be independent of the weights.

1. INTRODUCTION

Different ways of building semi–blind criteria are possible. Optimal semi–blind methods can take into account the knowledge of symbols even arbitrarily dispersed in the burst: in [1], we proposed optimal methods based on Maximum Likelihood (ML). This symbol configuration is in general undesirable as the associated semi–blind criteria will require computationally demanding algorithms because the structure of the blind problem is lost.

For grouped known symbols (training sequence) computionally low solutions can be built because the structure of the blind problem is kept. By neglecting some information about the known or unknown symbols, ML easily allows one to construct semi–blind criteria that are a linear combination of a blind and a training sequence based criterion, with optimal weights in the ML sense.

A third way to build semi–blind criteria is to linearly combine a given blind criterion and a TS based criterion. We study here the Subchannel Response Matching (SRM) and Subspace Fitting (SF) based semi–blind criteria which are of particular interest because they are quadratic and represent closed formed solutions; semi–blind SRM in particular has a low complexity. We show that the blind SRM criterion needs first to be denoised and then correctly combined to the TS criterion. In [2], the SRM case is treated but the criterion is neither denoised nor correctly weighted. The SF case is treated in [3, 4]. In [4], the optimal weights are found by optimizing the theoretical expression for the performance, which represents an increase in complexity. Here, we provide a simple solution to find the correct weights.

At last, we provide a performance study of deterministic quadratic semi–blind criteria. We especially characterize their performance in the case of an asymptotic number of unknown symbols M_U and finite number of known symbols M_K. In [4], a similar study was conducted for $M_K \ll M_U$, but with $M_K \to \infty$, which does not allow to draw the conclusions we get. Here, we give theoretically founded conditions for the semi–blind criteria performance to be independent of the weights of the linear combination.

2. PROBLEM FORMULATION

Consider a sequence of symbols $a(k)$ received through m channels of length N with coefficients $\boldsymbol{h}(i)$:

$$\boldsymbol{y}(k) = \sum_{i=0}^{N-1} \boldsymbol{h}(i) a(k-i) + \boldsymbol{v}(k), \quad (1)$$

$\boldsymbol{v}(k)$ is an additive independent white Gaussian noise and $r_{\boldsymbol{vv}}(k-i) = \mathrm{E}\, \boldsymbol{v}(k)\boldsymbol{v}(i)^H = \sigma_v^2 I_m\, \delta_{ki}$. Assume we receive M samples, concatenated in the vector $\boldsymbol{Y}_M(k)$:

$$\boldsymbol{Y}_M(k) = \mathcal{T}_M(h)\, A_M(k) + \boldsymbol{V}_M(k) \quad (2)$$

$\boldsymbol{Y}_M(k) = [\boldsymbol{y}^T(k-M+1) \cdots \boldsymbol{y}^T(k)]^T$, similarly for $\boldsymbol{V}_M(k)$, and $A_M(k) = [a(k-M-N+2) \cdots a(k)]^T$. $(.)^T$ denotes transpose and $(.)^H$ hermitian transpose. The channel transfer function is $\mathbf{H}(z) = \sum_{i=0}^{N-1} \boldsymbol{h}(i) z^{-i} = [\mathrm{H}_1^T(z) \cdots \mathrm{H}_m^T(z)]^T$. $\mathcal{T}_M(h)$ is a block Toeplitz matrix filled out with the channel coefficients grouped in $h = [\boldsymbol{h}^T(N-1) \cdots \boldsymbol{h}^T(0)]^T$. We shall simplify the notation in (2) with $k = M-1$ to:

$$\boldsymbol{Y} \;=\; \mathcal{T}(h)\, A + \boldsymbol{V}\;. \quad (3)$$

The space spanned by the columns of $\mathcal{T}(h)$ will be called signal subspace and its orthogonal complement, the noise subspace. We assume that a training sequence A_{TS} located at the beginning of the burst is present in the input burst. $\boldsymbol{Y}_{TS} = \mathcal{T}_{TS}(h) A_{TS} + V_{TS} = \mathcal{A}_{TS} h + \boldsymbol{V}_{TS}$ is the portion of the output burst containing only known symbols. $\boldsymbol{Y}_B = \mathcal{T}_B(h) A_B + V_B$ is the rest of the output burst (it contains the unknown symbols but also some known symbols which will be treated as unknown in the paper).

3. SEMI–BLIND SRM

SRM [5] is based on a linear parameterization $\mathbf{H}^{\perp}(z)$ of the noise subspace which satisfies $\mathcal{T}(h^{\perp})\mathcal{T}(h) = 0$ where $\mathcal{T}(h^{\perp})$ is the convolution matrix built from $\mathbf{H}^{\perp}(z)$ and spans the entire noise subspace. For $m = 2$, $\mathbf{H}^{\perp}(z) = [-\mathrm{H}_2(z) \;\; \mathrm{H}_1(z)]$. For $m > 2$ [6],

$$\mathbf{H}^{\perp}(z) = \begin{bmatrix} -\mathrm{H}_2(z) & \mathrm{H}_1(z) & 0 & \cdots & 0 \\ 0 & -\mathrm{H}_3(z) & \mathrm{H}_2(z) & \cdots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ \mathrm{H}_m(z) & 0 & \cdots & 0 & -\mathrm{H}_1(z) \end{bmatrix}. \tag{4}$$

In the noise free case, $\mathcal{T}(h^{\perp})\boldsymbol{Y}\;(= \mathcal{T}(h^{\perp})\mathcal{T}(h)A) = 0$. Using the commutativity of convolution $\mathcal{T}(h^{\perp})\boldsymbol{Y} = \mathcal{Y}h$, where $\mathcal{Y}$ is a structured matrix filled out with the elements of $\boldsymbol{Y}$: the channel coefficients can be identified uniquely from this equation as the minimal left eigenvector of $\mathcal{Y}$. When the received signal is noisy, h is obtained by solving the least–squares quadratic criterion $\min_{\|h\|=1} \|\mathcal{Y}h\|^2$.

Consider the following semi–blind cost function:

$$\alpha\, h^H \mathcal{Y}_B^H \mathcal{Y}_B h + \|\boldsymbol{Y}_{TS} - \mathcal{T}_{TS}(h) A_{TS}\|^2 . \tag{5}$$

Note that other TS based criteria can also be considered [7]. An intuitive way to weigh both TS and blind parts is to associate them with the number of data they are built from, as suggested in [3] for the semi–blind subspace fitting. In the SRM case, the optimal α would then be equal to 1. In figure 1, we show the NMSE for the channel averaged over 100 Monte–Carlo realizations of the channel (with i.i.d. coefficients) the noise and the input symbols. The NMSE is plotted w.r.t. the value of α in dotted lines. For $\alpha = 1$, semi–blind SRM gives worse performance than TS estimation.

The blind SRM criterion gives unbiased estimates only under a norm constraint for the channel [6]. As the semi–blind criterion is optimized without constraints, the blind SRM part gives biased estimates which renders the performance of the semi–blind algorithm poor. For the criterion to be unbiased, the term $\mathcal{Y}_B^H \mathcal{Y}_B$ needs to be denoised. Asymptotically in the number of data, $\mathcal{Y}_B^H \mathcal{Y}_B \to \mathcal{X}_B^H \mathcal{X}_B + \mathrm{E}\mathcal{V}_B^H \mathcal{V}_B$, where $\mathcal{X}_B$ is built from the noise free signal $\mathcal{T}_B(h) A_B$ and $\mathcal{V}_B$ from the noise $\boldsymbol{V}_B$; $\mathrm{E}\mathcal{V}_B^H \mathcal{V}_B \to \beta\sigma_v^2$ where β is a constant. The minimal eigenvalue of $\mathcal{Y}_B^H \mathcal{Y}_B$, $\lambda_{min}(\mathcal{Y}_B^H \mathcal{Y}_B)$, is an estimate of $\beta\sigma_v^2$ and the quantity $\mathcal{Y}_B^H \mathcal{Y}_B - \lambda_{min}(\mathcal{Y}_B^H \mathcal{Y}_B) I$ is the denoised SRM Hessian. Once the criterion is denoised, the choice for the constant α remains unsolved. To find it, we refer to semi–blind Deterministic ML for h [7]. The blind DML for h is:

$$\min_h \boldsymbol{Y}^H P^{\perp}_{\mathcal{T}(h)} \boldsymbol{Y} \tag{6}$$

where P_X is the orthogonal projection on the columns of X and $P_X^{\perp} = I - P_X$ is the projection on the orthogonal complement of X. By using the linear parameterization of the noise subspace, (6) becomes:

$$\begin{aligned} &\min_h \boldsymbol{Y}^H \mathcal{T}^H(h^{\perp}) \left(\mathcal{T}(h^{\perp})\mathcal{T}^H(h^{\perp})\right)^+ \mathcal{T}(h^{\perp})\boldsymbol{Y} \\ &\Leftrightarrow \min_h h^H \mathcal{Y}^H R^+(h) \mathcal{Y} h \end{aligned} \tag{7}$$

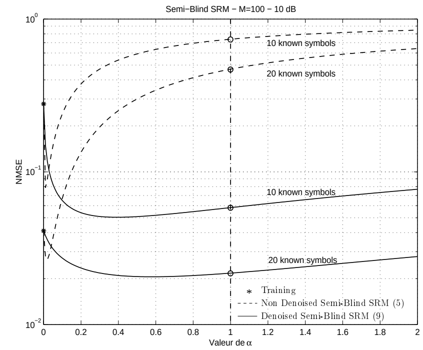

Figure 1: Semi–blind SRM built as a linear combination of blind SRM and TS based criteria.

Note that SRM appears as a non–weighted version of DML. In [7], the following semi–blind criterion based on DML was proposed:

$$h^H \mathcal{Y}_B^H \mathcal{R}_B^+(h) \mathcal{Y}_B h + \|\boldsymbol{Y}_{TS} - \mathcal{T}_{TS}(h) A_{TS}\|^2 . \tag{8}$$

Semi–blind DML gives the optimal weights between the blind and TS part: we build now semi–blind SRM as an approximation of DML. We approximate $\mathcal{R}_B(h)$ as a multiple of the identity matrix with multiple equal to the mean of the diagonal elements, *i.e.* $\frac{m}{2}\|h\|^2$. The norm of the channel can be estimated thanks to an estimate of the denoised second–order moment of a data sample $\mathrm{tr}(r_{yy}(0)) = \sigma_a^2 \|h\|^2$. After denoising, the semi–blind SRM criterion is:

$$\begin{aligned} \min_h \Big\{ \frac{2}{m} \frac{1}{\widehat{\|h\|^2}} \; h^H \left(\mathcal{Y}_B^H \mathcal{Y}_B - \lambda_{min}(\mathcal{Y}_B^H \mathcal{Y}_B)\right) h + \\ \|\boldsymbol{Y}_{TS} - \mathcal{T}_{TS}(h) A_{TS}\|^2 \Big\} . \end{aligned} \tag{9}$$

It can be shown that replacing $\|h\|^2$ by a consistent estimate does not change the asymptotic performance. This algorithm could also be interpreted as an approximation of the semi–blind Denoised IQML algorithm presented in [7]. In figure 1, in solid lines, we show the performance of the corrected semi–blind criterion. The scalar α scales the blind part in (9). The value $\alpha = 1$ gives approximately the optimal performance and in the neighborhood of $\alpha = 1$, the performance hardly depends on the value of α.

4. SEMI–BLIND SUBSPACE FITTING

Consider the received signal covariance matrix of length L: $R_{Y_L Y_L} = \sigma_a^2 \mathcal{T}_L(h) \mathcal{T}_L^H(h) + \sigma_v^2 I$. Let $R_{Y_L Y_L} = V_{\mathcal{S}} \Lambda_{\mathcal{S}} V_{\mathcal{S}}^H + V_{\mathcal{N}} \Lambda_{\mathcal{N}} V_{\mathcal{N}}^H$ be the eigendecomposition. $\Lambda_{\mathcal{S}}$ groups the $L+N-1$ (dimension of the signal subspace) largest eigenvalues of $R_{Y_L Y_L}$ and $\Lambda_{\mathcal{N}} = \sigma_v^2 I$ the smallest ones. The columns of $V_{\mathcal{S}}$ span the signal subspace and the columns of $V_{\mathcal{N}}$ the noise subspace. Signal Subspace Fitting (SSF) fits the column space of $\mathcal{T}(h)$ to that of $\mathcal{V}_S$ through the quadratic criterion:

$$\min_{\|h\|^2=1} \|P_{\widehat{\mathcal{V}}_{\mathcal{N}}} \mathcal{T}(h)\|^2 \Leftrightarrow \min_{\|h\|^2=1} h^H \mathcal{S}^H \mathcal{S}\, h \tag{10}$$

where $\widehat{\mathcal{V}}_{\mathcal{N}}$ is an estimate of $\mathcal{V}_{\mathcal{N}}$ obtained from the sample covariance matrix.

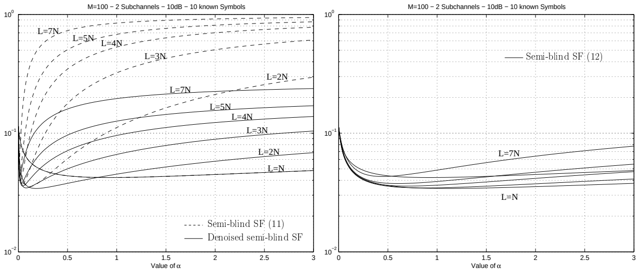

Figure 2: Semi–blind SF built as a linear combination of blind SF and TS based criteria.

Consider now the following semi–blind cost function:

$$\alpha M_U \; h^H \mathcal{S}_B^H \mathcal{S}_B \; h + \|\boldsymbol{Y}_{TS} - \mathcal{T}_{TS}(h) A_{TS}\|^2 \,. \qquad (11)$$

In [3], αM_U was chosen equal to the number of data on which the blind criterion is based, *i.e.* $\alpha = 1$. In figure 2 (left, dashed lines), we plot the NMSE of channel estimation w.r.t. α for different sizes L, for SNR=10dB, 10 known symbols. For $L = N$ (the solid line and dotted line are superposed), the semi–blind criterion is relatively insensitive to the value of α. For L larger than N however, it is visibly very sensitive to its value. The choice $\alpha = 1$ gives performance worse than that of training sequence based estimation for $L > N$. These simulations suggest that the linearly combined semi–blind algorithm is sensitive to the dimension of the noise subspace which varies when L varies.

We propose to "denoise" the blind part of the SF criterion, *i.e.* we force the smallest eigenvalue of $\mathcal{S}_B^H \mathcal{S}_B$ to 0 via $\mathcal{S}_B^H \mathcal{S}_B - \lambda_{min}(\mathcal{S}_B^H \mathcal{S}_B) I$. Note that theoretically we do not remove any noise contribution here, but clean the matrix $\mathcal{S}_B^H \mathcal{S}_B$. This strategy will be later discussed in section 5. The performance of the denoised semi–blind SF criterion w.r.t. α for 500 Monte–Carlo realizations of the channel, noise and input symbols are shown in figure 2 (left) in solid lines. We notice the significant effect of the denoising on the semi–blind algorithm performance, but this algorithm still does not give sufficiently good performance for $\alpha = 1$.

We propose here to scale the blind part of the semi–blind SF criterion by the dimension $\mathcal{N}$ of the noise subspace. The resulting criterion is:

$$\min_h \left\{ \alpha \frac{M_U}{\mathcal{N}} h^H \left(\mathcal{S}_B^H \mathcal{S}_B - \hat{\lambda}_{min} I \right) h + \|\boldsymbol{Y}_{TS} - \mathcal{T}_{TS}(h) A_{TS}\|^2 \right\} . \qquad (12)$$

In figure 2 (right), we show the performance of (12) in the case of random channels with 2 subchannels: the semi–blind SF criterion (12) gives satisfactory results for $\alpha = 1$.

5. PERFORMANCE STUDY

Consider the general semi–blind quadratic criterion:

$$\min_h \left\{ \alpha M_U h^H \widehat{Q}_B h + \|\boldsymbol{Y}_{TS} - \mathcal{T}_{TS}(h) A_{TS}\|^2 \right\} \,. \qquad (13)$$

$\widehat{Q}_B = \frac{2}{m\|h\|^2} \frac{1}{M_U} \left[\mathcal{Y}_B^H \mathcal{Y}_B - \lambda_{min}(\mathcal{Y}_B^H \mathcal{Y}_B) I \right]$ for semi–blind SRM and $\widehat{Q}_B = \frac{1}{\mathcal{N}} \left[\mathcal{S}_B^H \mathcal{S}_B - \hat{\lambda}_{min}(\mathcal{S}_B^H \mathcal{S}_B) I \right]$ for semi–blind SF. When the channel has $N_c - 1$ zeros (*e.g.* due to channel length overestimation), $\widehat{Q}_B$ tends asymptotically to Q_B which has N_c null eigenvalues. Let $\widehat{Q}_B = \widehat{\mathcal{W}}_1 \widehat{\Lambda}_1 \widehat{\mathcal{W}}_1^H + \widehat{\mathcal{W}}_2 \widehat{\Lambda}_2 \widehat{\mathcal{W}}_2^H$ be the eigencomposition of $\widehat{Q}_B$, $\widehat{\Lambda}_2 \to 0$ asymptotically. Note that the smallest eigenvalue of $\widehat{Q}_B$ is 0 for the semi–blind criteria proposed. One can show that the elements of $\widehat{\Lambda}_2$ (not exactly equal to 0) are of order $1/M_U$ for both SRM and SF.

The solution of (13) is:

$$\hat{h} = \left(\alpha M_U \widehat{Q}_B + \mathcal{A}_{TS}^H \mathcal{A}_{TS} \right)^{-1} \mathcal{A}_{TS}^H \boldsymbol{Y}_{TS} \qquad (14)$$

We study the performance of the semi–blind criterion (13) for the asymptotic cases:

- M_U and M_K infinite, with condition $\frac{\sqrt{M_U}}{M_K} \to 0$, which accounts for the fact that the TS part of the criterion should not be negligible w.r.t. the blind part [8].
- M_U infinite, M_K finite.

A similar analysis exists in [4] for SF where the last asymptotic condition is $M_K \ll M_U$, with M_K infinite however.

5.1. M_U and M_K infinite

In that case, it can be shown as in [8] that:

$$C_{\Delta h \Delta h} = \left(\alpha M_U Q_B + \mathcal{A}_{TS}^H \mathcal{A}_{TS} \right)^{-1} \left(\alpha^2 M_U^2 \mathrm{E}(\widehat{Q}_B h^o h^{oH} \widehat{Q}_B^H) + \sigma_v^2 \mathcal{A}_{TS}^H \mathcal{A}_{TS} \right) \left(\alpha M_U Q_B + \mathcal{A}_{TS}^H \mathcal{A}_{TS} \right)^{-1} . \qquad (15)$$

We do not provide here expressions for $\mathrm{E}(\widehat{Q}_B h^o h^{oH} \widehat{Q}_B^H)$ for lack of space. The performance in that case depends on the value of α. In order to optimize the performance, it would be necessary to find the optimal α, which should be avoided, because of the additional computational cost.

5.2. M_U infinite, M_K finite

Let $\mathcal{R}^{1/2} \mathcal{R}^{H/2}$ be the Cholesky decomposition of $\mathcal{A}_{TS}^H \mathcal{A}_{TS}$.

$$\left(\alpha M_U \widehat{Q}_B + \mathcal{A}_{TS}^H \mathcal{A}_{TS} \right)^{-1} = \mathcal{R}^{-H/2} \left(\alpha M_U \mathcal{R}^{-1/2} \widehat{Q}_B \mathcal{R}^{-H/2} + I \right)^{-1} \mathcal{R}^{-1/2} . \qquad (16)$$

Let the eigendecomposition of $\mathcal{R}^{-1/2} \widehat{Q}_B \mathcal{R}^{-H/2} = \widehat{Q}'_B$ be:

$$\widehat{Q}'_B = \widehat{\mathcal{W}}' \widehat{\Lambda}' \widehat{\mathcal{W}}'^H = \widehat{\mathcal{W}}'_1 \widehat{\Lambda}'_1 \widehat{\mathcal{W}}'^H_1 + \widehat{\mathcal{W}}'_2 \widehat{\Lambda}'_2 \widehat{\mathcal{W}}'^H_2 \,. \qquad (17)$$

$\widehat{\Lambda}'_2 \to 0$ asymptotically in the number of data.

$$\left(\alpha M_U \mathcal{R}^{-1/2} \widehat{Q}_B \mathcal{R}^{-H/2} + I \right)^{-1} = \widehat{\mathcal{W}}'_1 \left(\alpha M_U \widehat{\Lambda}'_1 + I \right)^{-1} \widehat{\mathcal{W}}'^H_1 + \widehat{\mathcal{W}}'_2 \left(\alpha M_U \widehat{\Lambda}'_2 + I \right)^{-1} \widehat{\mathcal{W}}'^H_2 = \widehat{\mathcal{W}}'_2 \left(\alpha M_U \widehat{\Lambda}'_2 + I \right)^{-1} \widehat{\mathcal{W}}'^H_2 \qquad (18)$$

at first order in $1/\sqrt{M_U}$. When $\widehat{\Lambda}'_2 \neq 0$, as the non–zero element of $\widehat{\Lambda}'_2$ are of order $1/M_U$, $\alpha M_U \widehat{\Lambda}'_2$ is of the same order as I, and the performance depends on α. When $\widehat{\Lambda}'_2 = 0$, $\widehat{\mathcal{W}}'_2 \left(\alpha M_U \widehat{\Lambda}'_2 + I \right)^{-1} \widehat{\mathcal{W}}'^H_2 = \widehat{\mathcal{W}}'_2 \widehat{\mathcal{W}}'^H_2$ and we can show that the performance are independent of α, and at first order in $1/M_U$, the covariance matrix of the estimation error $\Delta h = \hat{h} - h^o$ (h^o is the true value of the the channel) is:

$$C_{\Delta h \Delta h} = \sigma_v^2 \mathcal{W}_2 \left(\mathcal{W}_2^H \mathcal{A}_{TS}^H \mathcal{A}_{TS} \mathcal{W}_2 \right)^{-1} \mathcal{W}_2^H \qquad (19)$$

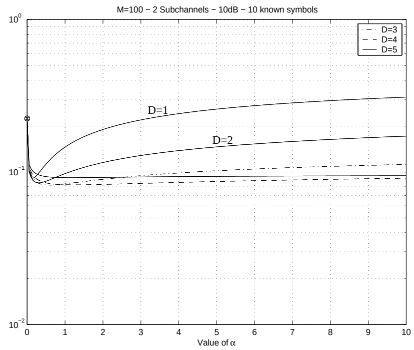

Figure 3: Semi–blind SRM: D=1–5 eigenvalues forced to 0.

This expression can be interpreted as the performance of the estimation of the zeros of the channel by training sequence with perfect knowledge of the irreducible part of the channel.

For $\widehat{\Lambda}'_2 = 0$, when considering expression (15), with M_K finite, we find expression (19): the asymptotic expression (15) in M_K and M_U is valid when M_K is finite. So there is a continuity between both expressions (15) and (19) and expression (15) is valid in any case and should be used to characterize the performance of the semi–blind criteria. This is not true when the N_c smallest eigenvalues of $\widehat{Q}_B$ are not exactly equal to 0: there is a discontinuity in the expressions, and both analyses do not coincide. In general, in this last case, it may not be obvious to know which analysis between M_K infinite or M_K finite is appropriate.

In practice the performance for small M_K is not constant w.r.t. α even when $\widehat{\Lambda}'_2 = 0$. For a randomly chosen channel, in general, the matrix $\widehat{Q}_B$ exhibits more than N_c "small" eigenvalues (the extra small eigenvalues are sufficiently small to influence the performance). We take the example of semi–blind SRM. To force n smallest eigenvalues to 0, we consider the quantity $\mathcal{Y}_B^H \mathcal{Y}_B - \lambda_n I$, where λ_n is the largest of the n smallest eigenvalues of $\mathcal{Y}_B^H \mathcal{Y}_B$, and force the negative eigenvalues to 0. For randomly chosen channels of length 5 (so a priori irreducible channels) we force 1 to 5 eigenvalues to zero: in figure 3, we show the resulting NMSE for 100 runs of the channel, noise and input symbols. For a number of 3 to 5 eigenvalues forced to 0, we see that the performance are not dependent on the value of α. The proposed DML, SRM and SF based algorithms force to zero only 1 eigenvalue and as already stated have their performance dependent on α. However the algorithms were constructed (weighted) such that the optimal α is approximately equal to 1: this solution is preferable because it is less complex. This analysis shows us the importance of the eigenvalues of $\widehat{Q}_B$ in the behavior of the semi–blind algorithms.

5.3. Optimal Weighted

Blind SRM and SF criteria are of the form $\min_h h^H \mathcal{U}_B^H \mathcal{U}_B h$. The optimally weighted version is $\min_h h^H \mathcal{U}_B^H \mathcal{W}_B^+(h) \mathcal{U}_B h$ with $\mathcal{W}_B^+(h) = \mathrm{E}\,\mathcal{U}_B h^o h^{oH} \mathcal{U}_B^H$ and gives better performance than the non–weighted version. When M_K and M_U are considered infinite, it can be easily shown that the optimally weighted semi–blind criterion is:

$$\min_h \left\{ h^H \mathcal{U}_B^H \mathcal{W}_B^+ \mathcal{U}_B h + \frac{1}{\sigma_v^2} \| \boldsymbol{Y}_{TS} - \mathcal{T}_{TS}(h) A_{TS} \|^2 \right\} \quad (20)$$

For these asymptotic conditions, finding the right scale factor α to optimize the performance of the non–weighted part of the criterion is difficult as w mentioned in section 5.1, however finding the right weighting matrix is easier.

In fact, semi–blind DML (6) is built as an optimally weighted combination of the blind and TS criteria and SRM can be seen as an approximation of this weighted criterion. We have not tested the weighted version of the SF criterion: the introduction of $\mathcal{N}$ may perhaps be explained by the blind weighting matrix.

6. CONCLUSION

In this paper, we have presented two quadratic semi–blind channel estimation criteria based on a linear combination of a blind and a TS criterion. We have seen that it may be difficult to construct a semi–blind criterion this way. A performance analysis has shown the importance of the small eigenvalues of the Hessian of the blind part of the criterion. We have studied conditions for the semi–blind performance to be independent from the weights associated to the blind and TS parts.

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