Optimizing Feedback in Energy Harvesting MISO Communication Channels

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***Abstract*—In this work,[1] we consider the optimization of feedback in a point-to-point MISO channel with an energy harvesting (EH) receiver (RX). The RX is interested in feeding back the channel state to the transmitter (TX) to help improve the transmission rate, yet must spend the harvested energy wisely to do so. The objective is to maximize the throughput by a deadline, subject to EH constraints at the RX. The throughput metric considered is an upper bound on the ergodic capacity of beamforming with limited feedback. The optimization problem is shown to be concave and a simple algorithm for obtaining the optimal feedback bit allocation policy is devised. Numerical results show that the optimal feedback policy obtained for the modified problem outperforms the naive scheme for the original problem.**

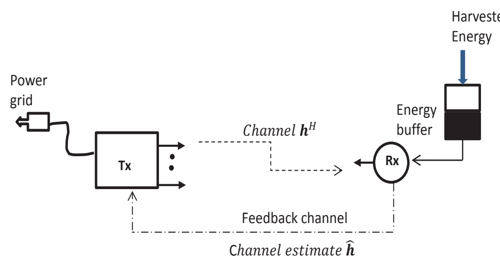

Fig. 1. System model.

I. Introduction

In traditional wireless networks, nodes get their energy from the power grid by always or periodically connecting to it. While it is easy to connect the terminals to the grid in some networks, in others, such as sensor networks, it cannot be done once after the deployment. Therefore, in such networks a node's lifetime and hence the network lifetime is constrained by the limited initial energy in the battery. One way to alleviate this problem is to provide the nodes with EH capabilities [1]. An EH node can scavenge energy from the environment (typical sources are solar, wind, thermal, etc.) [2]. With EH nodes in the network, in principle one can get perpetual lifetime without the need of replacing batteries.

However, EH poses a new design challenge as the energy sources are typically sporadic and random. The main challenge lies in ensuring Quality of Service (QoS) constraints of the network given the random and time varying energy sources. In [3], the authors consider a point-to-point fading channel with an EH transmitter (TX) and formulate the following problems: maximization of the throughput by a deadline and minimization of the transmission completion time. This approach has been extended to the broadcast channel [4], [5], relay channel [6], and imperfect battery [7] scenarios. See [8] for a more extensive overview.

A common aspect of most works in EH communication networks is that the TX is provided with perfect channel state information (CSI). However, recent studies have demonstrated that feedback resources (although feedback enhances the system performance) are limited and must be spent wisely [9]. This problem is particularly relevant in multiple antenna aided transmission systems. In such systems, although more accurate CSI feedback increases the throughput, it consumes more energy at the RX. As a result, an important question arises: How do the EH constraints affect the design of feedback enabled wireless networks?

In this paper we introduce the problem of feedback design with EH constraints in the context of a simple multiple antenna system, namely MISO channel, where feedback can be used to improve the rate through array gain. First, we consider optimizing an upper bound on the throughput which is shown to be concave. Then, using some results from majorization theory, we show that the optimization problem can be considerably simplified. Interestingly, the feedback bit allocation follows a similar structure to that of transmit power allocation in the throughput maximization problem considered in [3].

II. System model

We consider a point-to-point MISO fading channel as shown in Fig. 1. The TX is connected to the power grid (so it has an uninterrupted power supply), whereas the RX harvests energy from the environment. We assume that all the harvested energy at the RX is used for communication purposes.

A. Energy Harvesting Model

The total observation time is divided into K equal length EH intervals. At the beginning of the k-th ($k \in \{1, \ldots, K\}$) EH interval, a new energy packet of size E_k units arrives at the RX. This energy can be stored in an infinite size battery and it is used only for future communication purposes. We assume that all E_k's are known in advance at the beginning.

[1]This work has been performed in the framework of the European research project E-CROPS.

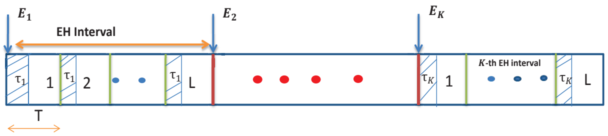

Fig. 2. Energy harvesting time frame structure.

This model is suitable for an energy harvesting system where the amount of harvested energy can be predicted in advance. The time frame structure is shown in Fig. 2.

B. Communication System Model

An EH interval consists of L data frames, each of length T. We assume a block fading channel model, where the channel is constant over T channel uses and i.i.d. with elements $\mathcal{CN}(0,1)$ from frame to frame. The TX has $M > 1$ antennas. The received signal in a given channel use is given by

$$y = \boldsymbol{h}^{\mathrm{H}}\boldsymbol{x} + \eta, \tag{1}$$

where $\boldsymbol{h} \in \mathbb{C}^{M}$ represents the vector of channel coefficients from TX antenna array to the RX, $\boldsymbol{x} \in \mathbb{C}^{M\times 1}$ represents the transmit symbol (i.i.d Gaussian) with $\mathrm{E}[\|\boldsymbol{x}\|^2] = P$ and $\eta \sim \mathcal{CN}(0,1)$ represents the complex circular-symmetric additive Gaussian noise at the RX.

C. Feedback Model

We assume that the receiver perfectly knows the channel at the beginning of each frame and feed backs the quantized CSI to the TX within the same frame. In the k-th EH interval, the frame structure is as follows: The RX in τ_k channel uses sends the CSI through a feedback channel (uplink) which is modeled as AWGN. In the remaining $T-\tau_k$ channel uses, TX sends data to the RX (downlink). The feedback model is adopted from [10], where the CSI acquisition and data transmission are performed in the same fading block. In Frequency-Division-Duplex (FDD) systems uplink and downlink takes place in different frequency bands. Therefore, a more realistic model that captures the FDD system is the one in which acquisition of CSI consumes uplink resources. Future work consists of considering more realistic feedback models along the lines of [10]. In the k-th EH interval, quantization of the channel is performed using a codebook $\mathcal{C}_k$ known at both the TX and RX. The receiver uses Random Vector Quantization. The codebook consists of M-dimensional unit vectors $\mathcal{C}_k \triangleq \{\boldsymbol{w}_1, \ldots, \boldsymbol{w}_{2^{B_k}}\}$, where B_k is the number of bits used for quantization. The quantization of the channel $\boldsymbol{h}$ in the k-th EH interval is found according to $\hat{\boldsymbol{h}} = \arg\max_{\boldsymbol{w}\in\mathcal{C}_k} |\boldsymbol{h}^{\mathrm{H}}\boldsymbol{w}|^2$. We assume that the length of the EH interval is very large compared to the channel coherence time (i.e., L very large). As a result, the achievable ergodic rate in the k-th EH interval is given by

$$R_k = \left(1-\frac{\tau_k}{T}\right)\times \mathrm{E}_{\boldsymbol{h},\mathcal{W}_k}\left[\log_2\left(1+\frac{P}{\left(1-\frac{\tau_k}{T}\right)}\|\boldsymbol{h}\|^2\cos^2\left(\angle(\boldsymbol{h},\hat{\boldsymbol{h}})\right)\right)\right]. \tag{2}$$

Even though there is no closed-form expression for the ergodic rate in (2), an equivalent numerically computable expression is given in [11]. However, this offers little insight into the convexity of the problem which is required to reduce the complexity of optimization. This motivates the use of a bound on the ergodic rate as an objective function. By using Jensen's inequality and the bounds on the quantization error [12], an upper bound on the ergodic rate is given by,

$$R_k^U = \left(1-\frac{\tau_k}{T}\right)\log_2\left[1+\frac{PM}{\left(1-\frac{\tau_k}{T}\right)}\left(1-\left(\frac{M-1}{M}\right)2^{\frac{-B_k}{M-1}}\right)\right] \tag{3}$$

By using the AWGN feedback channel model, the number of feedback bits B_k can be translated into the energy spent at the RX Q_k and the number of channel uses τ_k as follows,

$$B_k = \tau_k \log_2\left(1+\frac{Q_k}{\tau_k\sigma^2}\right). \tag{4}$$

For analytical tractability we neglect the practical constraint that B_k should be an integer. From (3) and (4) the ergodic rate upper bound is expressed as

$$R_k^U = \left(1-\frac{\tau_k}{T}\right)\times \log_2\left[1+\frac{PM}{\left(1-\frac{\tau_k}{T}\right)}\left(1-\frac{M-1}{M}\left(1+\frac{Q_k}{\tau_k\sigma^2}\right)^{\frac{-\tau_k}{M-1}}\right)\right]. \tag{5}$$

III. Throughput Maximization

In this section we study the problem of maximizing the throughput by a deadline (i.e., by the end of the K-th EH interval). The optimization problem is given by

$$\max_{Q_k,\tau_k} \quad \mathcal{U} = \sum_{k=1}^{K} R_k^U \tag{6a}$$

$$\text{s.t.} \quad L\sum_{i=1}^{l} Q_i \le \sum_{i=1}^{l} E_i, l = 1, ..., K, \tag{6b}$$

$$0 \le \tau_k \le T, \quad \text{and} \quad Q_k \ge 0, \quad k = 1, ..., K. \tag{6c}$$

As the objective function is monotonic in Q_k, the constraint in (6b) must be satisfied with equality for $l = K$, otherwise, we can always increase Q_K, and hence the objective, without violating any constraints. We represent the feasible set as

$$\mathfrak{F} = \{\boldsymbol{Q}, \boldsymbol{\tau} | Q_k, \tau_k \text{ satisfy (6b), (6c)}\}, \tag{7}$$

where $\boldsymbol{Q} = [Q_1, \ldots, Q_K]$ and $\boldsymbol{\tau} = [\tau_1, \ldots, \tau_K]$. To show that the above problem is a convex optimization problem, we make use of the following lemma.

Lemma 1. *If the function $f(x,t)$ is convex (resp. concave) $\forall (x,t) \in \mathbb{R}_+^2, t \in [0,T)$, $g(y)$ is convex (resp. concave) and monotonically increasing $\forall y \in \mathbb{R}_+$, then the function $h(x,t) = (T-t)\, g\left(\frac{f(x,t)}{T-t}\right)$ is convex (resp. concave) $\forall (x,t) \in \mathbb{R}_+^2, t \in [0,T)$.*

Proof: See Appendix. ■

Proposition 1. *The objective function in the optimization problem (6) is concave.*

Proof: Since the objective function is the summation of R_k^U's, showing that R_k^U is concave for $Q_k \geq 0, \tau_k \in [0,T)$ for any given $k \in \{1,\ldots,K\}$ is enough to prove the proposition. We can write

$$R_k^U = \frac{T-\tau_k}{T} g\left(\frac{f(Q_k,\tau_k)}{T-\tau_k}\right), \tag{8}$$

where $f(Q_k,\tau_k) \triangleq PMT\left(1 - \frac{M-1}{M}\left(1+\frac{Q_k}{\tau_k\sigma^2}\right)^{\frac{-\tau_k}{M-1}}\right)$ and $g(y) = \log_2(1+y)$. Since B_k is concave in Q_k and τ_k, it can be easily seen that $2^{-\frac{B_k}{M-1}} = \left(1+\frac{Q_k}{\tau_k\sigma^2}\right)^{\frac{-\tau_k}{M-1}}$ is convex, and hence, $f(Q_k,\tau_k)$ is concave. Using Lemma 1 we can see that R_k^U is concave. ■

Since the objective function in (6) is concave and the constraints are linear, it has a unique maximizer [13]. Using the concavity of the objective function and some results from multivariate majorization theory, we show that the optimal energy allocation vector consist of finding the most majorized feasible energy vector. To prove this, we need the following results from multivariate majorization theory.[2]

Definition 1. *[14, Definition 12.34] Let* $\mathbf{A} = [\boldsymbol{a}_1,\ldots,\boldsymbol{a}_n]^{\mathrm{T}}, \mathbf{B} = [\boldsymbol{b}_1,\ldots,\boldsymbol{b}_n]^{\mathrm{T}}, \mathbf{A},\mathbf{B} \in \mathbb{R}^{k\times n}, k \geq 2, n \geq 2$. $\mathbf{A}$ *is majorized by* $\mathbf{B}$ *in the multivariate sense* ($\mathbf{A} \preceq^m \mathbf{B}$) *if there exists a* $n\times n$ *doubly stochastic matrix* $\mathbf{P}$ *such that* $\mathbf{A} = \mathbf{BP}$.

Theorem 1. *[14, Theorem 12.38] Let* $\mathbf{X},\mathbf{Y} \in \mathbb{R}^{k\times n}$ *as defined above. If* $\mathbf{X} \preceq^m \mathbf{Y}$, *then* $f(\mathbf{X}) \geq f(\mathbf{Y})$ *holds for all* $f: \mathbb{R}^{k\times n} \to \mathbb{R}$ *which are symmetric and concave in the sense that (i)* $f(\mathbf{X}) = f(\mathbf{XT})$ *for all permutation matrices* $\mathbf{T}$ *and (ii)* $f(\alpha\mathbf{U} + (1-\alpha)\mathbf{V}) \geq \alpha f(\mathbf{U}) + (1-\alpha) f(\mathbf{V}) \forall \alpha \in [0,1]$ *and* $\mathbf{U},\mathbf{V} \in \mathbb{R}^{k\times n}$.

Proposition 2. *If* $\exists\, \boldsymbol{Q}^* \in \mathfrak{F}$ *such that* $\boldsymbol{Q}^* \preceq \boldsymbol{Q}, \forall(\boldsymbol{Q},\boldsymbol{\tau}) \in \mathfrak{F}$ *then* $(\boldsymbol{Q}^*,\boldsymbol{\tau}^*)$ *is the global optimum of (6), where*

$$\boldsymbol{\tau}^* = \arg\max_{\tau_k} \quad \sum_{k=1}^{K} R_k^U(Q_k^*,\tau_k)\, s.t.\ \forall k, (Q_k^*,\tau_k) \in \mathfrak{F}. \tag{9}$$

Proof: We first prove that the optimal energy vector must be $\boldsymbol{Q}^*$. Assume that there exists an optimal energy vector $\boldsymbol{Q}_t$ such that $\boldsymbol{Q}_t \neq \boldsymbol{Q}^*$. Since $\boldsymbol{Q}^* \preceq \boldsymbol{Q}_t$, there exists a doubly stochastic matrix $\mathbf{P}$ such that $\boldsymbol{Q}^* = \boldsymbol{Q}_t\mathbf{P}$ [14]. We can easily check that the objective function $\mathcal{U}$ is symmetric and concave in the sense of Theorem 1, we have

$$\mathcal{U}\left(\boldsymbol{Q}^*,\boldsymbol{\tau}^{(a)}\right) \geq \mathcal{U}(\boldsymbol{Q}_t,\boldsymbol{\tau}), \qquad \forall(\boldsymbol{Q}_t,\boldsymbol{\tau}) \in \mathfrak{F}, \tag{10}$$

where $\boldsymbol{\tau}^{(a)} = \boldsymbol{\tau}\mathbf{P}$. Now, it needs to be verified that $\boldsymbol{\tau}^{(a)} \in \mathfrak{F}$. The k-th element of $\boldsymbol{\tau}^{(a)}$ is denoted by $\tau_k^{(a)}$, and $\tau_k^{(a)} = \sum_{j=1}^{K} \tau_j\{\mathbf{P}\}_{j,k}$. Since $\mathbf{P}$ is doubly stochastic, we have $\sum_{j=1}^{K}\{\mathbf{P}\}_{j,k} = 1$. From the above two arguments we can see that $0 \leq \tau_k^{(a)} < T, \forall k$ i.e., $\boldsymbol{\tau}^{(a)} \in \mathfrak{F}$. Therefore, an energy allocation vector $\boldsymbol{Q}_t$ such that $\boldsymbol{Q}_t \neq \boldsymbol{Q}^*$ cannot be optimal. Since the optimal energy allocation vector is $\boldsymbol{Q}^*$, the optimal vector $\boldsymbol{\tau}^*$ is obtained by (9). ■

[2]Due to the space limitation we could not provide some basic definitions in majorization theory that are used here. Please refer to [14].

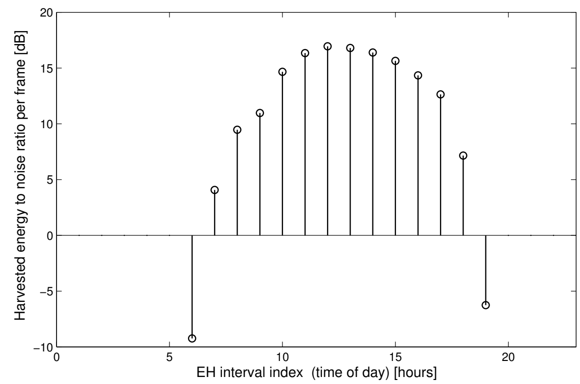

Fig. 3. Model for solar energy harvesting profile.

There is an existing algorithm in literature that constructs the optimal energy allocation policy, i.e., $\boldsymbol{Q}^* \preceq \boldsymbol{Q}, \forall(\boldsymbol{Q},\boldsymbol{\tau}) \in \mathfrak{F}$ given the EH constraints [3], [15]–[17]. The proof that the algorithm constructs the most majorized feasible energy vector is given in [17]. Due to the space limitation, details of the algorithm are not included here. Although the original problem is simplified to (9), there is no closed form expression for $\boldsymbol{\tau}^*$. Therefore we use numerical methods to obtain $\boldsymbol{\tau}^*$.

IV. Results

In this section, we compare different feedback bit allocation schemes using numerical results. The RX is equipped with a solar EH device. We take solar irradiance data from a database [18]. Each EH interval is of duration 1 hour, $T = 100$ ms, and $L = 36000$ frames. A hypothetical solar panel of variable area is assumed. The area of the panel is adjusted such that we have an EH profile shown in Fig. 3. In Fig. 3 harvested energy to noise ratio per frame in each EH interval $\frac{E_k}{L\sigma^2}$ is shown.

The feedback bit allocation policy obtained from the optimization serves two purposes:

(a) If used in evaluating (6a), gives an upper bound on the throughput.
(b) If used in evaluating the exact ergodic capacity expression [11, (27)], gives a lower bound on the throughput.

We compare this with a greedy scheme where optimization is performed only on $\boldsymbol{\tau}$ given $Q_k = E_k/L$. Fig. 4 shows the throughput of all the above mentioned policies for different system SNRs. In Fig. 5, feedback bit allocation is shown for the above mentioned policies for a system SNR of 10dB.

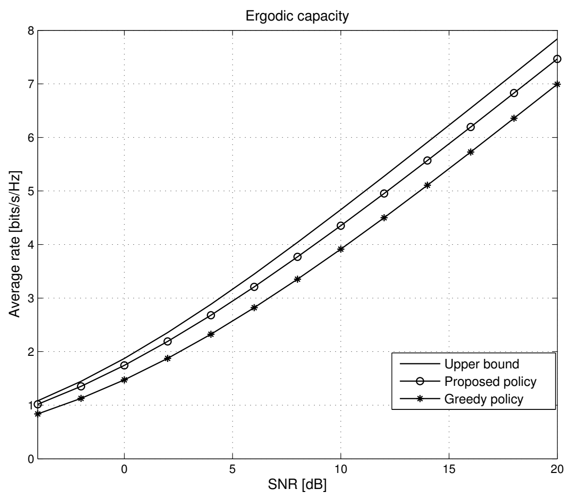

Fig. 4. Comparison of ergodic capacity for different policies.

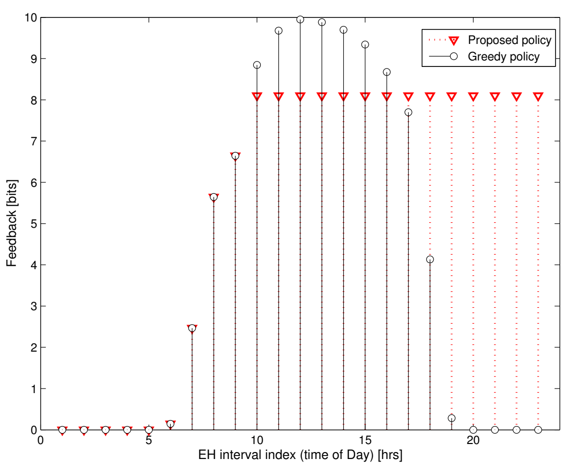

Fig. 5. Feedback load at SNR of 10 dB.

V. Conclusion

In this paper, we formulated the problem of feedback design with EH constraints in a point-to-point MISO channel with an EH RX. The proposed policy not only outperforms the greedy policy, but also achieves the performance which is quite close to the upper bound. Finally, the extension to multiuser settings, namely broadcast channel (where CSIT plays an even more important role), and to MISO channel with both EH TX and RX is currently under investigation.

Appendix

The proof is similar to that of showing the perspective of a convex function is convex. Here the proof is given for the concave case, the convex case follows similar steps. Let $X_1 = [x_1\ t_1]^{\mathrm{T}}, X_2 = [x_2\ t_2]^{\mathrm{T}}$, we have

$$\begin{aligned} h\left(\lambda X_1 + (1-\lambda) X_2\right) & \\ &\overset{(a)}{\geq} \Theta g\left(\frac{\lambda f(X_1) + (1-\lambda) f(X_2)}{\Theta}\right) \\ &= \Theta g\left(\frac{\Theta_1}{\Theta}\frac{f(X_1)}{(T-t_1)} + \frac{\Theta_2}{\Theta}\frac{f(X_2)}{(T-t_2)}\right) \\ &\overset{(b)}{\geq} \Theta_1 g\left(\frac{f(X_1)}{T-t_1}\right) + \Theta_2 g\left(\frac{f(X_2)}{T-t_2}\right) \\ &= \lambda h(X_1) + (1-\lambda) h(X_2), \end{aligned} \tag{11}$$

where $\Theta = \Theta_1 + \Theta_2, \Theta_1 = \lambda (T - t_1)$ and $\Theta_2 = (1-\lambda)(T-t_2)$, and

(a) follows from the fact that $f(x, t)$ is concave and $g(.)$ is monotonically increasing.
(b) follows from the fact that $\frac{\Theta_1}{\Theta} + \frac{\Theta_2}{\Theta} = 1$ and $g(.)$ is concave.

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