On the Value of Spectrum Sharing among Operators in Multicell Networks

Rajeev Gangula and David Gesbert
Mobile Communications Department, EURECOM
Sophia Antipolis, France

Johannes Lindblom and Erik G. Larsson
Dept. of Electrical Engineering (ISY), Linköping University
Linköping, Sweden

Abstract—**This work considers the benefits of allowing spectrum sharing among co-located wireless service providers operating in the same multicell network. Although spectrum sharing was shown to be valuable in some scenarios where the created interference can be eliminated, the benefits have not clearly shown for multicell networks with aggressive reuse. We explore this question and show that spectrum sharing is preferred for just a certain subset of the users defined by their distance from the serving bases, while beyond this distance, an orthogonal division of resources between operators gives better results. The claims are backed with theoretical analysis matching our simulations.**

I. INTRODUCTION

In current cellular systems, network operators are allocated exclusive frequency bands so that inter-operator interference is trivially avoided. As the radio spectrum is a valuable and finite resource, recent activities promote the idea of sharing the spectrum among otherwise competing service providers resulting in significant interference yet potentially increased efficiency [1].

Although multiple antenna techniques can be used to combat this extra interference, it is not clear whether spectrum sharing will bring gains overcoming the signal degradation resulting for inter-operator interference [2], [3]. For example, [3] indicates that there is marginal or no gain in spectrum sharing compared to that of a non sharing scenario. In [3] spectrum sharing is modeled as a Multiple-Input Single-Output (MISO) IC. The intuition behind the marginal gain is that even though by sharing each operator gets double bandwidth, each one has to sacrifice some degrees of freedom (DoF) per bandwidth unit to suppress the inter-operator interference thus making the effective DoF same in both sharing and non-sharing scenarios. This type of studies serve to demonstrate the difficulty of establishing real benefits for spectrum sharing and the high level of dependence of deployment parameters, such as the number of antennas, the topology of the network etc. Note that spectrum sharing comes with an extra overhead of transfer of Channel State Information (CSI) among the operators. Therefore to get practical results the gains resulting should overweigh the CSI overhead.

A common trait of most previous evaluations of spectrum sharing was the consideration of a single cell scenario. However, in reality operators must deploy base stations in multiple cells so as to reach a satisfactory Bits/Hz/m2 performance. Furthermore, base stations in multiple cells typically reuse the same frequencies for greater efficiency, causing additional inter-cell interference. This fundamentally alters the gain analysis for spectrum sharing as two types of interference now plague the system: inter-operator and inter-cell interference. In this paper we thus consider the multi-cell case specifically.

Towards this we examine a system consisting of two cells, where in each cell spectrum is shared among network operators. We show that the spectrum sharing gain depends on the user's position in the cell. Depending on the user position orthogonal division of spectrum may outperform the spectrum sharing strategy. Based on this we propose an adaptive spectrum sharing policy.

II. SYSTEM MODEL

We consider a system consisting of two cells, where in each cell communication services are offered by K coexisting network operators[1]. Let $\mathcal{O} = \left\{ o^{(1)}, o^{(2)}, \ldots, o^{(K)} \right\}$ denote the set of network operators. Orthogonal Frequency Division Multiple Access (OFDMA) is considered. The total bandwidth available in the system is $KB = KN_s\Delta f$, where KN_s is the number of subchannels and Δf is the bandwidth of a subchannel. On a given subchannel, each operator with one transmitter (TX) communicates with a unique receiver (RX) at a time. Each TX and RX are equipped with M antennas. We assume that in each cell the TXs belonging to different operators are collocated. This represents the scenario where the base stations belonging to different operators share the same cell cite. The channel matrix between the TX of operator y in the j-th cell and the RX of operator x in the i-th cell on a subchannel q, $\mathbf{H}_{x_i y_j, q} \in \mathbb{C}^{M \times M} \forall i, j \in \{1, 2\}$ and $x, y \in \mathcal{O}$ is given by

$$
\mathbf{H}_{x_i y_j, q} = \begin{cases} \sqrt{\alpha_{x_i y_j}} \mathbf{G}_{x_i y_j, q} & \text{If } x_i, y_j \text{ operate on} \\ & \quad \text{the subchannel } q, \\ \mathbf{0} & \text{otherwise.} \end{cases} \quad (1)
$$

The elements of the matrix $\mathbf{G}_{x_i y_j, q}$ are i.i.d $\mathcal{CN}(0,1)$, $\alpha_{x_i y_j}$ is a positive real number modeling the distance based long term attenuation.

---

[1]The reason behind choosing a twocell network is because it is easier to evaluate the performance of spectrum sharing techniques analytically and get some insights. However, the techniques used and the conclusions drawn are not specific to the twocell asumption. In principle similar results should be obtained in the multicell scenario.
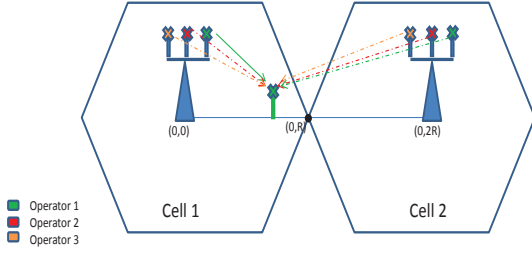
Fig. 1. A two cell network, each cell has 3 operators. In each cell the TXs are mounted on the same cell cite.

III. ORTHOGONAL SPECTRUM ALLOCATION

This represents the traditional way of allocating the spectrum, where the spectrum is divided into chunks and exclusively assigned to the licensed to operators. Here we assume that the total bandwidth of KN_s subchannels is divided equally among the operators so that each operator gets N_s subchannels. Since the spectrum is divided orthogonally, there will be no inter-operator interference. Now the only source of interference is that of from the TXs of the neighboring cells of the same operator, operating in the same frequency band. In the following a brief description of the some well known frequency reuse schemes will be given. Using these basic schemes later we design the adaptive spectrum sharing scheme.

A. Frequency Reuse

The inter-cell interference can simply be avoided by using a reuse-$1/2$ scheme, where the bandwidth available is equally divided between the two cells. In this case, inter-cell interference is avoided at the cost of reduced bandwidth. On a given subchannel the transmission scheme is modeled as a point-to-point MIMO channel. The average rate of operator o in the i-th cell in bits/sec/Hz is given by [2]

$$R_F = \frac{1}{2K} \mathbb{E}\left[\log_2 \det\left(\mathbf{I} + \frac{P}{M\sigma^2}\mathbf{H}_{o_i o_i}\mathbf{H}_{o_i o_i}^\dagger\right)\right], o \in \mathcal{O}, \tag{2}$$

where σ^2 is the noise variance. The factor $1/2K$ in front is because in each cell an operator gets $N_s/2$ subchannels from the total of KN_s subchannels.

In universal frequency reuse, users in each cell have access to the entire bandwidth available to the operator i.e. N_s subchannels. This scheme makes efficient use of the spectrum but users in the cell are affected by out-of-cell interference. We assume that there is neither cooperation between TXs nor RXs have interference cancellation capabilities. Therefore each RX treats out-of-cell interference as noise. The average rate of operator o in the i-th cell in bits/sec/Hz is given by

[2] In order to simplify notation, we omit the subchannel index when we focus on a single subchannel.

$$R_U = \frac{1}{K} \mathbb{E}\left[\log_2 \det\left(\mathbf{I} + \frac{P}{M}\mathbf{R}^{-1}\mathbf{H}_{o_i o_i}\mathbf{H}_{o_i o_i}^\dagger\right)\right], \tag{3}$$

where

$$\mathbf{R} = \sigma^2\mathbf{I} + \frac{P}{M}\mathbf{H}_{o_i o_{\bar{i}}}\mathbf{H}_{o_i o_{\bar{i}}}^\dagger, o \in \mathcal{O} \tag{4}$$

is the interference plus noise covariance matrix at the RX, $\bar{i} = mod\,(i, 2) + 1$. The factor $1/K$ in front is because in each cell an operator gets N_s subchannels from the total of KN_s subchannels. Note that the power allocated for a stream is $\frac{P}{M}$ and hence the total power emitted on a subchannel is P.

IV. NON ORTHOGONAL SPECTRUM SHARING

In this scenario the total bandwidth of KN_s subchannels is available to all the operators. As a result of sharing the spectrum, there is a significant amount of interference among the operators. Since the TXs belong to different operators, we assume that they are not allowed to exchange users data symbols, giving rise to an MIMO Interference Channel (MIMO IC). However, all the TXs have the knowledge of the CSI of the interference channels and hence can coordinate their transmissions. We model this as symmetric (M, M, K) MIMO IC. Interference Alignment (IA) is used within each cell among the operators to combat the inter-operator interference. In order to get meaningful results in multicell network, first the parameters K and M are to be properly chosen such that spectrum sharing is indeed beneficial in the single cell scenario.

Corollary 1. [4] The DoF of a two user MIMO interference channel with each TX, RX having M antennas is M.

Corollary 2. [5] In an (M, M, K), $K \geq 3$ IC with desired DoF $d = 1$ per user, IA is feasible if and only if $M \geq \frac{K+1}{2}$.

Remark 1: If spectrum sharing between two operators, each operator gets double bandwidth compared to the orthogonal division. However, from corollary 1 we can see that each operator has to sacrifice half of it's spatial DoF per bandwidth unit to suppress the inter-operator interference thus making the effective DoF same in both sharing and non-sharing scenarios. Therefore, we model the spectrum sharing scenario as (M, M, K) IC, $K \geq 3$ with desired DoF $d = 1$ per operator. We consider a tight feasibility setting i.e. $K = 2M - 1$ such that all spatial DoFs of the operators are consumed in dealing with the inter-operator interference and then evaluate the performance in the presence of inter-cell interference.

In the multicell scenario the total bandwidth of KN_s subchannels is available to all the operators in each cell. Transmitter i of the operator o uses a linear precoder $\boldsymbol{v}_i^{(o)} \in \mathbb{C}^{M \times 1}$ to map the symbol $s_i^{(o)}$ to its transmit antennas,

$$\mathbf{x}_i^{(o)} = \boldsymbol{v}_i^{(o)} s_i^{(o)}, \tag{5}$$

where the transmitted symbols $s_i^{(o)}, \forall i \in \{1, 2\}, o \in \mathcal{O}$ are i.i.d $\mathcal{CN}(0, 1)$. The precoder is normalized such that $\left\|\boldsymbol{v}_i^{(o)}\right\|^2 = \frac{P}{K}$. Since we do not know how the future regulatory

requirements for spectrum sharing scenarios look like, we will keep the power emitted in both orthogonal allocation and spectrum sharing scenarios equal.

The received signal at the i-th RX of the o-th operator is

$$\boldsymbol{y}_i^{(o)} = \mathbf{H}_{o_i o_i} \mathbf{x}_i^{(o)} + \sum_{t \in \mathcal{O} \setminus o} \mathbf{H}_{o_i t_i} \mathbf{x}_i^{(t)} + \sum_{u \in \mathcal{O}} \mathbf{H}_{o_i u_{\bar{i}}} \mathbf{x}_{\bar{i}}^{(u)} + \boldsymbol{\eta}_i^{(o)},$$
(6)

where the first summation term in (6) represents the inter-operator interference, second summation term represents the intercell interference and $\boldsymbol{\eta}_i^{(o)}$ represents the additive white Gaussian noise at the RX o_i and its elements are i.i.d $\mathcal{CN}(0, \sigma^2)$.

Analytical expressions for the precoders and RX filters for IA are not known (expect for the 3-user MIMO IC) but there exist numerical methods for designing them [6], [7], [8].

A. Interference Alignment

A simple precoding scheme that we will consider in the simulations is based on interference leakage [7]. A brief description of the main steps in the algorithm is given here for the sake of completeness. First arbitrary RX filters $\boldsymbol{u}_i^{(o)}$ for each receiver are used for initialization. Then at each step, the precoders and RX filters are updated as

$$\boldsymbol{v}_i^{(o)} = \lambda_{min} \left(\sum_{t \in \mathcal{O} \setminus o} \mathbf{H}_{t_i o_i}^{\dagger} \mathbf{u}_i^{(t)} \mathbf{u}_i^{(t)\dagger} \mathbf{H}_{t_i o_i} \right),$$

$$\boldsymbol{u}_i^{(t)} = \lambda_{min} \left(\sum_{o \in \mathcal{O} \setminus t} \mathbf{H}_{t_i o_i} \mathbf{v}_i^{(o)} \mathbf{v}_i^{(o)\dagger} \mathbf{H}_{t_i o_i}^{\dagger} \right),$$
(7)

respectively until a stopping criteria is reached. This algorithm converges to an alignment solution when IA is feasible and removes the inter-operator interference within the cell. The signal to interference plus noise ratio SINR is given by

$$\text{SINR} = \frac{\left| \boldsymbol{u}_i^{(o)\dagger} \mathbf{H}_{o_i, o_i} \boldsymbol{v}_i^{(o)} \right|^2}{\sum_{u \in \mathcal{O}} \left| \boldsymbol{u}_i^{(o)\dagger} \mathbf{H}_{o_i, u_{\bar{i}}} \boldsymbol{v}_{\bar{i}}^{(o)} \right|^2 + \sigma^2}.$$
(8)

Remark 2: Different algorithms such as Max-SINR, MMSE have superior performance compared to the leakage algorithms [7]. But due to their iterative nature, the exact dependence of TX, RX filters on the channel matrices are unknown hence there distribution is difficult to get. Although these algorithms are expected to give better performance than the considered one here, to simulate these algorithms on a large system as we will consider in Sec.V is computationally prohibitive. Therefore, we consider the IA Leakage algorithm so that we can provide analytical results that gives insights into the performance of spectrum sharing techniques.

1) Statistical properties: Here we give some simple statistical properties of the signal and interference terms resulted from the IA algorithm. Later they are used to compute the average rate achieved by the operators in the spectrum sharing scenario.

Lemma 1. *Denote* $T_k \triangleq \boldsymbol{u}^{\dagger} \mathbf{G}_k \boldsymbol{v}$ *where* $\boldsymbol{v}, \boldsymbol{u} \in \mathbb{C}^{M \times 1}$ *are unit vectors independent of* $\mathbf{G}_k \in \mathbb{C}^{M \times M}$. *The matrices* $\mathbf{G}_k$ *are independent and they have i.i.d* $\mathcal{CN}(0, 1)$ *entries. Then* $T_k \forall k$ *are i.i.d with* $T_k \sim \mathcal{CN}(0, 1)$.

Proof: Since the distribution of $\mathbf{G}_k$ is rotationally invariant and $\boldsymbol{u}, \boldsymbol{v}$ are independent of $\mathbf{G}_k$, we can perform change of basis such that $T_k = \boldsymbol{e}_i^{\dagger} \mathbf{G}_k \boldsymbol{e}_j$, where $\boldsymbol{e}_i, \boldsymbol{e}_j$ are standard orthonormal basis [9]. After performing the change of basis, the distribution of T_k is same as that of the element $\mathbf{G}_k(i, j)$, which are i.i.d $\mathcal{CN}(0, 1)$. ∎

Lemma 2. *Let* $Z \triangleq \frac{\gamma_1 X}{1 + \gamma_2 Y}$ *be a random variable where* X, Y *are independent and* $X \sim \chi_2^2$, $Y \sim \chi_{2L}^2$, *where* χ_n^2 *denotes the chi-square distribution with* n *degrees of freedom. Defining* $R_s \triangleq \mathbb{E}_Z [\log_2 (1 + Z)]$, *we have*

$$R_s (L, \gamma_1, \gamma_2) =$$

$$\log_2 (e) \left(\frac{\gamma_1}{\gamma_2} \right)^L \left[A_{11} I_1 \left(\frac{1}{\gamma_1} \right) + \sum_{r=1}^{L} A_{2r} I_2 \left(r, \frac{1}{\gamma_1}, \frac{\gamma_1}{\gamma_2} \right) \right],$$

where $I_1 (.) I_2 (., ., .)$ *is given in (16), the terms* A_{11}, A_{2r} *are obtained by solving (14).*

Proof: See Appendix. ∎

Proposition 1. *The average rate of operator o in the spectrum sharing scenario is given by* $R_s (K, \gamma_1, \gamma_2)$, *where* $\gamma_1 = \frac{P \alpha_{o_i o_i}}{(K \sigma^2)}$ *and* $\gamma_2 = \frac{P \alpha_{o_i o_{\bar{i}}}}{(K \sigma^2)}$.

Proof: Since the TXs of the operators are assumed to be mounted on the same cell cite $\alpha_{o_i u_{\bar{i}}} = \alpha_{o_i o_{\bar{i}}} \forall u \in \mathcal{O}$. The SINR expression in (8) can be rewritten as

$$\text{SINR} = \frac{\frac{P \alpha_{o_i o_i}}{(K \sigma^2)} \left| \boldsymbol{u}_i^{(o)\dagger} \mathbf{G}_{o_i, o_i} \frac{\boldsymbol{v}_i^{(o)}}{\|\boldsymbol{v}_i^{(o)}\|} \right|^2}{\frac{P \alpha_{o_i o_{\bar{i}}}}{(K \sigma^2)} \sum_{u \in \mathcal{O}} \left| \boldsymbol{u}_i^{(o)\dagger} \mathbf{G}_{o_i, u_{\bar{i}}} \frac{\boldsymbol{v}_{\bar{i}}^{(o)}}{\|\boldsymbol{v}_{\bar{i}}^{(o)}\|} \right|^2 + 1}.$$
(9)

At each step of the alternating minimization method, direct channel links do not appear in the computation of TX, RX filters and are also independent of out-of-cell interfering links (7). Using Lemma 1, $\text{SINR} \triangleq \frac{\gamma_1 S}{1 + \gamma_2 I}$, where S and I are independent with $S \sim \chi_2^2$ and $I \sim \chi_6^2$, $\gamma_1 = \frac{P \alpha_{o_i o_i}}{(K \sigma^2)}$ and $\gamma_2 = \frac{P \alpha_{o_i o_{\bar{i}}}}{(K \sigma^2)}$. The average rate is obtained by substituting K, γ_1 and γ_2 in the function $R_s (., ., .)$ of lemma 2. ∎

In Fig. 2 we compare the performance of above discussed spectrum allocation policies. Spectrum is shared among $K = 3$ operators and each TX, RX have $M = 2$ antennas. We plot the average rate of the operator $o^{(1)}$ in cell 1. Since the system is symmetric, similar results can be obtained for others. Referring to Fig. 1, RX $o_1^{(1)}$ is moved from the center of cell 1 towards the edge along the straight line connecting the TXs of two cells. The precoders and RX filters in each cell are obtained by using the IA algorithm mentioned in Sec.IV-A. At each position of RX $o_1^{(1)}$, simulation results are averaged over 1000 small scale fading realizations. In the spectrum
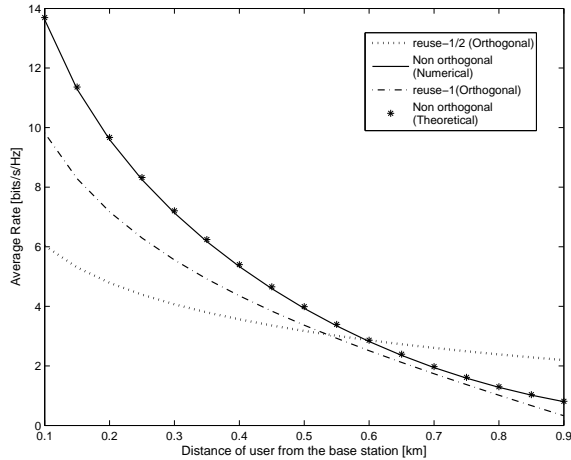
Fig. 2. Average rate of different strategies as a function of user position. User of an operator in Cell 1 moves from center to towards the cell edge.

sharing scenario, in addition to averaging over fading realizations results are also averaged over RXs $o_1^{(k)}, k = 2, 3.$ and $o_2^{(k)}, k = 1, 2, 3.$ positions which are generated uniformly and independent of RX $o_1^{(1)}$. We assume no power control at the TXs so the average rate of $o^{(1)}$ depends only on the position of RX $o_1^{(1)}$ and independent on other RX positions. The cell radius is 1 Km. The pathloss between the TX and RX which are separated by distance dt Km is $128 + 37.6 \log_{10}(dt)$. Average cell edge SNR is 20 dB.

V. SPECTRUM PARTITIONING AND RESOURCE ALLOCATION

From the above section we see that there is no scheme outperforming the others at all user positions. Hence, it is natural to think of an adaptive transmission strategy based on the user locations. In this section we compare two schemes, a well known FFR scheme in orthogonal spectrum allocation scenario and the Fractional Spectrum Sharing (FSS) scheme in the non orthogonal sharing case.

A. Fractional Frequency Reuse

FFR techniques are widely used in multicell networks to improve the cell-edge user performance. Cell interior users are allocated a common set of frequencies while the cell-edge user's bandwidth is partitioned across the cells based on a reuse factor Δ. One of the most important design parameters is the interior radius r_{int}, which determines the size of the frequency partitions. One common and practical method is to classify users based on their average received SINR. The base station can classify users with average SINR less than a predetermined threshold as edge users, while users with average SINR greater than the threshold are classified as interior users. First the total available bandwidth is divided equally among the operators so that each operator gets N_s subchannels and then each operator applies FFR scheme. Within each operator the subchannels allocations are given

by [10], [11]

$$N_{int} = \left\lceil N_s \left(\frac{r_{int}}{R} \right) \right\rceil,$$
$$N_{ext} = \lfloor (N_s - N_{int}) / \Delta \rfloor, \tag{10}$$

where N_{int} and N_{ext} denotes the number of subchannels allocated to interior users and exterior users and Δ is the frequency reuse factor. Here we assume $\Delta = 2$. In each cell users are uniformly distributed on the straight line connecting the TXs in the two cells. The system is fully loaded meaning that at any given time there are enough users to serve and all the subchannels are used.

B. Fractional Spectrum Sharing

From Fig. 2, we can see that for cell edge users orthogonal allocation and reuse-$1/2$ scheme outperforms the spectrum sharing scheme. Therefore based on the user positions we can chose different strategies. Similar to the FFR, we introduce the concept of FSS where operators share only a fraction of the total spectrum and the rest is divided orthogonally. The important design parameter is the interior radius r_{int} which decides the amount of spectrum shared among the operators. The frequency partitions are given by

$$N_{shr} = \left\lceil K N_s \left(\frac{r_{int}}{R} \right) \right\rceil,$$
$$N_{ext} = \lfloor (K N_s - N_{shr}) / K \Delta \rfloor, \tag{11}$$

were N_{shr} denotes the number of subchannels shared among the operators and they are used for interior users. The remaining $K N_s - N_{shr}$ subchannels are divided among the operators orthogonally i.e. each operator gets $(K N_s - N_{shr}) / K$ subchannels. In these subchannels there is no inter-operator interference. Here we assume reuse-$1/2$ scheme is applied within the operator in these subchannels.

In Fig. 3 we compare the performance of an operator o in cell 1 when using traditional FFR and proposed FSS schemes as a function of interior radius r_{int}. Since the system is symmetric, similar results can be obtained for other operators. The total bandwidth of 6.48 MHz corresponding to 432 subchannels with each subchannel having a bandwidth $\Delta f = 15$ kHz is considered. At each value of r_{int}, number of interior and edge users are given by (10), (11) for FFR and FSS schemes respectively. Then the user positions are generated uniformly according to the definition of interior and edge users. In order to simulate this scenario, first TX, RX filters are to be computed by using IA algorithm in two cells and then the rate is to be calculated using SINR given in (8). This procedure has to be done on each subchannel and for large number of subchannels it is computationally prohibitive. Thanks to proposition 1, at each Monte Carlo iteration, we avoid the computation of TX, RX filters by generating statistically equivalent terms in the SINR expression.

VI. CONCLUSION

Spectrum sharing among the operators in a two-cell network is analyzed. Spectrum sharing is shown to greatly improve the average rates of cell interior users but not for the cell
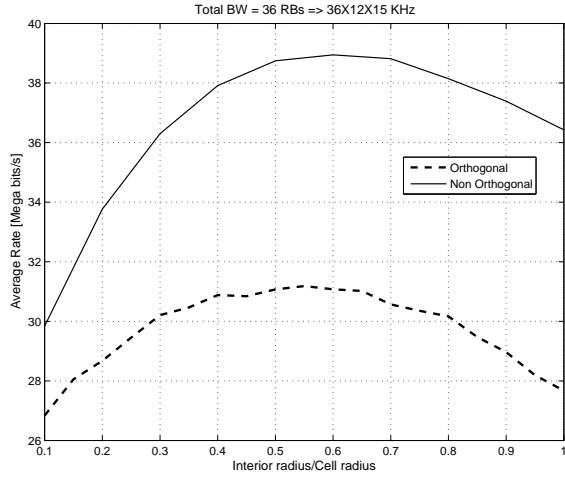
Fig. 3. Average rate of FFR and FSS schemes as a function of r_{int}.

edge users. A fractional spectrum sharing scheme is proposed in which only certain portion of the total spectrum is shared among the operators and the remaining spectrum is divided orthogonally.

ACKNOWLEDGMENT

APPENDIX

A. Proof of Lemma 2

The cumulative distribution function (cdf) of Z can be easily derived as

$$F_Z(z) = 1 - \frac{e^{-z/\gamma_1}}{(z + \gamma_2/\gamma_1)^L}. \tag{12}$$

The expectation of $ln(1+z)$ on Z is derived as follows,

$$\begin{aligned}
\mathbb{E}_Z[\ln(1+Z)] &= \int_0^\infty \ln(1+z)\, dF_Z \\
&= \int_0^\infty \frac{1 - F_Z(z)}{z+1}\, dz \\
&= \left(\frac{\gamma_1}{\gamma_2}\right)^L \int_0^\infty \frac{e^{-z/\gamma_1}}{(z + \gamma_1/\gamma_2)^L (z+1)}\, dz
\end{aligned} \tag{13}$$

Using partial fractions,

$$\frac{1}{(z + \gamma_1/\gamma_2)^L (z+1)} = \sum_{i=1}^2 \sum_{r=1}^{J_i} \frac{A_{ir}}{(z + a_i)^r} \tag{14}$$

where $J_1 = 1, J_2 = L, a_1 = 1$ and $a_2 = \frac{\gamma_1}{\gamma_2}$.

$$\begin{aligned}
\mathbb{E}_Z[\ln(1+Z)] &= \left(\frac{\gamma_1}{\gamma_2}\right)^L \\
&\times \left[A_{11} \int_0^\infty \frac{e^{-z/\gamma_1}}{z+1}\, dz + \sum_{r=1}^L A_{2r} \int_0^\infty \frac{e^{-z/\gamma_1}}{(z+\gamma_1/\gamma_2)^r}\, dz \right] \\
&= \left(\frac{\gamma_1}{\gamma_2}\right)^L \left[A_{11} I_1\left(\frac{1}{\gamma_1}\right) + \sum_{r=1}^L A_{2r} I_2\left(r, \frac{1}{\gamma_1}, \frac{\gamma_1}{\gamma_2}\right) \right],
\end{aligned} \tag{15}$$

where the functions I_1, I_2 are given by [12, 3.352.4], [12, 3.353.2]

$$\begin{aligned}
I_1(\mu) &= e^\mu \operatorname{Ei}(-\mu), \\
I_2(r, \mu, \beta) &= \\
\frac{1}{(r-1)!} &\sum_{k=1}^{r-1} (k-1)! (-\mu)^{r-k-1} \beta^{-k} - \frac{(-\mu)^{r-1}}{(r-1)!} e^{\mu\beta} \operatorname{Ei}(-\mu\beta).
\end{aligned} \tag{16}$$

REFERENCES

[1] SAPHYRE, "Sharing physical resources." [Online]. Available: http://www.saphyre.eu.

[2] E. A. Jorswieck, L. Badia, T. Fahldieck, M. Haardt, E. Karipidis, J. Luo, and R. Pisz, "Spectrum sharing improves the network efficiency for wireless operators," submitted to IEEE Comm. Mag.,2012,. [Online]. Available: http://www.saphyre.eu.

[3] J. Lindblom and E. Larsson, "Does Non-Orthogonal Spectrum Sharing in the same Cell Improve the Sum-Rate of Wireless Operators?" in SPAWC 2012.

[4] S. Jafar and M. Fakhereddin, "Degrees of freedom for the mimo interference channel," *IEEE Transactions on Information Theory*, vol. 53, no. 7, pp. 2637 –2642, july 2007.

[5] G. Bresler, D. Cartwright, and D. Tse, "Settling the feasibility of interference alignment for the mimo interference channel: the symmetric square case." [Online]. Available: http://arxiv.org/abs/1104.0888

[6] K. Gomadam, V. Cadambe, and S. Jafar, "A distributed numerical approach to interference alignment and applications to wireless interference networks," *IEEE Transactions on Information Theory*, vol. 57, no. 6, pp. 3309 –3322, june 2011.

[7] S. Peters and R. Heath, "Cooperative algorithms for mimo interference channels," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 1, pp. 206 –218, jan. 2011.

[8] Z. Ho and D. Gesbert, "Balancing egoism and altruism on interference channel: The mimo case," in *IEEE International Conference on Communications (ICC), 2010*, may 2010.

[9] N. Jindal, J. Andrews, and S. Weber, "Rethinking mimo for wireless networks: Linear throughput increases with multiple receive antennas," in *Proc ICC 2009*, 2009, pp. 1 –6.

[10] T. Novlan, J. Andrews, I. Sohn, R. Ganti, and A. Ghosh, "Comparison of Fractional Frequency Reuse Approaches in the OFDMA Cellular Downlink," in *Proc GLOBECOM 2010*, 2010.

[11] M. Assaad, "Optimal Fractional Frequency Reuse (FFR) in Multicellular OFDMA System," in *Proc VTC 2008.*, 2008.

[12] I. Gradshteyn and I. Ryzhik, *Table of Integrals series and products*, 6th ed. New York: Academic press.