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About Contractivity Constraints when Coding Still Grey-Level Images using Fractal Transforms

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Abstract

Using Iterated Function Systems for data compression, some convergence problems appear because of an iterative decoding process. Several works have tried to slove them using sufficient (i.e. sub-optima) conditions based on a contractivity criterion. Unfortunately, no efficient convergence condition has been found in practice. This paper is an overview of the relevant existing hypotheses which ensure the convergence, in particlular with regard to the contractivity of the fractal code...

1 Introduction

Since the late 80s, fractal coding based on the Iterated Function Systems (I.F.S.) theory [1] has been used in the field of loosy data compression. Research work on existence conditions has been done, but no answer allows us to conclude rigorously about the iterative process convergence, in the general discrete case.
So, this paper aims at reviewing the main results in this field about a contractivity sufficient condition.

2 Jacquin's Coding using Local I.F.S. - a brief reminder

Let us note Λ the space of digitized images with arbitrary same size, which can be modelled by $\mathbb{R}^K$ for some positive integer K. Jacquin's coding scheme [7] is based on a local self-similarities research. The image $\mu \in \Lambda$ that we want to encode, is subdivided into N

non-overlapping squares (partition), called the **range cells** of the image and denoted by R_i for $1 \leq i \leq N$ ($\mu = \cup_{i=1}^{N} \mu_{|R_i}$). In order to encode independently each range block $\mu_{|R_i}$, a dictionary of transformations is defined ($\mu_{|R_i}$ gives the restriction of the image μ to the cell R_i). Each range block $\mu_{|R_i}$ will be associated with another block in the original image relatively to some cell $D_{q(i)} \in \mathcal{D} = \{D_1, \ldots, D_Q\}$ called a **domain cell**, *via* a matching τ_i belonging to the catalog called the **local fractal code** of the corresponding range block ($\mathcal{D}$ is the set of all allowed domain cells). This local code is built using a least squares criterion in order to minimize the local collage error $\varepsilon_{c,i} = d_2(\mu_{|R_i}, \tau_i(\mu_{|D_{q(i)}}))$ where d_2 is the Euclidian metric (the global **collage error** ε_c is given by $\varepsilon_c = \sum_{1 \leq i \leq N} \varepsilon_{c,i}$).
The **fractal code** τ of the whole original image is defined by the collection of all the local fractal codes τ_i. More precisely, each local fractal code $\tau_i : D_{q(i)} \to R_i$ is the composition of the following applications :

- A reduction $r_{i,n} : D_{q(i)} \to R_i$ by a factor 2^n such that $r_{i,n}(\mu_{|D_{q(i)}})$ is a block which has the same size of its associated range block $\mu_{|R_i}$. In this study, we consider domain blocks which are 2^n-times bigger than the range blocks, where n is a positive integer. If $R \times R$ (respectively $D \times D$) is the range blocks (*resp.* domain blocks) size, we suppose that $D = 2^n \cdot R$. To decimate the domain block, a reduction by an averaging is used.
  Let us consider the reduction $r_i = r_{i,1}$ by a factor 2 (such that the reduction $r_{i,n}$ can be expressed by the n-flod composition of the transformation r_i by itself). This reduction $r_i : D_{q(i)} \to R_i$ can be expressed by:

$$\left[r_i(\mu_{|D_{q(i)}})\right]_{k,l} = \frac{\sum\limits_{0 \leq s,t \leq 1} \left[\mu_{|D_{q(i)}}\right]_{2k+s,2l+t}}{4} \tag{1}$$

  for $k, l \in \{0, \ldots, R-1\}$, where $[\nu]_{k,l}$ shows the value of the pixel located on the row k and the column l in the subimage ν.

- A geometrical transformation $I_{e(i)}$ chosen among 8 discrete isometries $\{I_e\}_{e \in \{1,\ldots,8\}}$ such as identity, first and second diagonal symetries, horizontal and vertical symetries, rotations by $90°$, $180°$ and $-90°$.

- An affine mapping A_{α_i,β_i} on the luminance values such as $A_{\alpha_i,\beta_i} = \alpha_i \cdot id + \beta_i$, where id is the identity.

So, the local fractal code τ_i can be written as: $\tau_i = A_{\alpha_i,\beta_i} \circ I_{e(i)} \circ r_{i,n}$ and the global fractal code as: $\tau = \cup_{i=1}^{N} \tau_i$.
After the computation of τ during the coding stage, the loosy restitution μ_a of the original image μ is obtained using an iterative process : for an arbitrary initial image μ_0, the τ's attractor μ_a is computed with the recursive sequence $\mu_1 = \tau(\mu_0)$, $\mu_2 = \tau(\mu_1)$, $\ldots$, $\mu_a = \lim_{k \to \infty} \tau^k(\mu_0) = \tau(\mu_a)$. The main problem is to master the coding process using a good parameter choice, especially the upper bound α_m of the scales α_i.

3 State of the Art - Sufficient Convergence Conditions/Contractivity Constraints

The authors emphasize that all the following results about convergence depend on the metric used in each space. Moreover, in the following sections only results regarding strict contractivity are described although that is not necessary. In fact, the less restrictive concept of eventually contractive maps [3, 6] (i.e. mappings τ such that τ^k is a contractive mapping for some positive integer k) is sufficient, but it brings up much unchecked difficulties.

3.1 In the continuous case

Let us note $\mathcal{M}$ the space of non-empty compact subsets of the metric space $(\mathcal{R}, m)$ ($\mathcal{R} = I\!R^2$ for the binary images modeling). This space is a complete metric space [1] according to a Hausdorff metric h_m defined by:

$$\forall \mu, \nu \in \mathcal{M},\ h_m(\mu, \nu) = \max\{\tilde{m}(\mu, \nu), \tilde{m}(\nu, \mu)\} \tag{2}$$

where m is some metric on $\mathcal{R}$ ($m = d_2$ for example), and:
$\tilde{m}(\mu, \nu) = \max\{\min\{m(x, y), y \in \nu\}, x \in \mu\}$.
Let $\{\tau_i\}_{i=1}^N$ be an **Iterated Function System** (i.e. a finite set of contractive mappings τ_i), s_i the τ_i's contractivity factor and $\tau : \mathcal{M} \rightarrow \mathcal{M}$ an image transformation such that $\forall \mu \in \mathcal{M}, \tau(\mu) = \cup_{i=1}^N \tau_i(\mu)$. Then, one can proove that τ is also contractive according to the Hausdorff metric h_m (i.e. $\forall \mu, \nu \in \mathcal{M},\ h_m(\tau(\mu), \tau(\nu)) \leq s \cdot h_m(\mu, \nu)$) with the contractivity factor $s = \max_{1 \leq i \leq N} s_i$.
Therefore, the Fixed Point theorem implies that the sequence $\{\tau(\mu_0),\ \tau^2(\mu_0),\ \tau^3(\mu_0), \ldots\}$ (for an arbitrary initial image μ_0 belonging to $\mathcal{M}$) converges torwards an unique object μ_a in $\mathcal{M}$, called the **attractor** of τ, such that $\mu_a = \tau(\mu_a)$.
Hence, in the continuous case, one has:

$$\forall i,\ s_i < 1 \Rightarrow \forall \mu_0 \in \mathcal{M},\ \tau^k(\mu_0) \stackrel{k \to \infty}{\longrightarrow} \mu_a \tag{3}$$

In practice, a particular case of I.F.S. such as the **Partitioned Iterated Function System** (P.I.F.S.) is used. Moreover, the local fractal codes τ_i are not applied to the entire image, but only to a restriction such as the domain cell $D_{q(i)}$. A real grey-level image μ could be viewed as the graph of a Lebesgue measurable function f. Indeed, let $G = [0, 255]$ be the range of all allowed grey-level values, and $\mathcal{R} = I\!R^2$ be the space of the image supports, and $D_q(i)$, R_i be square cells in $\mathcal{R}$. Then, an image $\mu = \{(x, y, z = f(x, y)) : (x, y) \in \mathcal{S}\}$ belongs to the space $\mathcal{R} \times G$ ($\mathcal{S} = \cup_{1 \leq i \leq N} R_i$ is the compact support in $\mathcal{R}$ of the image μ) and $\tau_i : D_{q(i)} \times G \rightarrow R_i \times G$ is defined by:

$$\tau_i(x, y; z) = \begin{pmatrix} \zeta_{i,1} & \zeta_{i,2} & 0 \\ \zeta_{i,3} & \zeta_{i,4} & 0 \\ 0 & 0 & \alpha_i \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} t_{i,1} \\ t_{i,2} \\ \beta_i \end{pmatrix}$$

where α_i is the scale factor, β_i is the shift factor, $(t_{i,1}, t_{i,2})^T$ a translation vector in $\mathcal{R}$ which maps D_i onto R_i, and $(\zeta_{i,j})_j$ the matrix associated to the transformation $I_{e(i)} \circ r_{i,n}$ in the continuous case (the T superscript represents the transpose).
Let us notice that in this continuous model, the geometric (on the support) and massic (on the luminance) transformations are independent and then the contractivity constraint only affects (in case of P.I.F.S.) the z-axis (i.e. the luminance transform A_{α_i,β_i}). But in practice this is not the case because $r_{i,n}$ alters the luminance values.

3.2 In the discrete case

In practice, using P.I.F.S. for digitized image coding, the convergence of the restitution process is needed and the choice of the coding parameters plays an important role on this aspect. Although it is sub-optimum, the idea is to adapt the result (3), which could easily be applied during the coding stage, in the discrete case. So, several papers try to produce some hypotheses about the coding parameters ranges which can ensure the convergence during the decoding stage, mainly based on the upper bound of the scales.

3.2.1 Experimental approach

Several upper bounds on the scales have been used for fractal coders. These bounds are often found by a statistical approach such as testing the convergence of the process or the reconstruction quality [4] on a "large" enough set of digitized images using some sampled values of the maximum allowed scale. In [5] for example, some upper bounds α_m are given and it seems that $\alpha_m = 1.2$ or $\alpha_m = 1.5$ yields the best quality versus compression, but values like $\alpha_m = 1.0$ or $\alpha_m = 1.2$ are sometimes prefered because of a smaller number of iterations during the decoding stage (less than ten). Other authors [3] give out also $\alpha_m = 1.3$ or even $\alpha_m = 2.0$...
These bounds are less restrictive than the following ones, but are not based on a theoretical proof, and so induce polemics.

3.2.2 A *post* global sufficient condition in the general case - matrix notation

Using a linear algebra approach of the I.F.S. theory in the discrete case, a sufficient condition which ensures the convergence is proposed [4, 9]. But this condition cannot be applied during the coding stage to choose the coding parameters ranges.
The global fractal code $\tau = \cup_{i=1}^{N} \tau_i$, thanks to its affine form, can be written as:

$$\tau(\nu) = L \cdot \nu + b \tag{4}$$

where ν is some image belonging to $\Lambda \cong I\!R^K$, L is a K rows by K columns matrix and b is a vector of $I\!R^K$. Let us note σ_L the spectral radius of the matrix L (i.e. the largest eigenvalue of L). Then:

$$\boxed{\sigma_L < 1} \Rightarrow \forall \mu_0 \in \Lambda,\ \tau^k(\mu_0) \overset{k\to\infty}{\longrightarrow} \mu_a \tag{5}$$

Indeed, the τ's attractor μ_a is given by the formula:

$$\mu_a = \lim_{k\to\infty} \left(L^k \cdot \mu_0 + \sum_{l=0}^{k} L^l \cdot b \right) \tag{6}$$

where μ_0 is an arbitrary initial image in Λ.
So, the condition $\sigma_L < 1$ is only a *post* condition which only allows to theoretically verify whether the iterative process is convergent.

Particular case: Let us consider the set $\mathcal{D}$ which uses only *non-overlapping* domain cells D_q. Let note $I = \{1, \ldots, Q\}$ the set of all allowed domain cells indices ($\mathcal{D} = \cup_{q=1}^{Q}\{D_q\}$) and $I_q = \{i : (\tau_i : R_i \to D_q)\} = \{i : q(i) = q\} \subseteq \{1, \ldots, N\}$ the set of indices i such that the range cells R_i use D_q as a domain cell. Then, Lundheim [3] prooves that the global contractivity s of the fractal code τ is given by:

$$s = \frac{R}{D} \cdot \sqrt{\max_{q\in I} \sum_{i\in I_q} {\alpha_i}^2} \tag{7}$$

where s is defined in this context by $s = ||L||_2 = \sqrt{\sigma_{L^T L}}$ when using the euclidian norm $||\cdot||_2$ on Λ and $\sigma_{L^T L}$ is the spectrum radius of the matrix $L^T \cdot L$. So, one has:

$$\boxed{\max_{q\in I} \textstyle\sum_{i\in I_q} {\alpha_i}^2 < 4^n} \Rightarrow \forall \mu_0 \in \Lambda,\ \tau^k(\mu_0) \overset{k\to\infty}{\longrightarrow} \mu_a \tag{8}$$

3.2.3 No constraint is needed in a particular case - orthogonalization of Collage subspace bases

Øien & *al.* [3, 8] have developped a modified version of the Jacquin fractal coding algorithm. Using their method, based on an orthogonalization of decimated domain blocks with respect to the translation basis, no contractivity constraint is needed if we assume that *each domain block consists in an integer number of complete range blocks* (i.e. $\forall q, \exists J_q \subseteq \{1, \ldots, N\} : D_q = \cup_{i\in J_q} R_i$).
Let us note $\mathbf{1} = (1, \ldots, 1)^T$ the unit vector of $I\!R^{R^2}$, $\mathbf{R}_i$ the vector $\mu_{|R_i}$ where $\mu \in \Lambda$ is the digitized image to encode, and $\mathbf{D}_{i;q,e} = I_e \circ r_{i,n}(\mu_{|D_q})$ a decimated domain vector. Hence, the local collage error is defined by $\varepsilon_{c,i} = \varepsilon_{i;q(i),e(i)}^{\alpha_i,\beta_i}$ where $\varepsilon_{i;q,e}^{\alpha,\beta} = ||A_{\alpha,\beta}(\mathbf{D}_{i;q,e}) - \mathbf{R}_i||_2$.
Then, the Jacquin's coding algorithm can be expressed by : $\forall i \in \{1, \ldots, N\}$, $\tau_i = A_{\alpha_i,\beta_i} \circ I_{e(i)} \circ r_{i,n}$ and $\tau_i : D_{q(i)} \to R_i$ where α_i, β_i, $e(i)$, $q(i)$ verify:

$$\begin{aligned}
&\varepsilon_{c,i} = \inf_{e,q,\alpha,\beta} \varepsilon_{i;q,e}^{\alpha,\beta} = \inf_{e,q,\alpha,\beta} ||\alpha \cdot \mathbf{D}_{i;q,e} + \beta \cdot \mathbf{1} - \mathbf{R}_i||_2 \\
\Leftrightarrow\ & \begin{cases} \mathbf{R}_i - A_{\alpha_i,\beta_i}\left(\mathbf{D}_{i;q(i),e(i)}\right) \perp \overline{\{\mathbf{D}_{i;q(i),e(i)}, \mathbf{1}\}} \\ \varepsilon_{c,i} = \inf_{q,e} \varepsilon_{i;q,e}^{\alpha_i,\beta_i} \end{cases} \\
& \text{(projection theorem)} \\
\Leftrightarrow\ & \begin{cases} \left\langle \mathbf{R}_i - A_{\alpha_i,\beta_i}\left(\mathbf{D}_{i;q(i),e(i)}\right), \mathbf{1} \right\rangle = 0 \\ \left\langle \mathbf{R}_i - A_{\alpha_i,\beta_i}\left(\mathbf{D}_{i;q(i),e(i)}\right), \mathbf{D}_{i;q(i),e(i)} \right\rangle = 0 \\ \varepsilon_{c,i} = \inf_{q,e} \varepsilon_{i;q,e}^{\alpha_i,\beta_i} \end{cases}
\end{aligned} \tag{9}$$

where $\langle \mathbf{v}, \mathbf{w} \rangle = \sum_{s=1}^{R^2} v_s \cdot w_s$ for all $\mathbf{v} = (v_s)_s, \mathbf{w} = (w_s)_s \in I\!R^{R^2}$, $\sqrt{\langle \mathbf{v}-\mathbf{w}, \mathbf{v}-\mathbf{w} \rangle} = d_2(\mathbf{v}, \mathbf{w})$ and $\overline{\{\mathbf{v}, \mathbf{w}\}}$ is the 2-dimensional linear subspace spanned by the basis vectors $\mathbf{v}, \mathbf{w}$.

Moreover, for reasons of computational complexity, Øien makes all decimated domain blocks $\mathbf{D}_{i;q,e}$ orthogonal to the 1-dimensional linear subspace $\overline{\{\mathbf{1}\}}$. This corresponds to left-multiplying each vector $\mathbf{D}_{i;q,e}$ with the orthogonalization matrix $O = id - \mathbf{1} \cdot \mathbf{1}^T$. Then, the basis $\{O \cdot \mathbf{D}_{i;q,e}, \mathbf{1}\}$ replaces $\{\mathbf{D}_{i;q,e}, \mathbf{1}\}$ for the collage approximation. The new fractal code is given by:

$$\left\{ \begin{array}{l} \tilde{\tau}(\mu) = \tilde{L} \cdot \mu + \tilde{b} = \cup_{i=1}^{N} \tilde{\tau}_i \left(\mu_{|D_{q(i)}} \right) \\ \tilde{\tau}_i = A_{\tilde{\alpha}_i, \tilde{\beta}_i} \circ O \circ I_{e(i)} \circ r_{i,n} \end{array} \right. \tag{10}$$

In such a way, we only have to compute the best scales $\tilde{\alpha}_i$ for each domain block because the shifts $\tilde{\beta}_i$ computation is independent, for each range block, of the domain blocks used :

$$\left\{ \begin{array}{l} \tilde{\beta}_i = \langle \mathbf{R}_i, \mathbf{1} \rangle \\ \tilde{\alpha}_i = \frac{\langle \mathbf{R}_i, \mathbf{D}_{i;q(i),e(i)} \rangle - \tilde{\beta}_i \langle \mathbf{D}_{i;q(i),e(i)}, \mathbf{1} \rangle}{||\mathbf{D}_{i;q(i),e(i)}||_2 - \langle \mathbf{D}_{i;q(i),e(i)}, \mathbf{1} \rangle} \end{array} \right. \tag{11}$$

Let us remark that $\alpha_i = \tilde{\alpha}_i$, $\beta_i = \tilde{\beta}_i - \frac{\langle \mathbf{1}, \mathbf{D}_{i;q(i),e(i)} \rangle}{\alpha_i}$ and luckily τ and $\tilde{\tau}$ have the same attractor μ_a.

Moreover, this study proposes the number M of decoder iterations which only depends on domain and range blocks sizes. For example, when using an uniform range partition with $R = 2^r$ and $D = 2^d$ ($d > r$ and $n = d - r$), M is given by:

$$M = 1 + \left\lceil \frac{r}{n} \right\rceil \tag{12}$$

where $\lceil x \rceil$ denotes the smallest integer larger than x.

Hence, the τ's attractor μ_a is simply given by:

$$\mu_a = \sum_{k=0}^{M-1} \tilde{L}^k \cdot \tilde{b} \tag{13}$$

In particular, one has a non-iterative decoding process if $M = 2$ (i.e. when $r \le n \Leftrightarrow R^2 \le D$) and then $\mu_a = \tilde{L} \cdot \tilde{b} + \tilde{b} = \tilde{\tau}(\tilde{b})$.

3.2.4 A *pre* local sufficient condition when using non-overlapped range blocks

This section gives a bound α_m [6] on the scales α_i which ensures the **euclidian contractivity** aspect of all the local fractal codes τ_i i.e.

$$\begin{aligned} \forall i, |\alpha_i| < \alpha_m \quad \Rightarrow & \quad \forall i, \tau_i \text{ is contractive} \\ & \Leftrightarrow \forall i, s_i < 1 \end{aligned} \tag{14}$$

where s_i is the τ_i's contractivity factor. The metrics used in this section are the Euclidian metric d_2 for the blocks and a sup-metric d_∞ for the whole image. These metrics are defined by:

$$\begin{aligned} d_{2,i}(\mu_{|R_i}, \nu_{|R_i}) &= \sqrt{\sum_{0\leq k,l\leq R-1} ([\mu_{|R_i}]_{k,l} - [\nu_{|R_i}]_{k,l})^2} \\ d_\infty(\mu,\nu) &= \max_{1\leq i\leq N} d_{2,i}(\mu_{|R_i}, \nu_{|R_i}) \end{aligned} \tag{15}$$

where μ, ν are arbitrary images of Λ.
Let us note Λ_{R_i} the space of images obtained by restricting the space Λ to the cell R_i. Then, we consider here that $\tau_i : \Lambda \to \Lambda_{R_i}$, even if τ_i only alters the pixels belonging to the cell D_i ($\tau_i = A_{\alpha_i,\beta_i} \circ I_{e(i)} \circ r_{i,n}$ where $r_{i,n} : \Lambda \to \Lambda_{R_i}$ and $I_{e(i)}, A_{\alpha_i,\beta_i} : \Lambda_{R_i} \to \Lambda_{R_i}$). In this case, the global code $\tau : \Lambda \to \Lambda$ is equal to $\tau(\mu) = \sum_{1\leq i\leq N} \tau_i(\mu) = \cup_{1\leq i\leq N} \tau_i(\mu)_{|R_i}$. Therefore, we consider the following metric spaces: (Λ, d_∞) and $(\Lambda_{R_i}, d_{2,i})$ for $i \in \{1,\ldots,N\}$.
In order to find α_m, we have to accurately upper bound the contractivity s_i of the local code τ_i in function of α_i and impose the local contractivity constraint $\forall i,\ s_i < 1$.
The contractivity factor s_i is defined by:

$$\forall \mu,\nu \in \Lambda, d_{2,i}(\tau_i(\mu), \tau_i(\nu)) \leq s_i \cdot d_\infty(\mu,\nu) \tag{16}$$

Moreover, s_i is given [6] by the formula:

$$s_i = s_{r_{i,n}} \cdot s_{I_{e(i)}} \cdot s_{A_i} = s_{r_{i,n}} \cdot |\alpha_i| \tag{17}$$

where $s_{r_{i,n}}$ is the contractivity factor of the reduction $r_{i,n}$, $s_{I_{e(i)}}$ of the isometry $I_{e(i)}$ and s_{A_i} of the affine transformation A_i. Then, using equations (14) and (17), a sufficient local contractivity constraint is given by:

$$\alpha_m = \frac{1}{s_{r_{i,n}}} \tag{18}$$

Therefore, we only have to upper bound the contractivity factor $s_{r_{i,n}}$. Moreover, $s_{r_{i,n}}$ is given (using a suitable n-fold composition of r_i by itself) by:

$$\forall i,\ s_{r_{i,n}} = {s_{r_i}}^n \tag{19}$$

The r_i's contractivity factor s_{r_i} is then found by considering the following inequality:

$$d_{2,i}(r_{i,n}(\mu), r_{i,n}(\nu)) \leq d_\infty(\mu,\nu) \tag{20}$$

i.e. $s_{r_i} = 1$, and, using equation (19), we obtain $s_{r_{i,n}} = 1$.
Hence, using equation (18), the contractivity of the local codes τ_i is ensured if:

$$\alpha_m = 1 \Leftrightarrow \forall i \in \{1,\ldots,N\}, \boxed{\alpha_i < 1} \tag{21}$$

4 Conclusion

In this paper, we summerized the main results about coding hypotheses which ensure the convergence of the iterative decoding process. Unfortunately, the results (5) and (8) cannot be used in practice to constrain the coding parameters. Athough it gives a sub-optimum constraint for convergence, only the strict contractivity aspect of all the local fractal codes (ensured using the upper bound on the scales (21)) is theoretically proven and can be applied efficiently during the coding stage. Of course, the best result would be the global euclidian contractivity...

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