

POWER CONTROL SCHEMES FOR TDD SYSTEMS WITH MULTIPLE TRANSMIT AND RECEIVE ANTENNAS

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ABSTRACT

This paper investigates the performance of narrowband, slowly-fading, and delay-limited multiple-antenna systems where *channel state information* (CSI) is available at the transmission end. This situation can arise in *time-division duplex* (TDD) based two-way systems where channel state estimation can be performed using the signal received from the opposite link. Power control methods which attempt to keep the transmission rate constant at the expense of randomizing the transmit power are considered. It is shown that significant savings in average transmit power (sometimes on the order of tens of dB) can be expected compared to systems which keep the total transmit power constant. Several practical channel coding examples using are illustrated and their bit and frame error rate performance are discussed.

1. INTRODUCTION

The aim of this paper is to investigate the performance improvements which can be gained by employing *channel state information* (CSI or side information) in the form of *power control* at the transmission end of a multi-antenna radio communication system. The first question that one may ask is how realistic is it to assume that quasi-perfect CSI can be made available at the transmission end. The answer depends strongly on the system architecture. If we employ the same antenna array for transmission and reception (which is not always the case in current FDD systems) in a TDD system then channel reciprocity allows us to use channel estimates obtained during reception for transmission. We will show that the performance improvements over modern antenna processing schemes are dramatic when CSI is exploited. This provides a strong argument for employing TDD, when possible, in next generation wireless systems.

This paper approaches the problem more from the point of view of information theory rather than signal processing. Recent papers on antenna processing us-

ing a similar approach include [1, 2, 3, 4, 5]. We will make use of some key results from these studies in order to analyse the benefits of employing CSI at the transmitter. In this work we focus on delay-sensitive systems and/or non-ergodic channels. For such cases (see e.g. [6, 3, 7, 8]) the instantaneous average mutual information between the transmitted and received signals depends on the channel state and is therefore random. In practice, a situation such as this will arise when the number of antennas is small and little or no time/frequency variation occurs in the duration/band of the transmitted signal. Mathematically speaking, the number of significant degrees of freedom characterizing the randomness of the channel process is small enough that ergodic arguments cannot be invoked in analyzing system performance. To counter the randomness of the channel, generalized power control can be employed to maintain a constant average mutual information between transmitter and receiver, at the expense of rendering the transmit power random. The direct consequence of this will be that for certain channel realizations the expended transmit power will be higher than for others. Our figure of merit, therefore, is the average transmit power to maintain a constant average mutual information (or maximum information rate for which reliable communication is possible) between the transmitter and receiver.

2. MULTIAN TENNA TRANSMISSION/RECEPTION

Consider a multi-antenna signaling scheme which uses M antennas in the transmitter and N antennas in the receiver. The same antenna is used for transmission and reception at the same carrier frequency so that channel reciprocity holds. We assume that the M transmitted signals, $s_i(t)$, $i = 1, \dots, M$ are narrowband QAM

signals of the form

$$\tilde{s}_i(t) = \sum_k \sqrt{P_i(\mathbf{A})} u_{i,k} g(t - kT), \quad i = 1, 2, \dots, M$$

where $u_{i,k}$ is the k^{th} complex symbol on the i^{th} transmit antenna and $g(t)$ is a signaling pulse such that its Fourier transform, $G(f)$ is zero for frequencies $f > W/2$, where W is the bandwidth of the transmitted signal. We assume that $g(t) * g^*(-t)$ satisfies Nyquist's criterion for zero intersymbol interference and that the average energy of the $u_{i,k}$ is $E |u_{i,k}|^2 = 1$. The constants $P_i(\mathbf{A})$ are the instantaneous power allocated to each transmit antenna element. The real transmitted signals are $s_i(t) = \text{Re}(\tilde{s}_i(t) e^{j2\pi f_c t})$, $i = 1, 2, \dots, M$ where f_c is the carrier frequency.

The $s_i(t)$ are transmitted over a static L -path multipath channel with NM impulse responses $h_{ij}(t) = \sum_{l=1}^L a_l \delta(t - d_l(i, j))$, where a_l and $d_l(i, j)$ are the gain and delay of the l^{th} path between transmitter i and receiver j . The channel remains static during the transmission of long codewords but can change from codeword to codeword. We assume that $W(d_l(i, j) - d_1(i, j)) \ll 1$ so that we may approximate the complex baseband equivalent signal seen by the j^{th} receiver by

$$\tilde{r}_j(t) = \sum_{i=1}^M \sum_k \sqrt{P_i(\mathbf{A})} A_{i,j} u_{i,k} g(t - kT) + \tilde{z}_j(t),$$

$$j = 1, 2, \dots, N$$

where A_{ij} is the complex gain of the i^{th} transmitter and j^{th} receiver given by

$$A_{ij} = \sum_{l=1}^L a_l e^{j2\pi f_c d_l(i, j)}.$$

The $\tilde{z}_j(t)$ are circularly symmetric additive white (in the band of the signal) Gaussian noise with power spectral density N_0 .

As is common in the literature we take the A_{ij} to be complex Gaussian random variables with variance σ_A^2 and mean μ_{ij} . We will assume that no direct path between the transmitter and receiver exists so that $\mu_{ij} = 0$. Assuming that the receiver employs maximum-likelihood detection using filters $g^*(-t)$ sampled at instants $t = kT$ we have the discrete-time channel model

$$r_{k,j} = \sum_{i=1}^M \sqrt{P_i(\mathbf{A})} A_{i,j} u_{k,i} + z_{k,j}, \quad j = 1, \dots, M$$

or in vector form

$$\mathbf{r}_k = \mathbf{A} \text{diag}(\mathcal{P}_i(\mathbf{A})) \mathbf{u}_k + \mathbf{z}_k$$

where \mathbf{A} is an $N \times M$ matrix of complex channel gains.

2.1. Receiver and Transmitter Channel State Information

The A_{ij} are assumed to be known perfectly to the receiver. This can be achieved by inserting training sequences (possibly a different one for each transmit antenna!) which allow for quasi-perfect estimation of the A_{ij} and at the same time do not significantly reduce information rates. As mentioned in the introduction, the results of this work are intended to provide a strong argument for employing time-division duplex with the same antennas used for transmission and reception. We will therefore also assume that the A_{ij} may be known to the transmitter. The channel state information used during transmission will be taken from the signal received during the previous time-slot.

2.2. Parallel Channel Decomposition

As in [3, 2] which generalize the continuous-time frequency-selective channel described in [9, Chap. 8] to a discrete-time multi-antenna system, we decompose \mathbf{A} using its *singular value decomposition* $\mathbf{A} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^*$ where \mathbf{U} and \mathbf{V} are $N \times N$ and $M \times M$ unitary matrices and

$$\mathbf{\Sigma} = \begin{cases} \begin{bmatrix} \sqrt{\Lambda} & \mathbf{0} \end{bmatrix} & M \geq N \\ \begin{bmatrix} \sqrt{\Lambda} \\ \mathbf{0} \end{bmatrix} & M < N \end{cases}$$

and Λ is a $\min(N, M)$ -dimensional diagonal matrix containing the eigenvalues of $\mathbf{A} \mathbf{A}^*$ or $\mathbf{A}^* \mathbf{A}$. Since \mathbf{A} is known to the transmitter, $\mathbf{U}, \mathbf{\Sigma}$ and \mathbf{V} can, at least in principle, be computed before transmission or reception. We may therefore transform the detection problem, without loss of generality, as

$$\mathbf{r}'_k = \mathbf{U}^* \mathbf{r}_k = \mathbf{\Sigma} \text{diag}(\mathcal{P}'_i(\mathbf{A})) \mathbf{u}'_k + \mathbf{z}'_k$$

where $\text{diag}(\mathcal{P}'_i(\mathbf{A})) \mathbf{u}'_k = \mathbf{V}^* \text{diag}(\mathcal{P}_i(\mathbf{A})) \mathbf{u}_k$ and \mathbf{u}'_k has unit variance components. The $\mathcal{P}'_i(\mathbf{A})$ are the power of the transmitted signal components in the transform domain. Note that either the transformed input or transformed output may be reduced in dimension if \mathbf{A} is not square. We have the equivalent parallel channel representation

$$r'_{i,k} = \sqrt{\lambda_i} \mathcal{P}'_i(\mathbf{A}) u'_{i,k} + z'_{i,k}, \quad i = 1, \dots, \min(M, N).$$

3. $N \times 1$ AND $1 \times M$ ANTENNA DIVERSITY

We first focus on the case where the parallel channel decomposition results in a single channel representation. Assuming we transmit long codewords and the channel is static during the codeword, the instantaneous average mutual information (channel capacity) as a function of λ_1 and $\mathcal{P}(\lambda_1)$ is given by

$$C(\mathcal{P}(\lambda_1), \lambda_1) = W \log_2 \left(1 + \frac{\mathcal{P}(\lambda_1) \lambda_1}{W N_0} \right) \quad \text{bits/s}$$

assuming an average symbol energy of one and minimum bandwidth pulse shapes $g(t)$. It is achieved if the $u'_{1,k}$ are zero-mean, independent, circular symmetric complex Gaussian random variables [9, Chap. 7]. The meaning of this quantity is that reliable communication is impossible if $R < C(\mathcal{P}(\lambda_1), \lambda_1)$, or that R must be adjusted as a function of $\mathcal{P}(\lambda_1)\lambda_1$. If we wish to maintain a constant channel capacity (or practically transmit at a constant rate) then the power controller must satisfy

$$\mathcal{P}(\lambda_1) = \frac{WN_0(2^{W^{-1}R} - 1)}{\lambda_1}$$

and the average transmitted power (taken over all realizations of λ_1) is

$$\bar{P} = WN_0(2^{W^{-1}R} - 1) \int_0^\infty \frac{f_{\Lambda_1}(u)}{u} du$$

In Rayleigh fading, it is simple to show that λ_1 is Chi-square distributed with N (transmit antennas) or M (receive antennas) degrees of freedom. This yields

$$\bar{P} = \frac{WN_0(2^{W^{-1}R} - 1)}{X - 1}, X = M, N$$

For a transmit diversity system, this corresponds to a *beamforming* network (i.e. where the phase of each antenna element is chosen such that all paths combine coherently at the receive antenna). For receive diversity it is a *maximal ratio combiner*. We remark that for $N = M$, the performance of transmit and receive diversity systems is the same when power control is used. This is not the case without power control (see e.g. [3]).

3.1. Comparison with Space-Time Coding

In the case where equal power \bar{P}/M is assigned to each transmit antenna, the information outage probability is [3, 4]

$$P_{\text{out}}(R) = \Pr \left(\log \det \left(\mathbf{I} + \frac{\bar{P}}{MWN_0} \mathbf{A}^* \mathbf{A} \right) < W^{-1}R \right)$$

This outage rate indicates the practical FER performance of *space-time codes*. For a $1 \times M$ system in Rayleigh fading this simplifies to

$$P_{\text{out}}(R) = 1 - Q_M(0, \sqrt{\beta})$$

where $\beta = \frac{MWN_0}{\bar{P}}(2^{W^{-1}R} - 1)$ and $Q_M(\cdot)$ is the *Marcum Q function of order M* [10]. The effect of beamforming is evident since we obtain gains on the order of 12dB compared to space-time coding for frame error-rates around 10^{-2} . Much more impressive gains can be had at lower FER. We may conclude, therefore, that under the assumption that fairly simple codes *designed*

for the AWGN channel can bring us to within a few dB from channel capacity, we may obtain huge reductions in average transmit power with respect to an *optimal* space-time coding scheme.

This is demonstrated in Figure 2 where we show the simulated *bit-error rate (BER)* and *frame-error rate (FER)* of a dual transmit antenna ($L = 2$) QPSK 4-state space-time code (taken from [1]) with fixed transmit power and uncoded QPSK with power control for the same *average* transmitted $\mathcal{E}_b/N_0 = \bar{P}/RN_0$, where $R = 2$ is now the number of bits per symbol. This was carried out under the assumption of independent Rayleigh fading for each antenna element. We chose a block length of 130 QPSK symbols in both cases. Note that even with uncoded modulation, the effect of power control is a dramatic reduction in average transmit power. Moreover, since power control keeps the *received* SNR constant, any standard coding scheme for AWGN channels can be employed while maintaining the same amount of coding gain.

4. GENERALIZED POWER-CONTROL

Let us now consider the more general case where $M, N > 1$.

For the set of $\min(M, N)$ parallel Gaussian channels in (2.2) we have that the instantaneous channel capacity is given by [9, Chap 7]

$$C(\mathcal{P}'_i(\mathbf{\Lambda}), \mathbf{\Lambda}) = \sum_{i=1}^{\min(M, N)} \log_2 \left(1 + \frac{\mathcal{P}'_i(\mathbf{\Lambda})\lambda_i}{WN_0} \right) \quad (1)$$

and is achieved by using input symbols u'_{ik} which are independent Gaussian random variables. Moreover, we are interested in choosing the $\mathcal{P}'_i(\mathbf{\Lambda})$ such that the total power is minimum for each realization of the $\{\lambda_i\}$ subject to the fixed rate constraint $\sum_{i=1}^{\min(M, N)} R_i = R$, where $R_i = \log_2 \left(1 + \frac{\mathcal{P}'_i(\mathbf{\Lambda})\lambda_i}{WN_0} \right)$. Note that this is not exactly the standard water-filling optimization [9, Chap. 7]. Here we assure that we always transmit reliably at a fixed rate while minimizing the long-term average power. This optimization is a special case of the general multiuser framework considered in [11]. In addition it is also an important special case of the single-user parallel channel information outage probability minimization [8]. Specifically we have

$$\min \sum_{i=1}^{\min(M, N)} \frac{2^{R_i} - 1}{\lambda_i} \quad \text{subject to} \quad \sum_{i=1}^{\min(M, N)} R_i = R \quad (2)$$

which is a standard concave optimization problem (see e.g. [9, Chap 4]). Applying the Kuhn-Tucker condi-

tions yields the following power controllers

$$\begin{cases} \mathcal{P}'_1(\mathbf{A}) = W N_0 \frac{2^R - 1}{\lambda_1}, \mathcal{P}'_2(\mathbf{A}) = 0, & \lambda_1 \geq 2^R \lambda_2 \\ \mathcal{P}'_1(\mathbf{A}) = 0, \mathcal{P}'_2(\mathbf{A}) = W N_0 \frac{2^R - 1}{\lambda_1}, & \lambda_2 \geq 2^R \lambda_1 \\ \mathcal{P}'_1(\mathbf{A}) = W N_0 \frac{2^{R/2}}{\sqrt{\lambda_1 \lambda_2}} - \frac{1}{\lambda_1}, \mathcal{P}'_2(\mathbf{A}) = \frac{2^{R/2}}{\sqrt{\lambda_1 \lambda_2}} - \frac{1}{\lambda_2} \\ \text{otherwise} \end{cases}$$

In the optimal power control scheme, we see that if one channel (eigenvector) is much stronger (depending on the rate constraint) than the other, only the stronger one is used. Otherwise, both are used. Note that in the latter case, the received SNR on each channel is not constant, but depends on the relative strengths of the two channels. Unlike the single channel case, this implies that standard AWGN codes need not be effective for this type of system, and special codes must be designed. A simple sub-optimal modification (Scheme I) of this scheme would be to use a rate R code on the stronger channel when one channel is K times stronger than the other and a rate $R/2$ code on each channel otherwise. K is a parameter to be determined. When $K = 1$ we simply select the best channel (selection diversity). In this power control scheme, we could keep the received SNR constant and use AWGN codes with predictable performance.

4.1. Numerical Results

In order to determine the average transmit power needed to communicate reliably at rate R , we must determine $f_{\lambda_1, \lambda_2}(u, v)$. For i.i.d. Gaussian components in \mathbf{A} the distribution of the ordered eigenvalues is known in closed-form [12, 3] and that of the unordered eigenvalues in [3]. The ordered p.d.f. is given by

$$f_{\lambda_{\max}, \lambda_{\min}}(u, v) = K_2(D) e^{-u-v} u^D v^D (u - v)^2,$$

where $K_2(D)$ is a normalizing constant. Although the average powers for both schemes can be computed analytically, the expressions involve sums of hypergeometric numbers, which are difficult to compute. They are more easily computed using Monte Carlo averaging. It is straightforward to show that $K = 1$ minimizes the average transmit power in the sub-optimal scheme, which yields a very simple one-dimensional transmission technique, since the transmit signal lies solely in the dimension of the eigenvector corresponding to the largest eigenvalue of $\mathbf{A}\mathbf{A}^*$.

Another even simpler sub-optimal signaling scheme (Scheme II) which does not require an eigenvalue decomposition is as follows. On the end with $U = \max(M, N)$ antennas, we perform either beamforming (transmission) or maximal ratio combining (reception) to obtain the largest gain. On the end with $L = \min(M, N)$ antennas we select the antenna yielding the strongest received signal-to-noise ratio. In this case the single channel gain is $\alpha = \max_i \|\mathbf{a}_i\|^2, i = 1, \dots, L$ and

\mathbf{a}_i is the i^{th} U -dimensional row or column of \mathbf{A} . In Rayleigh fading the average transmit power can be computed in closed-form.

In figure 3 we show the BER/FER of uncoded QPSK using the two suboptimal power control schemes outlined above with $N = M = 2$ and block sizes of 130 QPSK symbols. We also show the outage probabilities for optimal space-time coding schemes. Although the gains due to beamforming are smaller than in the $1 \times N$ antenna diversity case, power reductions on the order of 5 dB can be expected.

5. CONCLUSIONS

In this paper, we examined dynamic power control schemes based on quasi-perfect channel gain estimates for narrowband, delay-limited multi-antenna systems with slow Rayleigh fading. The principal results is that significant reductions in average transmit power can be expected compared to systems where the transmit power is fixed. Specifically when a single antenna is present on one end, the power reduction can be greater than 10dB even with a small array (≤ 4 elements). Moreover, the performance is independent of whether the array is transmitting or receiving, which is not the case in fixed power systems [3]. This is because CSI at the transmitter allows us to employ *beamforming* which is analogous to *maximal-ratio combining* at the transmitter, in conjunction with power-control.

We then considered generalized beamforming systems where multiple-antennas are present both in the transmitter and receiver. Here the performance is again independent of the direction of communication, which is not the case in systems with fixed transmit power. We show that when the minimum number of elements is two, considerable power savings can be expected, however less than the case where a single-antenna is present on one end.

Issues such as the effects of outdated and/or noisy channel estimates and extensions for multiuser systems are currently under investigation.

6. REFERENCES

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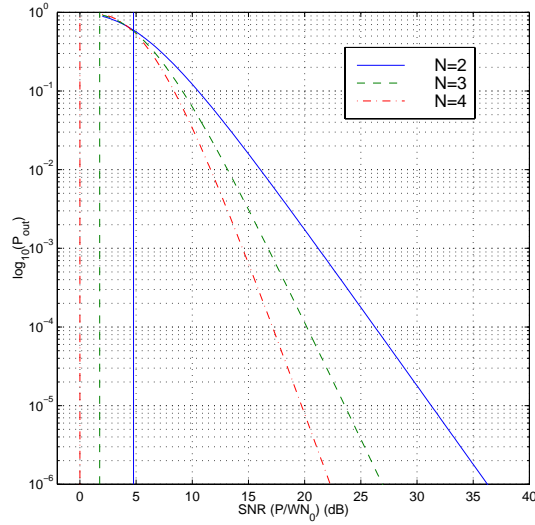


Figure 1: Space-time Codes Comparison

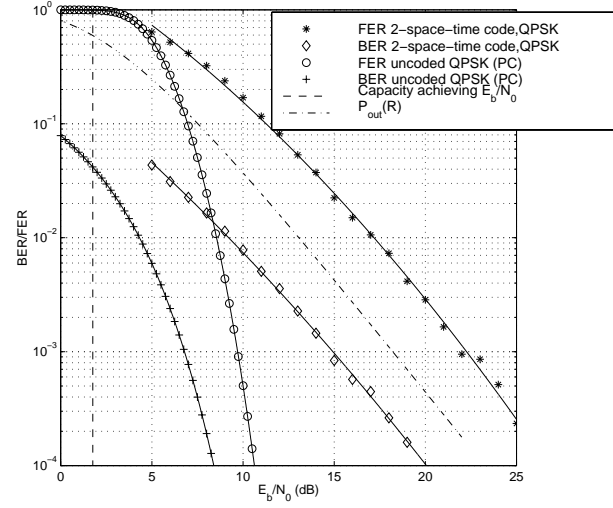


Figure 2: BER/FER Comparison of Power Controlled QPSK and 2-Space-time QPSK Codes (4-state)

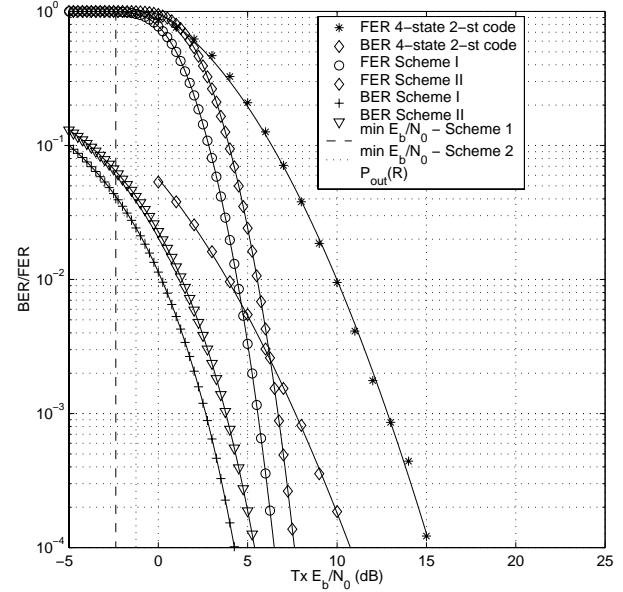


Figure 3: BER/FER Comparison ($R = 2$ bits/s/Hz)