Optimized Linear Receivers and Power Allocation for Two Multi-User MIMO Downlink Schemes with Linear Precoding

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Abstract— **In this paper, we propose some ameliorations and a comparison of linear closed form precoding techniques for Multi-User MIMO (MU-MIMO) systems. In fact, the sum-rate, one of the most important parameters in the design of communication systems, is analysed through the study of two existing linear precoding schemes. These schemes are the PU-MMSE precoder named PU-MMSE and the Max SJNR scheme (Signal to Jamming plus Noise ratio). Two major improvements are introduced to the existing algorithms in order to increase the throughput of the system. The first modification consists in proposing an alternate receiver namely an MMSE receiver to enhance the performance. The second modification consists in introducing a power optimization involving a user selection procedure at the transmitter. Finally, these improvements have been validated by means of simulations. Indeed, important performance improvements are obtained thanks to these modifications. Moreover, this allows us to perform a comparison of these two linear precoding techniques. The comparision demonstrates that the SJNR precoder is offering better system throughputs than the PU-MMSE precoder.**

I. INTRODUCTION

Multiuser MIMO (MU-MIMO) downlink system known in the information theory as the broadcast channel system represents today one of the most important research fields in wireless communications because of the potential for improving both reliability and capacity of the system. Some theoretical analysis of the capacity demonstrated that the capacity of a broadcast MU-MIMO channel can be achieved by applying a Dirty-Paper Coding (DPC) [1, 2] algorithm as a pre-coder. Nevertheless, a DPC precoding is difficult to compute and is high resource consuming. Some suboptimal linear algorithms with low implementation complexity exist and can be divided into two families: the iterative [3, 4] and the closed form solutions [5, 6, 7].

In this paper we are going to focus on the closed form linear solutions for two reasons. The first one is that such solutions are a one formula based algorithm making it possible to generate a MU-MIMO pre-coder through one iteration. Such an algorithm is therefore requiring low computational resources. The second reason is that some iterative algorithms may converge to a local minimum [4] or even some times diverge.

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Among the closed form linear solutions presented in the literature, two seem to be interesting. The first algorithm is a Per User Minimum Mean Square Error (PU-MMSE) based linear pre-coder [5] and the second one is a maximum Signal to Jamming and Noise Ratio (SJNR) [6] linear precoder.

In these papers, the authors presented two couples of precoder/receiver in order to enhance the system performance. Their approach is take into consideration the inter-user interference and build a precoder minimizing the received amount of interference for all users. However, the main work is focused on the design of the transmitter neglecting the receiving part. Indeed, the choice of the receivers has been made assuming that the precoders succeed in completely eliminating the interference of all other users. The reality is that by analysing the received signal for user k, some residual interference is still present.

Moreover, in a MIMO system, more than one receiving antenna is present at the terminals generating reception diversity that can be exploited to ameliorate the signal to interference and noise ratio. This property of the MIMO system is not fully used in the presented algorithms of [5] and [6]. Another important parameter to take into consideration for in inter-user interference cancellation at the transmitter is to make a power allocation control while distributing it among the different streams. In fact, the first precoding algorithm (PU-MMSE) is performing through the precoders calculation process an indirect power allocation. On the other hand the second algorithm with the SJNR precoder does not define any power distribution. This approach degrades dramatically the performance of the system.

The next section is introducing the model of the MU-MIMO system considered in our work, followed by a presentation of the two considered precoding algorithms, the PU-MMSE of [5] and the SJNR of [6]. Section III describes the two enhancements that we propose to these algorithms. The section after presents simulation context and obtained results for the comparison of the two techniques and the gain obtained thanks to our improvements.

II. SYSTEM MODEL

Lets consider in our study a multi-user MIMO communication system with N_T transmission antennas at the base station and K different users with N_{R_k} receiving antennas for each user k.

We assume that the base station has a perfect knowledge of the channel state information (CSI) of all K users. Let S_k a $Q_k \times 1$ vector representing the transmitted data symbols for user k where Q_k is the number of transmission streams for the same user. In our paper we are interested in the case of one stream per user $Q_k = 1$.

The total transmit power at the base station is supposed to be constant and equal to P_T. The noise variance N_0 is equal to 1. For the channel part, H_k denotes the MIMO channel for user k which is a $N_{R_k} \times N_T$ matrix.

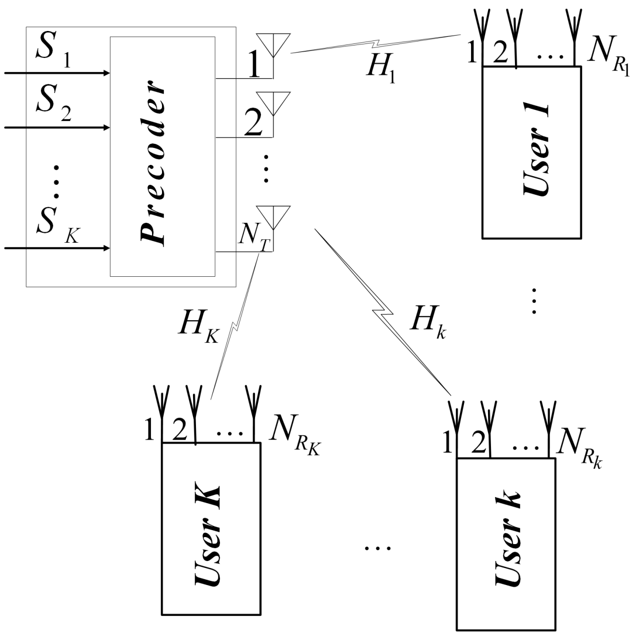

Fig. 1. System model.

A. PU-MMSE

The PU-MMSE algorithm proposed in [5] is a simplification of the successive MMSE (SMMSE). Here, a pre-coder is designed for each user taking into consideration all signals transmitted to the other ones as interference. Equation (1) is used to compute the pre-coder.

$$\hat{T}_k = \left(\tilde{H}_k^H \tilde{H}_k + \frac{N_{R_k}}{P_T} N_0 I\right)^{-1} \hat{H}_k^H \tag{1}$$

Where

$$\tilde{H}_k^T = \left[H_1^T \cdots H_{k-1}^T H_{k+1}^T \cdots H_K^T\right] \tag{2}$$

and

$$\hat{H}_k^T = \left[H_k^T \tilde{H}_k^T\right] \tag{3}$$

To maximize the transmitted power in the best direction, an SVD decomposition is applied to the virtual channel composed of the cascade of the channel and the transmitter $H_k \hat{T}_k$. And the eigenvector $V_k^{(1)}$ corresponding to the largest eigen value is considered. The final obtained pre-coder is then given by equation (4).

$$T_k = \beta \hat{T}_k V_k^{(1)} \tag{4}$$

where

$$H_k \hat{T}_k = U_k \Sigma_k V_k^H \tag{5}$$

and

$$\beta = \sqrt{\frac{P_T}{tr\left(\sum_{j=1}^{K} \hat{T}_j V_j^{(1)} \left(\hat{T}_j V_j^{(1)}\right)^H\right)}} \tag{6}$$

As a decoder for this system, the left hand eigenvector (LEV) matching the largest eigen value of the virtual channel decomposition is used. This ensures that the maximum power is extracted.

$$D_k = \left(U_k^{(1)}\right)^H \tag{7}$$

B. SJNR algorithm

The SJNR is defined as the ratio of the useful power for user k over the jamming and noise powers. The jamming power is the total power of interference caused by the user k and received by the other mobiles. This algorithm proposed in [6], considers the generalized eigen value of the SJNR expression given in equation (8).

$$SJNR_k = \frac{T_k^H H_k^H H_k T_k}{T_k^H \sum_{j=1, j\neq k}^{K} H_j^H H_j T_k + N_0 I} \tag{8}$$

The corresponding pre-coder for user k is then given in (9).

$$T_k = \sqrt{P_k}\zeta_m \left[\left(\sum_{j=1, j\neq k}^{K} H_j^H H_j + \frac{N_0}{P_k} I\right)^{-1} H_k^H H_k\right] \tag{9}$$

Where P_k denotes the transmitted power for user k with $\sum_{k=1}^{K} P_k = P_T$ and $\zeta_m[x]$ a function computing the largest eigenvector of x. The largest eigenvector is defined as the eigenvector corresponding to the largest eigen value of x. It must be noted that in the case of an hermitian semidefinite positive matrix the eigen decomposition is equivalent to an SVD(Singular Value Decomposition) and that the generated singular values are in an decreasing order.

As a receiver [6] proposes a matched filter (MF) given by equation (10).

$$D_k = \frac{(H_k T_k)^H}{\|H_k T_k\|} \tag{10}$$

The sum rate SR for this system is given by equation (11) [8, 9]. R_{S_k} here represent the covariance matrix of the transmitted signal S_k for user k. In this paper $R_{S_k} = I_{Q_k}$.

III. Proposed Improvements

In our study, we base our work on the two algorithms explained in section II, and propose two major modifications. The first modification is made at the receiver side; we propose an alternate receiver to optimize the sum rate SR of the system given by equation (11). In a MU-MIMO system, the major optimization is done at the transmission side. Nevertheless, in the presence of multiple receive antennas, an optimization at the reception can help in improving the sum-rate of the system. The second amelioration consists in introducing a management in the power allocation between the different users to counter balance the interference between users.

$$SR = \sum_{k=1}^{K} log_2 \left(det \left(I_{Q_k} + D_k H_k T_k R_{S_k} T_k^H H_k^H D_k^H \left(D_k \left(H_k \sum_{j=1, j\neq k}^{K} T_j R_{S_j} T_j^H H_k^H + N_0 I \right) D_k^H \right)^{-1} \right) \right) \quad (11)$$

A. Receiver design

The proposed receivers in the considered solutions are suboptimal receivers. In fact, the precoding procedure produces one stream per user by performing a beamforming at the transmission minimizing the inter-user interference. However, not all the interference coming from other users is completely eliminated. This shows that the different virtual streams can not be considered as orthogonal and thus makes these receivers suboptimal.

However, seen from the users' part, the system is as a one stream signal part with multi reception antennas and some interference. To maximize the Signal to Iterference and Noise Ratio of such a system the optimal receiver [3, 10, 11] is an MMSE one given by equation (12).

$$D_k = T_k^H H_k^H \left(H_k \left(\sum_{i=1}^{K} T_i T_i^H \right) H_k^H + N_0 I \right)^{-1} \quad (12)$$

That is why we propose a MMSE receiver to increase the total system performance by optimally exploiting the receiving diversity.

B. User selection procedure

Another way to deal with the inter-user interference is to apply power allocation algorithms. But the design of the system makes it difficult to perform a power allocation to optimize the system performance. Moreover, as the different streams are not orthogonal, the famous Waterfilling technique used in the case of single user MIMO can not be applied in this case. Therefore, we are considering a scheduling procedure to optimize indirectly the power allocation and minimize the impact of "bad" users.

So the second amelioration consists in introducing a power allocation optimization among all users. We select some streams among all the streams of our users, give these selected streams all the transmit power, neglect the non selected users and try to find the best set of streams that maximizes the sum rate of the system.

The algorithm turns on and off one by one the users and evaluates the sum rate obtained. The calculation of the pre-coders is then performed only on the basis of the selected users using formula (4) (respectively formula (9)) for the PU-MMSE pre-coder (respectively the SJNR pre-coder). This user selection procedure is feasible thanks to the low computational complexity of the closed form solutions.

IV. Simulations And Results

In all our simulations, we consider that we have only one stream per user $Q_k = 1$ and that the number of receiving antennas is the same for all users $N_{R_k} = N_R$ and in most cases $N_R = 2$. We choose a Rayleigh fading channel $H_k = (h_{i,j}^k)_{1\leq i\leq N_R, 1\leq j\leq N_T}$ such as $E\|h_{i,j}^k\|^2 = 1$. The simulation generates 10000 independent channel realizations for each user. To generate the total throughput of the system, we perform an average over all channel realizations on the quantity SR given in equation (11). For the SJNR precoder, we distribute the energy equally over all considered users according to $P_k = P_T / K$.

For the figures presented in this chapter, the *"TX MMSE RX LEV"* represents the MU-MIMO system with the PU-MMSE pre-coder and left hand eigen vector corresponding to formula (7); the *"TX MMSE RX MMSE"* is for the system with the PU-MMSE pre-coder and an MMSE receiver given in formula (12); *"TX SJNR RX MF"* represents the curves of the system with an SJNR pre-coder and a matched filter receiver given by (10); *"TX SJNR RX MMSE"* are the curves for an SJNR pre-coder and an MMSE receiver given in formula (12).

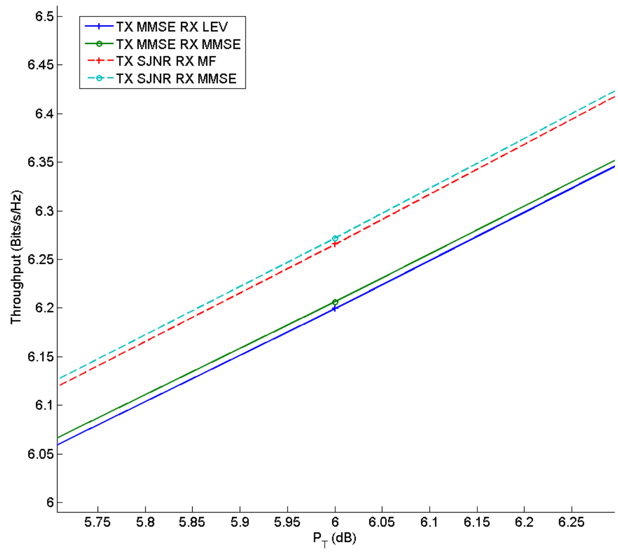

Fig. 2. Throughput in function of transmit power P_T for $N_T = 4$, $N_R = 2$ and $K = 2$.

Figures 2 and 3 compare the MMSE receiver with the proposed receivers in [5] and [6] for the case of $N_T = 4$ and demonstrate that MMSE receiver gives improvements on the two precoding procedures with different user parameters $K = 2$ and 3. This gain is very low for the case of two users as the interference is very low. In fact in a 2 user system with one stream per user using 4 transmitting antennas leaves 2 degrees of freedom. These degrees of freedom give the pre-coder the possibility to reduce further the interference part. In the case of 3 users, the gain introduced

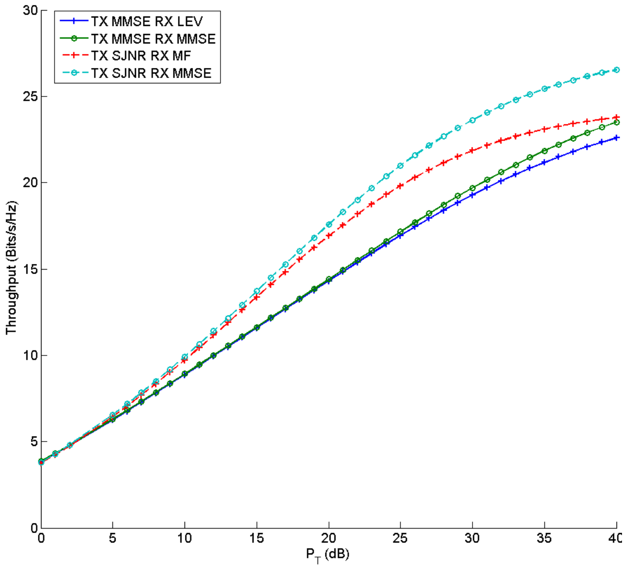

Fig. 3. Throughput for $N_T = 4$, $N_R = 2$ and $K = 3$.

by our receiver is higher as the interference increases and the degrees of freedom are decreased by 1. Nevertheless, the gain for the PU-MMSE pre-coder case remains low. We can also see that the sum rate of the SJNR solution is starting to saturate at very high transmit power although it remains better than the PU-MMSE pre-coder. This demonstrates the limits for using such a linear closed form pre-coder.

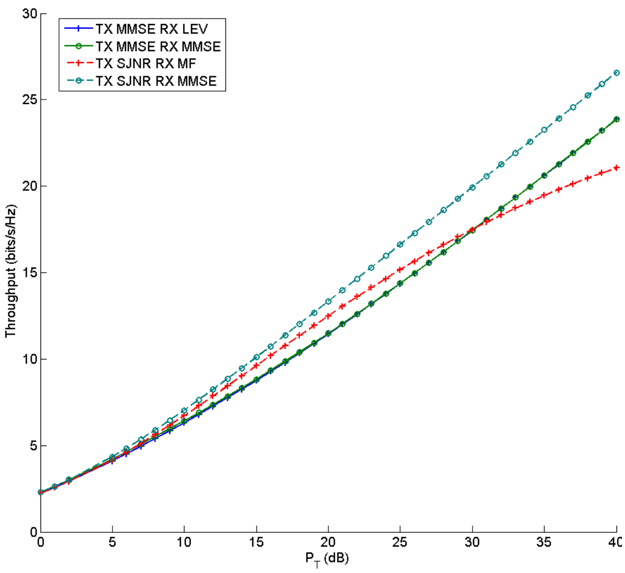

Fig. 4. Throughput for $N_T = 2$, $N_R = 2$ and $K = 2$.

Figure 4 illustrates the performance of the system for $N_T = 2$. The improvement introduced by the use of an MMSE receiver with a PU-MMSE precoder is low as it is the case in figure 3 and can only be perceived at low transmit power. On the other hand for the SJNR precoding, we note a very important gain by introducing the MMSE receiver. This gain is getting bigger at high transmit power.

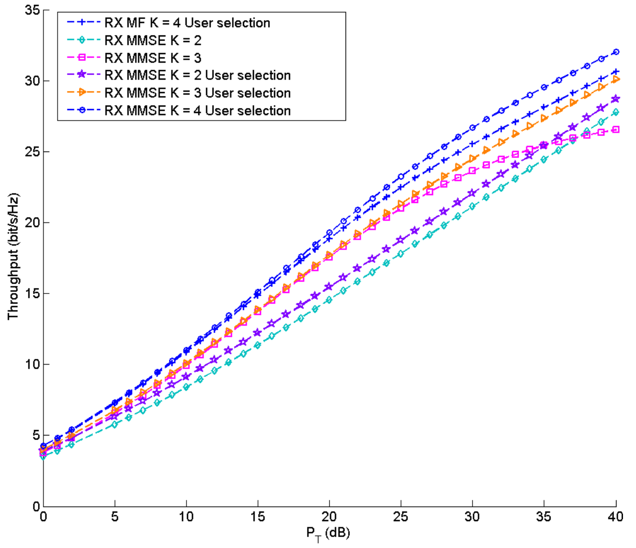

Fig. 5. Throughput with and without user selection optimization for SJNR precoder.

In figure 5, we consider the system after introduction of the user selection algorithm. The curves represented are for the SJNR precoder and for the case where $N_T = 4$ and $K = \{3, 4\}$. For comparison, we also report on the same figure the case $K = 2$ shown in figure 2 and $K = 3$ of figure 3. Comparing curves with and without user selection procedure for $K = 3$ we conclude that the selection procedure improves the system throughput.

Analyzing curves that applies the user selection procedure for $K = \{2, 3, 4\}$, shows that the throughput improves with an increasing number of users. This gain can be explained by the smart power allocation corresponding to shutting off the most interfering users.

The next figure, figure 6, represents a comparison of the PU-MMSE and the SJNR precoders for the same receiving structure that is an MMSE receiver. Simulation results demonstrate that in all simulated cases, the SJNR precoder outperforms the PU-MMSE. This observation has been verified to be true, as long as an MMSE receiver is used at the mobile side, for all simulated system parameters respecting the LTE standard [12] namely for $N_R \in \{1, 2, 4\}$ and $N_T \in \{1, 2, 4\}$. Moreover, the choice of the MMSE receiver has not only been motivated by the fact that it offers the best total throughput, but also because it was not possible to make a fair comparison between the considered precoding schemes with other types of receivers. Indeed, by using another type of receivers (MF or LEV), the two capacity curves of the SJNR and the PU-MMSE precoders intersect, i.e., no curve is always better than the other one for all transmit power region; this makes it impossible to draw conclusions.

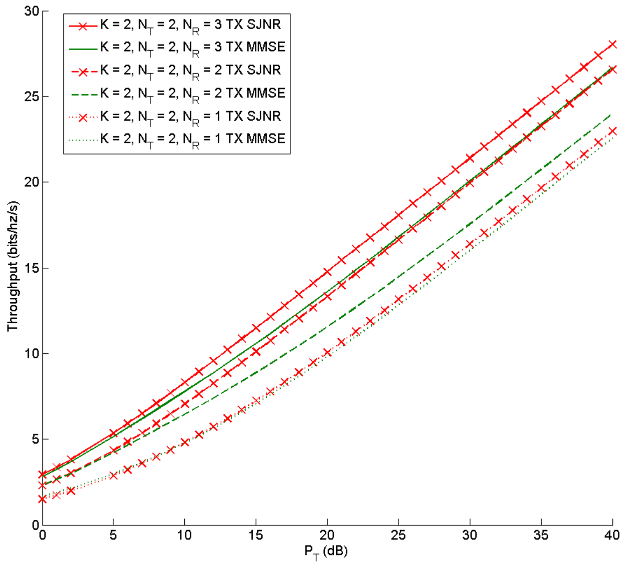

Fig. 6. Throughput for an MMSE receiver, $N_T = 2$, $N_R = \{1, 2, 3\}$ and $K = 2$.

V. Conclusion

In this paper, we put forward improvements on two closed form per-user linear transmitter precoding vectors design methods, namely PU-MMSE and SJNR. The paper describes the two proposed modifications which are the user power distribution on the selected users and the receiver structure optimization where an MMSE receiver is used.

These improvements have been validated by means of simulations. Important gains have been observed. This highlights the importance of the receiver structure optimization when multi-antennas are present at the receiver side. This principle can be generalized to all pre-coders as long as inter-user interference exists. This paper also points out that the SJNR precoder outperforms the PU-MMSE precoder when the optimum MMSE receiver is used.

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