

Distributed Power Control and Beamforming on MIMO Interference Channels

Zuleita K. M. Ho[†], Mariam Kaynia* and David Gesbert[†]

[†]EURECOM ,

2229 route des crêtes , BP 193 F-06560 Sophia-Antipolis cedex

Tel. : +33 (0)4 93 00 81 00 - Fax : +33 (0)4 93 00 82 00

Email: hokm@eurecom.fr, David.Gesbert@eurecom.fr

*Dept. of Electronics and Telecommunications, Norwegian University of Science and Technology,

O.S. Bragstads plass 2B, 7491 Trondheim, Norway

Phone: +47-73550398, Email: mariam.kaynia@iet.ntnu.no

Abstract— This paper considers the so-called Multiple-Input-Multiple-Output interference channel (MIMO-IC). We address the design of precoding (i.e. beamforming) vectors and power control at each data stream with the aim of striking a compromise between beamforming gain at the intended receiver (Egoism) and the mitigation of interference created towards other receivers (Altruism). Combining egoistic and altruistic beamforming has been shown previously to be instrumental to optimizing the rates in a Multiple-Input-Single-Output (MISO) interference channel [1], [2] and MIMO-IC [3], [4]. Here we extend these concepts to multi-stream scenarios and further improve the rate performance by allowing power control which is not addressed in previous interference alignment related works. The key idea behind power control in interference coordination schemes is that it can help restore feasibility conditions in the high SNR regime, thus avoiding a saturation of the sum rate. Our analysis and simulations attest improvement in terms of complexity and performance.

I. INTRODUCTION

In point-to-point wireless networks, such as multi-cell MIMO systems and cognitive radios, interference coordination is of utmost importance: with the lack of interference coordination, excess system interference saturates the sum rate as the SNR increases. On the other hand, if interference is mitigated completely, the sum rate scales indefinitely with SNR and the maximum degree of freedom can be achieved.

The linear combination of the egoistic (Maximum Ratio Transmission, MRT) and altruistic (Zero-forcing, ZF) beamformers is proved to be pareto optimal in the 2-user MISO IC [1], [2]. In [3], [4], we extend this idea to MIMO IC. The proposed egoism and altruism balancing beamforming design algorithm achieves sum rate close to rate optimization schemes [5], [6] and outperform IA techniques [7]–[9] in asymmetric networks (when some receivers suffer from out of coordination group interference.)

In rate optimization and IA works without power control, the sum rate performance saturates in the high SNR regime when IA is infeasible [8], [10]. To obtain sum rate that scales indefinitely with the SNR, the system must somehow be brought back to a scenario where IA is again feasible. This can be done by shutting down a subset of transmission streams in order to allow a perfect interference removal at the receiver

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side in the large SNR regime. In the finite SNR regime, we point out that it is not optimal to make IA fully feasible, as shutting down fewer links (than strictly necessary for IA) may be better in terms of sum rate. Binary power control can be seen as a low-complexity version for the continuous power control presented in previous MIMO interference channel contributions, such as [6]. This concept remains particularly useful in increasing the sum rate, compared with traditional IA techniques, which do not account for power control. Binary power control is shown to be sum rate optimal in the 2-user IC and close to optimal in multi-user IC [11].

In this paper, our contributions are as follows:

- We extend the game-theoretic egoistic and altruistic beamforming methods to multi stream MIMO-IC. We derive analytically the equilibria for so-called egoistic and altruistic bayesian games [12] where players (data streams) do not have access to complete channel state information (CSI), which is the situation in distributed precoding.
- With binary power control, we show that our algorithm scales indefinitely when SNR grows in the IA infeasible region, which is not addressed in the recent interesting iterative IA based methods such as alternated subspace optimization and iterative maximum SINR precoding [7]–[9].
- At finite SNR, we show improvements in terms of sum rate, especially in the case of asymmetric networks where IA methods are unable to properly weigh the contributions on the different interfering links to the sum rate.

A. Notations

The lower case bold face letter represents a vector whereas the upper case bold face letter represents a matrix. $(.)^H$ represents the complex conjugate transpose. \mathbf{I} is the identity matrix. $V^{(max)}(\mathbf{A})$ (resp. $V^{(min)}(\mathbf{A})$) is the eigenvector corresponding to the largest (resp. smallest) eigenvalue of \mathbf{A} . \mathcal{E}_B is the expectation operator over the statistics of the random variable B . $\mathbb{S} \setminus \mathbb{B}$ define a set of elements in \mathbb{S} excluding the elements in \mathbb{B} . $div(l, m)$ and $mod(l, m)$ give the quotient and remainder of the division of l by m . \mathbb{C} denotes the set of all complex numbers.

II. SYSTEM MODEL

Let \mathbb{N} and \mathbb{N}_c be the set of all transmitter-receiver (Tx-Rx) pairs and the set of coordinating pairs, $\mathbb{N}_c \in \mathbb{N}$. Denote the cardinality of \mathbb{N}_c and \mathbb{N} be N_c and N respectively. The Tx and Rx are randomly distributed in a restricted area. Each Tx and Rx has N_t and N_r antennas respectively. The channel from Tx i to Rx j $\mathbf{H}_{ji} \in \mathcal{C}^{N_r \times N_t}$ is given by:

$$\mathbf{H}_{ji} = \sqrt{\alpha_{ji}} \bar{\mathbf{H}}_{ji} \quad (1)$$

Each element in the channel matrix $\bar{\mathbf{H}}_{ji}$ is an independent identically distributed complex Gaussian random variable with zero mean and unit variance and α_{ji} denotes the slow-varying shadowing and pathloss attenuation.

For simplicity, each Tx is assumed to transmit the same number of data streams, $N_s \leq \min(N_t, N_r)$. The transmit beamforming matrix of Tx i is $\mathbf{W}_i \in \mathcal{C}^{N_t \times N_s}$ with column vectors $\mathbf{w}_{i1}, \dots, \mathbf{w}_{iN_s}$ and the receive beamforming matrix of Rx i is $\mathbf{V}_i \in \mathcal{C}^{N_r \times N_s}$ with column vectors $\mathbf{v}_{i1}, \dots, \mathbf{v}_{iN_s}$. As in several important contributions dealing with coordination on the interference channels, we assume linear precoding (beamforming) [1], [8], [13]–[16]. With the noise variance σ_i^2 at Rx i , the received signal-to-interference-and-noise ratio (SINR) of data stream k of Rx i is

$$\gamma_{ik} = \frac{|\mathbf{v}_{ik}^H \mathbf{H}_{ii} \mathbf{w}_{ik}|^2 P_{ik}}{\mathcal{I}_{ik} + \sigma_i^2}, \quad (2)$$

where P_{ik} is the transmit power of stream k of Tx i and \mathcal{I}_{ik} is the received interference power of data stream k of Rx i

$$\mathcal{I}_{ik} = \sum_{(j,m) \in \mathbb{I}_{ik}} |\mathbf{v}_{ik}^H \mathbf{H}_{ij} \mathbf{w}_{jm}|^2 P_{jm}, \quad (3)$$

with \mathbb{I}_{ik} being the set of streams that would interfere stream k of Tx i .

$$\mathbb{I}_{ik} = \{(j,m) | (j \neq i, m = 1, \dots, N_s)\} \cup \{(j,m) | j = i, m \neq k\} \quad (4)$$

The power constraint at each Tx i is

$$\sum_{k=1}^{N_s} P_{ik} \leq P. \quad (5)$$

A. Receiver design

The Rxs are assumed to employ maximum SINR (Max-SINR) beamforming throughout the paper so as to also maximize their rates [17]. The receive beamformer of stream k of Rx i is classically given by:

$$\mathbf{v}_{ik} = \frac{C_{Rik}^{-1} \mathbf{H}_{ii} \mathbf{w}_{ik}}{|C_{Rik}^{-1} \mathbf{H}_{ii} \mathbf{w}_{ik}|}, \quad (6)$$

where C_{Rik} is the covariance matrix of received interference and noise of stream k at Rx i :

$$C_{Rik} = \sum_{(j,m) \in \mathbb{I}_{ik}} \mathbf{H}_{ij} \mathbf{w}_{jm} \mathbf{w}_{jm}^H \mathbf{H}_{ij}^H P_{jm} + \sigma_i^2 \mathbf{I}. \quad (7)$$

Importantly, the noise will in practice capture thermal noise effects but also any interference originating from the rest of the network, i.e. coming from Txs located beyond the coordination cluster. Thus, depending on path loss and shadowing effects,

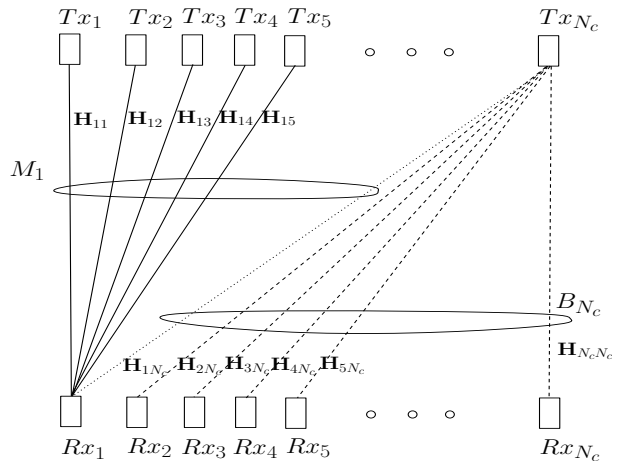


Fig. 1. Limited channel knowledge model for an example of transmitter, here Tx N_c , indicated by dotted lines, and an example of receiver, here Rx 1, indicated by solid lines.

the $\{\sigma_i^2\}$ may be quite different from each other [18]. We assume the out of cluster interference to be white due to the large number of Tx in the network and relatively small cluster size.

B. Limited Channel knowledge

To allow for overhead reduction and a better scalability of multi-cell coordination techniques when the number of coordinated links N_c is large, we seek solutions which can operate based on limited, preferably local, CSI. Although there may exist various ranges and definitions of local CSI, we assume the devices (Tx and Rx alike) are able to gain direct knowledge of those channel coefficients directly connected to them, as illustrated in Fig. 1.

The set of CSI locally available (resp. not available) at Tx i by \mathbb{B}_i (resp. \mathbb{B}_i^\perp) is defined by:

$$\mathbb{B}_i = \{\mathbf{H}_{ji}\}_{j=1, \dots, N_c}; \quad \mathbb{B}_i^\perp = \{\mathbf{H}_{kl}\}_{k,l=1 \dots N_c} \setminus \mathbb{B}_i. \quad (8)$$

Similarly, define the set of channels known (resp. unknown) at Rx i by \mathbb{M}_i (resp. \mathbb{M}_i^\perp) by:

$$\mathbb{M}_i = \{\mathbf{H}_{ij}\}_{j=1, \dots, N_c}; \quad \mathbb{M}_i^\perp = \{\mathbf{H}_{kl}\}_{k,l=1 \dots N_c} \setminus \mathbb{M}_i. \quad (9)$$

Additional receiver feedback: Because local CSI is insufficient to exploit all the degrees of freedom of the MIMO-IC [8], some additional limited feedback will be considered where indicated, in the form of feedback of the beamforming matrices \mathbf{v}_i used at the receiver. In reciprocal channels, the feedback requirement can be replaced by a channel estimation step based on uplink pilot sequences.

III. BAYESIAN GAMES ON INTERFERENCE CHANNEL

Bayesian games are a class of games in which players must optimize their strategy based on *incomplete state information* [12], and are hence particularly well-suited for distributed optimization problems. The definition of the bayesian game follows closely with [3], [4] and is summarized briefly in table I.

player l	data stream k of Tx i where $i = \text{div}(l, N_s) + 1, k = \text{mod}(l, N_s)$ $l \in \mathcal{M} = \{1, \dots, N_c \times N_s\}$
action set	$\mathbb{A} = \{\mathbf{w} \in \mathbb{C}^{N_c \times 1} : \mathbf{w} ^2 = 1\}$
strategy	beamforming vector of stream k of Tx i , $\mathbf{w}_{ik} \in \mathbb{A}, i = \text{div}(l, N_s) + 1, k = \text{mod}(l, N_s)$
utility function	$u : \mathbb{A}^{N_c N_s} \rightarrow \mathbb{R}$
egoistic :	received SINR of stream l (12)
altruistic :	negative interference caused by stream l (15)
Belief	unknown channel statistics \mathbb{B}_i^\perp as gaussian distribution
common knowledge	utility function and channel statistics

TABLE I
SUMMARY OF BAYESIAN GAME

A strategy of player l , here refers to beamforming design, \mathbf{w}_{ik} (see table I), is a deterministic choice of action given information \mathbb{B}_i . A strategy profile $\mathbf{W}^* = (\mathbf{w}_{ik}^*, \mathbf{w}_{-ik}^*)$ achieves the Bayesian Equilibrium if \mathbf{w}_{ik}^* is the best response of player l given strategies \mathbf{w}_{-ik}^* for all other players. The optimal transmit beamformer of player l , \mathbf{w}_{ik}^* , is characterized by the argument maximization of the expectation of the utility function $u(\cdot)$:

$$\mathbf{w}_{ik}^* = \arg \max_{\mathbb{B}_i^\perp} \mathcal{E}_{\mathbb{B}_i^\perp} \{u(\mathbf{w}_{ik}, \mathbf{w}_{-ik}^*, \mathbb{B}_i, \mathbb{B}_i^\perp)\}. \quad (10)$$

We can formulate the bayesian game as

$$G^B = [\mathcal{M}, \mathbb{A}, \mathbb{B}_i^\perp, \{u\}]. \quad (11)$$

Note that, intuitively, the player's strategy is optimized by averaging over the distribution of all missing CSI. The utility function $u(\cdot)$ as well as the statistics of the channels are assumed to be common knowledge.

In the following sections, we derive the equilibria for so-called *egoistic* and *altruistic* bayesian games respectively. These equilibria contribute extreme strategies which do not perform optimally in terms of the overall network performance, yet can be exploited as components of more general beamforming-based coordination techniques.

IV. BAYESIAN GAMES WITH RECEIVER BEAMFORMER FEEDBACK

We assume that each Tx has the local CSI and the added knowledge of receive beamformers through a feedback channel. Under these assumptions, we analyze the Egoistic and Altruistic beamforming solutions.

A. Egoistic Bayesian Game

Given receive beamformers as a common knowledge, the best response strategy of stream k of Tx i which maximizes the utility function, i.e. its own SINR,

$$u(\mathbf{w}_{ik}, \mathbf{w}_{-ik}, \mathbb{B}_i, \mathbb{B}_i^\perp) = \frac{|\mathbf{v}_{ik}^H \mathbf{H}_{ii} \mathbf{w}_{ik}|^2 P_{ik}}{\mathcal{I}_{ik} + \sigma_i^2}, \quad (12)$$

is the following:

Theorem 1: The best-response strategy of stream k of Tx i in the egoistic Bayesian game is

$$\mathbf{w}_{ik}^{Ego} = V^{(max)}(\mathbf{E}_{ik}) \quad (13)$$

where \mathbf{E}_{ik} will denote the *egoistic equilibrium matrix* for stream k of Tx i , given by

$$\mathbf{E}_{ik} = \mathbf{H}_{ii}^H \mathbf{v}_{ik} \mathbf{v}_{ik}^H \mathbf{H}_{ii}$$

and the corresponding Rx is given by $\mathbf{v}_{ik} = \frac{\mathbf{C}_{Rik}^{-1} \mathbf{H}_{ii} \mathbf{w}_{ik}^{Ego}}{|\mathbf{C}_{Rik}^{-1} \mathbf{H}_{ii} \mathbf{w}_{ik}^{Ego}|}$

Proof: The knowledge of receive beamformers decorrelates the maximization problem. The maximization problem can be written as

$$\mathbf{w}_{ik}^{Ego} = \arg \max_{|\mathbf{w}_{ik}| \leq 1} \mathcal{E}_{\mathbb{B}_i^\perp} \left\{ \frac{P_{ik}}{\mathcal{I}_{ik} + \sigma_i^2} \right\} \mathbf{w}_{ik}^H \mathbf{E}_{ik} \mathbf{w}_{ik}. \quad (14)$$

The egoistic-optimal transmit beamformer is the dominant eigenvector $\mathbf{w}_{ik}^{Ego} = V^{(max)}(\mathbf{E}_{ik})$. ■

B. Altruistic Bayesian Game

The altruistic utility of stream k at Tx i is defined here in the sense of minimizing the expectation of the sum of interference power towards other streams.

$$u(\mathbf{w}_{ik}, \mathbf{w}_{-ik}, \mathbb{B}_i, \mathbb{B}_i^\perp) = - \sum_{(j,m) \in \mathbb{I}_{ik}} |\mathbf{v}_{jm}^H \mathbf{H}_{ji} \mathbf{w}_{ik}|^2 \quad (15)$$

Theorem 2: The best-response strategy of stream k of Tx i in the altruistic Bayesian game is given by:

$$\mathbf{w}_{ik}^{Alt} = V^{(min)} \left(\sum_{(j,m) \in \mathbb{I}_{ik}} \mathbf{A}_{jmik} \right) \quad (16)$$

where \mathbf{A}_{jmik} will denote the *altruistic equilibrium matrix* for stream k of Tx i towards stream m of Rx j , defined by $\mathbf{A}_{jmik} = \mathbf{H}_{ji}^H \mathbf{v}_{jm} \mathbf{v}_{jm}^H \mathbf{H}_{ji}$. The corresponding receiver is $\mathbf{v}_{ik} = \frac{\mathbf{C}_{Rik}^{-1} \mathbf{H}_{ii} \mathbf{w}_{ik}}{|\mathbf{C}_{Rik}^{-1} \mathbf{H}_{ii} \mathbf{w}_{ik}|}$.

Proof: The altruistic utility can be rewritten as $-\mathbf{w}_{ik}^H (\sum_{(j,m) \in \mathbb{I}_{ik}} \mathbf{A}_{jmik}) \mathbf{w}_{ik}$. Since the \mathbf{v}_{jm} are known from feedback, the optimal \mathbf{w}_{ik} is the least dominant eigenvector of the matrix $\sum_{(j,m) \in \mathbb{I}_{ik}} \mathbf{A}_{jmik}$. ■

V. SUM RATE MAXIMIZATION WITH RECEIVE BEAMFORMER FEEDBACK

From the results above, it can be seen that balancing altruism and egoism can be done by trading-off between the dominant eigenvectors of the egoistic equilibrium \mathbf{E}_{ik} and negative altruistic equilibrium $\{\mathbf{A}_{jm}\}$ ($(j,m) \in \mathbb{I}_{ik}$) matrices. Interestingly, it can be shown that sum rate maximizing precoding for the MIMO-IC does exactly that. Thus we hereby briefly re-visit rate-maximization approaches such as [6] with this perspective.

Denote the sum rate by $\bar{R} = \sum_{i=1}^{N_c} \sum_{k=1}^{N_s} R_{ik}$ where $R_{ik} = \log_2 \left(1 + \frac{|\mathbf{v}_{ik}^H \mathbf{H}_{ii} \mathbf{w}_{ik}|^2 P_{ik}}{\mathcal{I}_{ik} + \sigma_i^2} \right)$ where \mathcal{I}_{ik} is the received interference of stream k of Rx i given in (3).

Lemma 1: The transmit beamforming vector which maximizes the sum rate \bar{R} is given by the following dominant eigenvector problem,

$$\left(\mathbf{E}_{ik} + \sum_{(j,m) \in \mathbb{I}_{ik}} \lambda_{jmik}^{opt} \mathbf{A}_{jmik} \right) \mathbf{w}_{ik} = \mu_{max} \mathbf{w}_{ik} \quad (17)$$

where real values $\lambda_{j\text{mik}}^{\text{opt}}$, μ_{max} are defined in the proof.

Proof: see appendix X-A ■

Note that the balancing between altruism and egoism in sum rate maximization is done using a simple *linear combination* of the altruistic and egoistic equilibrium matrices. The balancing parameters, $\{\lambda_{j\text{mik}}^{\text{opt}}\}$, coincide with the pricing parameters invoked in the iterative algorithm proposed in [6]. Clearly, these parameters plays a key role. However, their computation is a function of the *global* channel state information. Instead we seek a suboptimal egoism-altruism balancing technique which only requires statistical channel information, while exhibiting the right performance scaling.

A. Egoism-altruism balancing algorithm: DBA

We are proposing the following distributed beamforming algorithm with receiver feedback (DBA), to compute the transmit beamformers

$$\mathbf{w}_{ik} = V^{\text{max}} \left(\mathbf{E}_{ik} + \sum_{(j,m) \in \mathbb{I}_{ik}} \lambda_{j\text{mik}} \mathbf{A}_{j\text{mik}} \right). \quad (18)$$

DBA iterates between transmit and receive beamformers in a way similar to recent interference-alignment based methods such as e.g. [8], [9]. However here, interference alignment is *not* a design criterion. In [8], an improved interference alignment technique based on alternately maximizing the SINR at both sides is proposed. In contrast, the Max-SINR criterion is only used at the receiver side. This distinction is important as it dramatically changes performance in certain situations (see Section VII).

One important aspect of the algorithm above is whether it fully exploits the degree of freedom of the interference channel as shown in [8], i.e. whether it achieves the so-called interference alignment in the high SNR regime. The following theorem answers this question positively.

Definition 1: Interference is aligned when the following equations are satisfied simultaneously [8]:

$$\mathbf{v}_{ik}^H \mathbf{H}_{ij} \mathbf{w}_{jm} = 0 \quad \forall i, j, (j, m) \in \mathbb{I}_{ik} \quad (19)$$

Definition 2: Define the set of beamforming vectors solutions in downlink (respectively uplink) interference alignment to be [8]

$$\begin{aligned} IA^{DL} = \{ & (\mathbf{w}_{11}, \dots, \mathbf{w}_{N_c N_s}) : \\ & \left. \sum_{(j,m) \in \mathbb{I}_{ik}} \mathbf{H}_{ij} \mathbf{w}_{jm} \mathbf{w}_{jm}^H \mathbf{H}_{ij}^H \text{ is low rank, } \forall i, k \right\} \\ IA^{UL} = \{ & (\mathbf{v}_{11}, \dots, \mathbf{v}_{N_c N_s}) : \\ & \left. \sum_{(j,m) \in \mathbb{I}_{ik}} \mathbf{H}_{ji}^H \mathbf{v}_{jm} \mathbf{v}_{jm}^H \mathbf{H}_{ji} \text{ is low rank, } \forall i, k \right\}. \end{aligned} \quad (20)$$

Thus, for all $(\mathbf{w}_i, \dots, \mathbf{w}_{N_c}) \in IA^{DL}$, there exist receive beamformers $\mathbf{v}_i, i = 1, \dots, N_c$ such that (19) is satisfied.

Note that the uplink alignment solutions are defined for a virtual uplink having the same frequency and only appear here as technical concept helping with the proof.

Theorem 3: Assume the downlink interference alignment set is non empty (IA is feasible). Denote average SNR of

stream k of link i by $\bar{\gamma}_i = \frac{P \alpha_{ii}}{N_s \sigma_i^2}$. Let $\lambda_{j\text{mik}} = -\frac{1+\bar{\gamma}_i^{-1}}{1+\bar{\gamma}_j^{-1}} \bar{\gamma}_j$, then in the large SNR regime, $P \rightarrow \infty$, any transmit beamforming vector in IA^{DL} is a convergence (stable) point of DBA.

Proof: The proof is an extension and similar to [3], [4] and is included here for completeness.

We provide here a sketch of the proof. For full details, please refer to [3]. To prove that IA is a convergence point of DBA, we would prove that once DBA achieves interference alignment, DBA will not deviate from the solution.

Assumed interference alignment is reached and let $(\mathbf{w}_{11}^A, \dots, \mathbf{w}_{N_c N_c}^A) \in IA^{DL}$ and $(\mathbf{v}_{11}^A, \dots, \mathbf{v}_{N_c N_c}^A) \in IA^{UL}$. Let $\mathbf{Q}_{ik}^{DL} = \sum_{(j,m) \in \mathbb{I}_{ik}} \mathbf{H}_{ij} \mathbf{w}_{jm}^A \mathbf{w}_{jm}^{A,H} \mathbf{H}_{ij}^H$ and $\mathbf{Q}_{ik}^{UL} = \sum_{(j,m) \in \mathbb{I}_{ik}} \mathbf{H}_{ji}^H \mathbf{v}_{jm}^A \mathbf{v}_{jm}^{A,H} \mathbf{H}_{ji}$.

At the TxS: In high SNR regime, $\lambda_{j\text{mik}}$ becomes negative infinity and DBA gives $\mathbf{w}_{ik} = V^{\text{min}}(\mathbf{Q}_{ik}^{UL})$ (18). By (20), \mathbf{Q}_{ik}^{UL} is low rank and thus \mathbf{w}_{ik} is in the null space of \mathbf{Q}_{ik}^{UL} . In direct consequences, the conditions of IA (19) are satisfied. Thus, $(\mathbf{w}_{11}, \dots, \mathbf{w}_{N_c N_c}) \in IA^{DL}$.

At the RxS: The receive beamformer is defined as $\mathbf{v}_{ik} = \arg \max_{\mathbf{v}_{ik}} \frac{\mathbf{v}_{ik}^H \mathbf{H}_{ij} \mathbf{w}_{ik} \mathbf{w}_{ik}^H \mathbf{H}_{ij}^H \mathbf{v}_{ik}}{\mathbf{v}_{ik}^H \mathbf{Q}_{ik}^{DL} \mathbf{v}_{ik}}$. Since \mathbf{Q}_{ik}^{DL} is low rank, the optimal \mathbf{v}_{ik} would make the denominator zero and thus, \mathbf{v}_{ik} is in the null space of \mathbf{Q}_{ik}^{DL} . Hence, $\mathbf{v}_{ik} \in IA^{UL}$. Since both \mathbf{w}_{ik} and \mathbf{v}_{ik} stay within IA^{DL} and IA^{UL} , IA is a convergence point of DBA in high SNR. ■

VI. BINARY POWER CONTROL

In the high SNR regime, the residual interference saturates the sum rate performance when IA is infeasible. To scale the sum rate indefinitely in the IA infeasible region, binary power control is required to restore the feasibility of IA. Note that binary power control is shown to be sum rate optimal in 2 cells scenario and near-optimal in multi cell scenario [11]. In our scenario, a subset of the transmit streams are shut down in order to allow for a perfect interference removal at the receive side in the large SNR regime.

In order to obtain equations which are amenable to a simple power control scheme, we advocate a design guideline by which the residual interference at each Rx should be made on the same order of magnitude as the thermal noise (as opposed to making it zero, as the cost of degrees of freedom on the optimization of the beamforming coefficients). To check whether at least one stream should be turned off, we can easily check by comparing the received interference power to noise. Thus, according to our designing rule, the stream k of user i will be shut down when

$$P_{ik} = 0 \text{ if } \mathcal{I}_{ik} > \sigma_i^2 \text{ and } \gamma_{ik} < \gamma_{jm}, \forall (j, m) \in \mathbb{I}_{ik}. \quad (21)$$

To fulfill the transmit power constraint, equal power is allocated to the remaining streams at each Tx.

The proposed beamforming and power control algorithm can be summarized as follows:

- 1) Initialization: For each user $i \in \mathbb{N}_c$, initialize transmit power for each stream $k = 1 \dots N_s$ with equal power allocation $P_{ik} = \frac{P}{N_s}$. Initialize transmit beamformer \mathbf{w}_{ik} to a predefined vector and the receive beamformer \mathbf{v}_{ik} according to (6).

- 2) DBA: Start the iterative beamforming procedure using (18) and (6).
- 3) Power Control: When DBA converges, check power control criteria (21). If at least one stream is shut down, repeat DBA until power control criteria is satisfied.

A. Low Complexity of Binary Power Control

We include here briefly the pricing algorithm in [6] in the following, for details, please refer to section III B in [6].

- 1) Initialization of precoding matrices, interference prices, power profiles and receive filters.
- 2) Iteration: for each user,
 - a) optimize beamformers based on given interference prices and power profile.
 - b) optimize power profile by maximizing a non-convex surplus function .
 - c) recompute all interference prices and receive filters.
- 3) Repeat until convergence.

The Binary power optimization offers a complexity reduction advantage over a search over the continuous power domain proposed in [6].

B. Restoring IA feasibility in high SNR

Both [6] and DBA restore IA feasibility in high SNR regime. In high SNR or high interference regime, the individual rates become more sensitive towards the received interference. By definition, the prices are increased and force the transmit power of some Tx to decrease. In Fig. 5, we illustrate the sum rate performance of [6] with binary power allocation. As the sum rate scales indefinitely with SNR, the IA feasibility is restored. However, this binary power control in [6] can be affected by fast fading gains and thus in some channel realizations, some links remain transmitting even if the sum rate could be higher if they are shut down. Comparing to DBA, the binary power control criteria seems to be more effective and achieve a better sum rate in high SNR and IA unfeasibility region. (see later for details)

VII. SIMULATION RESULTS

In this section, we investigate the sum rate performance of DBA in comparison with several related methods, namely the *Max-SINR* method [8], the alternated-minimization (*Alt-Min*) method for interference alignment [9] and the sum rate optimization method (*SR-Max*) [6]. To ensure a fair comparison, all the algorithms in comparisons are initialized to the same solution and have the same stopping condition. We perform sum rate comparisons in asymmetric channels where links undergo different levels of out-of-cluster noise. Define the Signal to Interference ratio of link i to be $SIR_i = \frac{\alpha_{ii}}{\sum_{j \neq i} \alpha_{ij}}$. The SIR is assumed to be 1 for all links, unless otherwise stated. Denote the difference in SNR between two links in asymmetric channels by ΔSNR .

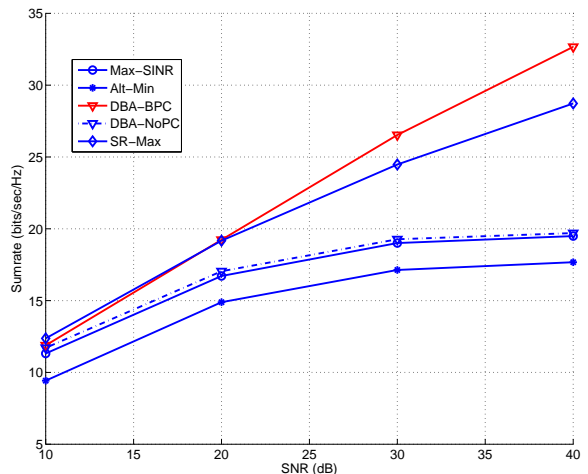


Fig. 2. Sum rate performance for asymmetric channel of 4 links system. One link suffers severe out-of-cluster noise. DBA with power control outperforms most algorithms.

A. Asymmetric channel with out-of-cluster noise

In Fig. 2, the sum rate performance is compared among schemes with and without binary power control on a 4 links system where each Tx and Rx is equipped with 2 antennas and each Tx sends 1 stream to its target Rx. The network is asymmetric in which Rx1 has additional out-of-cluster noise 10dB. In this scenario, IA is infeasible and turning off a suitable link, can restore the feasibility of IA and scale the sum rate indefinitely with SNR in high SNR regime. Note that DBA with binary power control outperforms *SR-Max* which has continuous power allocation in the high SNR regime. It is because the power control in *SR-Max* may be affected by fast fading channel coefficients and converge to a local optimal point.

In Fig. 3, we impose a more realistic settings in which the links suffer from different out-of-cluster noise. The noise of links are in the ratio 1 : 2.5 : 5 : 10. The remaining channel settings is the same as in Fig. 2. The sum rate performance of DBA is the highest among others in high SNR regime.

In Fig. 4, there are 3 links cooperating in the system. Each Tx and Rx has 2 antennas and has 1 stream transmission. Thus, IA is feasible. The noise at each Rx is the same. The system is asymmetric in a sense that the direct channel gain H_{11} of link 1 is 30dB weaker than other links in the network. This set up models a realistic environment where the user suffers strong shadowing. DBA achieves sum rate closed to *SR-Max* with continuous power allocation and much better than other IA based schemes *Max-SINR* and *Alt-Min*.

B. Symmetric channels

In Fig 5, the sum rate performance of *SR-Max* is compared with DBA in a IA unfeasibility region, namely a 4 links system with each Tx and Rx equipped with 2 antennas and 1 stream transmission. The system SNR is allowed to increase to a high value which is plotted as the x-axis. The link qualities in the

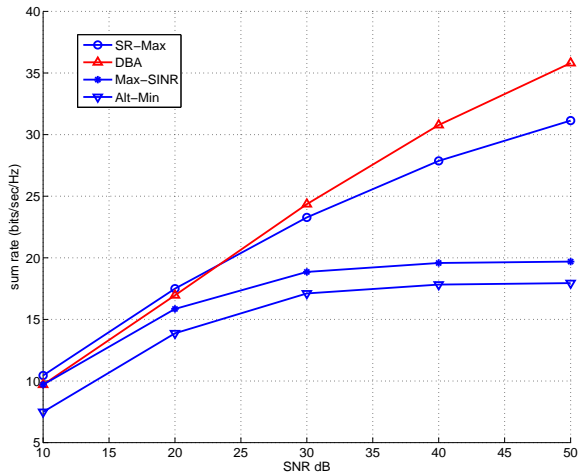


Fig. 3. Sum rate performance for asymmetric channel of 4 links system. Links suffer different out-of-cluster noise. *DBA* achieves a better scaling in high SNR regime.

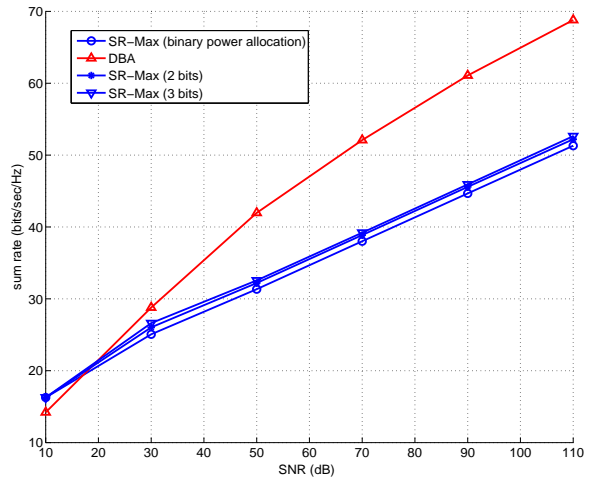


Fig. 5. Sum rate performance for symmetric channel of 6 links system. *DBA* achieves a higher sum rate than *SR-Max* with continuous power allocation in IA unfeasibility and high SNR regime.

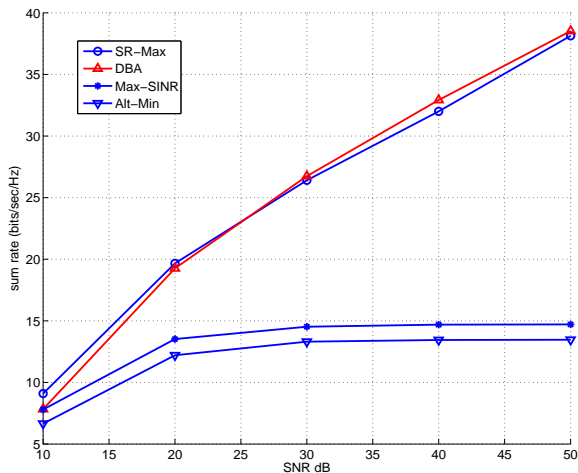


Fig. 4. Sum rate performance for asymmetric channel of 3 links system. The direct channel gain is 30dB weaker than other links. *DBA* achieves a better scaling in high SNR regime.

network are assumed to be equal, $\Delta SNR = 0$. To illustrate the design difference, we compare the performance of *SR-Max* with both continuous and binary power allocation. The continuous power allocation in [6] is a non-convex optimization. For implementation, the continuous power allocation is implemented as an exhaustive search over a quantized search space. We include here the performance of *SR-Max* with power control with 1 bit (binary), 2 bits and 3 bits quantization. As the system SNR increase, the sum rate becomes more sensitive towards the interference which increase the *price* in *SR-Max*. This forces some of the users to decrease their transmit power. However, the IA unfeasibility may not be fully restored in some channel realizations and may offer a lower performance compare with *DBA*.

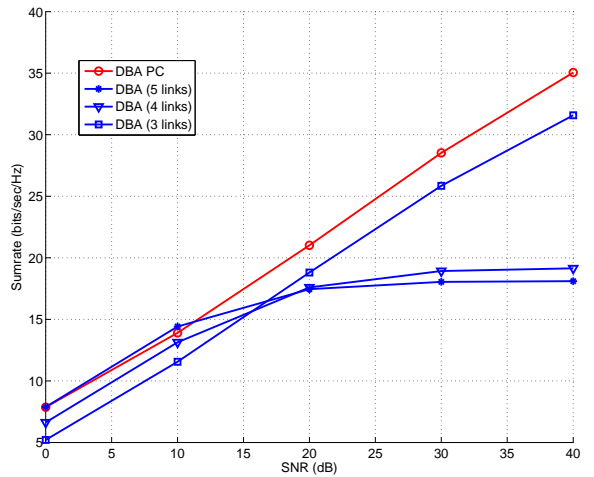


Fig. 6. Sum rate performance for asymmetric channel of 5 links system. *DBA* with power control improves the *DBA* without power control.

In Fig. 6, the sum rate performance of *DBA* is plotted with and without power control in a 5 links system with each Tx and Rx equipped with two antennas. As shown in the figure, the proposed scheme with power control improve the sum rate by turning off non-contributing links. As SNR grows, the scenario of IA feasibility has to be restored in order to have the maximum sum rate scaling. Depending on the system SNR, the proposed scheme adaptively turn off 1 or more non-contributing links and restore the sum rate scaling.

VIII. FUTURE WORK

In this paper, we have demonstrated that combining beamforming vectors and binary power control brings enormous gain to sum rate in IA unfeasibility and high SNR regime. Our future work is to investigate the optimal power control

in such regime. The difficulty of such work is the distributed CSI requirement.

IX. CONCLUSION

We derive the equilibria for the egoistic and altruistic bayesian games. We suggest a precoding technique based on balancing the egoistic and the altruistic behavior at each transmitter with the aim of maximizing the sum rate. Our simulations indicates good performance of *DBA*. It outperforms precious IA-based schemes that do not use power control in the unfeasibility region of alignment. Our method also achieves greater performance for the case of asymmetric channels thanks to a proper weighing of the contribution of each link towards the sum rate. The method also may outperform previous sum rate maximization schemes based on pricing although this difference is probably caused by the existence of local maxima in the sum rate objective function.

X. APPENDIX

A. Proof of Lemma 1

Define the Lagrangian of the sum rate maximization problem to be $\mathcal{L}(\mathbf{w}_{ik}, \mu) = \bar{R} - \mu(\mathbf{w}_{ik}^H \mathbf{w}_{ik} - 1)$. The necessary condition of Lagrangian $\frac{\partial}{\partial \mathbf{w}_{ik}^H} \mathcal{L}(\mathbf{w}_{ik}, \mu) = 0$ gives: $\frac{\partial}{\partial \mathbf{w}_{ik}^H} R_{ik} + \sum_{(j,m) \in \mathbb{I}_{ik}} \frac{\partial}{\partial \mathbf{w}_{ik}^H} R_{jm} = \mu \mathbf{w}_{ik}$. With elementary matrix calculus,

$$\begin{aligned} \frac{\partial}{\partial \mathbf{w}_{ik}^H} R_{ik} &= \frac{P_{ik}}{\mathcal{I}_{ik} + |\mathbf{v}_{ik}^H \mathbf{H}_{ii} \mathbf{w}_{ik}|^2 P_{ik} + \sigma_i^2} \mathbf{E}_{ik} \mathbf{w}_{ik} \\ \frac{\partial}{\partial \mathbf{w}_{ik}^H} R_{jm} &= -\frac{|\mathbf{v}_{jm}^H \mathbf{H}_{jj} \mathbf{w}_{jm}|^2 P_{jm}}{|\mathbf{v}_{jm}^H \mathbf{H}_{jj} \mathbf{w}_{jm}|^2 P_{jm} + \mathcal{I}_{jm} + \sigma_j^2} \\ &\quad - \frac{P_{ik}}{\mathcal{I}_{jm} + \sigma_j^2} \mathbf{A}_{jmik} \mathbf{w}_{ik}. \end{aligned}$$

Thus, λ_{jmik}^{opt} is a function of all channel states information and beamformer feedback.

$$\lambda_{jmik}^{opt} = -\frac{\mathcal{I}_{ik} + |\mathbf{v}_{ik}^H \mathbf{H}_{ii} \mathbf{w}_{ik}|^2 P_{ik} + \sigma_i^2}{|\mathbf{v}_{jm}^H \mathbf{H}_{jj} \mathbf{w}_{jm}|^2 P_{jm} + \mathcal{I}_{jm} + \sigma_j^2} \quad (22)$$

$$\frac{|\mathbf{v}_{jm}^H \mathbf{H}_{jj} \mathbf{w}_{jm}|^2 P_{jm}}{\mathcal{I}_{jm} + \sigma_j^2}.$$

And μ_{max} is therefore

$$\mu_{max} = \frac{\mathcal{I}_{ik} + |\mathbf{v}_{ik}^H \mathbf{H}_{ii} \mathbf{w}_{ik}|^2 P_{ik} + \sigma_i^2}{P_{ik}} \mu. \quad (23)$$

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